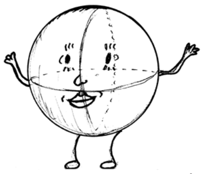


• Right Instruction Binder — Level 1 •

Trainees must complete the "New Hire" portion (sections 1-14) of the Right Instruction Binder before starting the "Initial Instructor Certification" (sections 15-21) part.

Instructions for:

Training Facilitators: Provide a copy of the Binder for each new trainee. The Right Instruction Binder is located in Business Documents. Additionally, a Facilitator's Guide and the Right Instruction Binder Answer Key are able for your reference in Business Documents.

Lead Instructors, Assistant Centre Directors and Centre Directors: The Right Instruction Binder is a great tool for anyone who wants to learn more about Mathnasium Methodologies. Your training facilitator will instruct you on how to use this binder for your training.

Franchisees/Centre Directors: All new hires must complete the Right Instruction Binder in its entirety within the first 6 weeks of employment. The "New Hire" part of the Binder must be completed before Instructors can work independently with students.

The Binder may also be used to refresh current employees on Mathnasium methodologies. In this case, you might strategically choose sections of Binder for the trainee to complete. The educational documents in the Binder are also a great resource for staff meetings and continued learning.

Printing:

Print double-sided. Use the table of contents to organise the contents into the divider sections. Each section has a unique footer like a PK. Use page 3 as the binder cover page. See Facilitator's Guide for more details.

Digital PDF:

The PDF has bookmarks for easy navigation of the document. Instructors will be prompted to complete six training videos throughout the binder.

Additional Resources (found in Business Documents):

- Right Instruction Binder Facilitator's Guide
- Right Instruction Binder Answer Key
- RIB Instructor Performance Evaluation
- Instructor Prescriptives

Note: If you are modifying the Right Instruction Binder remove the table of contents page from the trainee's binder.

Right Instruction Binder – Level 1

Learning Center Basics

Team Teaching

A First Look at Curriculum and Instruction

Numerical Fluency

Multiplication Fact Fluency

DeskTools and Manipulatives

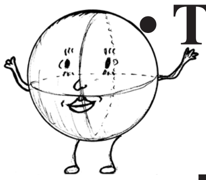
Instructions for Instructors:

Use this binder to lead you through your Mathnasium Level 1 training. The Binder will prompt you to complete training videos along the way. In between the videos there is Instructor Development Activities, Reflective and Critical Questions, Observations, Shadowing, Leading Instruction, Reflections and Progress Checks. The Progress Checks signal you to check in with your training facilitator.

The "New Hire" material must be completed before Instructors can work independently with students on the instructional floor.

The materials found in the first part of the binder support Instructor Level 1 – "New Hire" courses: **Learning Center Basics**, **Team Teaching** and the **First Look at Curriculum and Instruction**. Your facilitator will administer a practical exam upon completion of this material.

The materials found in the second part of the Binder support the Instructor Level 1 – Initial Instructor Certification courses: **Numerical Fluency**, **Multiplication Fact Fluency** and **Desk Tools and Manipulatives**. Your facilitator will administer a final practical exam upon completion of the entire binder.

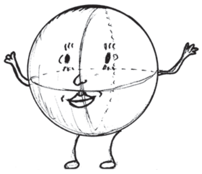


• Table of Contents – Right Instruction Binder •

"New Hire" (Sections 1–14)

"Initial Instructor Certification" (Sections 15–21)

The bold font denotes MU courses and Practical Exams.



Orientation

• Orientation •

The Centre Director will lead you through an orientation to the centre and your new role as an Instructor. Below is an overview of the topics to be discussed.

Objectives:

1. Instructors will be able to explain the scheduling and payroll processes.
2. Instructors will be able to summarize their interpretation of professional behaviour.
3. Instructors will be able to describe the responsibilities of a Mathnasium Instructor.
4. Instructors will know what the Instructor training path looks like.
5. Instructors will have a basic understanding of daily centre operations.

Employee Handbook Procedures:

- Scheduling– schedule process and instructions for requesting time off
- Payroll– checking-in, pay periods and frequency of checks
- Professional Behaviour– dress code, food and drink, mobile phones, personal conversations and contributing to the Centre's appearance and atmosphere

Instruction Job Description and Performance Evaluation:

- Instruction Job Description– role, responsibilities and tasks
- Instructor Performance Evaluation– criteria to be successful in the role

Instructor Training:

- Initial training– stages and time frame
- Ongoing training– staff meetings, etc.

Daily Centre Operations:

- Location of key areas
- Daily student process
- Centre rules and policies on homework; use of games and manipulatives
- Effective practices for keeping students safe

Learning Center Basics Course
&
Instructor Development

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Learning Center Basics Course

• Learning Center Basics •

Watch the Right Instruction - Teach the Mathnasium Way: Learning Center Basics training video. Then, read and complete the following pages:

The Student Binder and Learning Plan
The Mathnasium Hour
The Workout Plan
Rewards Cards

The Learning Center Basics course introduces the basic knowledge and syntax required to understand the Mathnasium Method of teaching and explains concepts in the Mathnasium context.

Objectives:

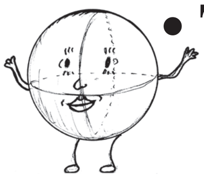
In this course, you will learn about:

1. The role of the Learning Centre Instructor.
2. Teaching the Mathnasium Way.
3. The Mathnasium Hour.
4. The Curriculum and Instructional Icons.
5. The Student Binder setup.
6. The Student Incentive Program.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the "Training Videos" section, click on "Right Instruction - Teach the Mathnasium Way: Learning Center Basics".

Notes:



• The Student Binder and Learning Plan •

The Student Binder is divided into 5 sections. Answer the following using the sample Student Binder provider by your facilitator.

The Student Binder

TAB 1	Written assessment, Learning Plan, any other progress-related information
TAB 2	3 Prescriptives – Black Mathnasium Divider – 3 Prescriptives Each PK is separated by a red sheet.
TAB 3	Workout Book Chapter
TAB 4	Completed Prescriptives are filed here upon completion of MC.
TAB 5	Completed Workout Book Chapters

Learning Plan

A student's Learning Plan (LP) is generated based on their assessment results. Each Prescriptive (PK) has a unique number that can be found on the bottom right corner of each page. These PK numbers match up with the PK listed on the student's LP.

Try these: **Use the sample Student Binder to answer the following questions.**

1) Looking at the student's Learning Plan, how many PKs are assigned? List the PK numbers. Do they match the PKs listed on the Learning Plan?

____

____

2) What is different about the Workout Book (WOB) Chapter in Tab 3 compared to the PKs in Tab 2?

____

____

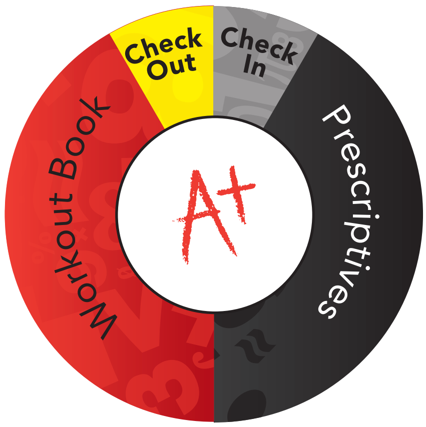
3) What is the purpose of the Black Mathnasium Divider in Tab 2?

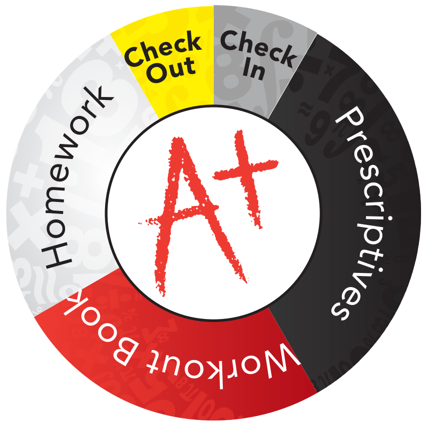
____

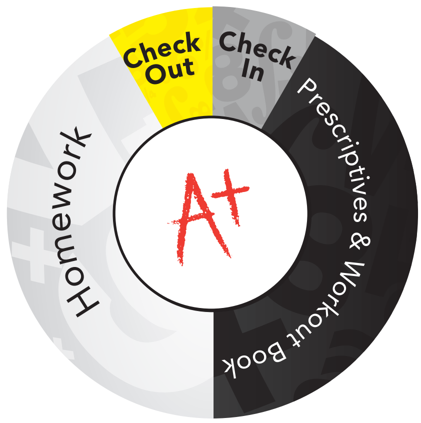
____

• Mathnasium Hour •

Students follow the Mathnasium Hour during their sessions to keep them on track.







***Try these*:**

1) How much time do students spend on Prescriptives and their Workout Book in the top version of the Mathnasium Hour?

____

____

2) How does the Mathnasium Hour differ in the bottom two versions compared to the top version of the Hour?

____

____

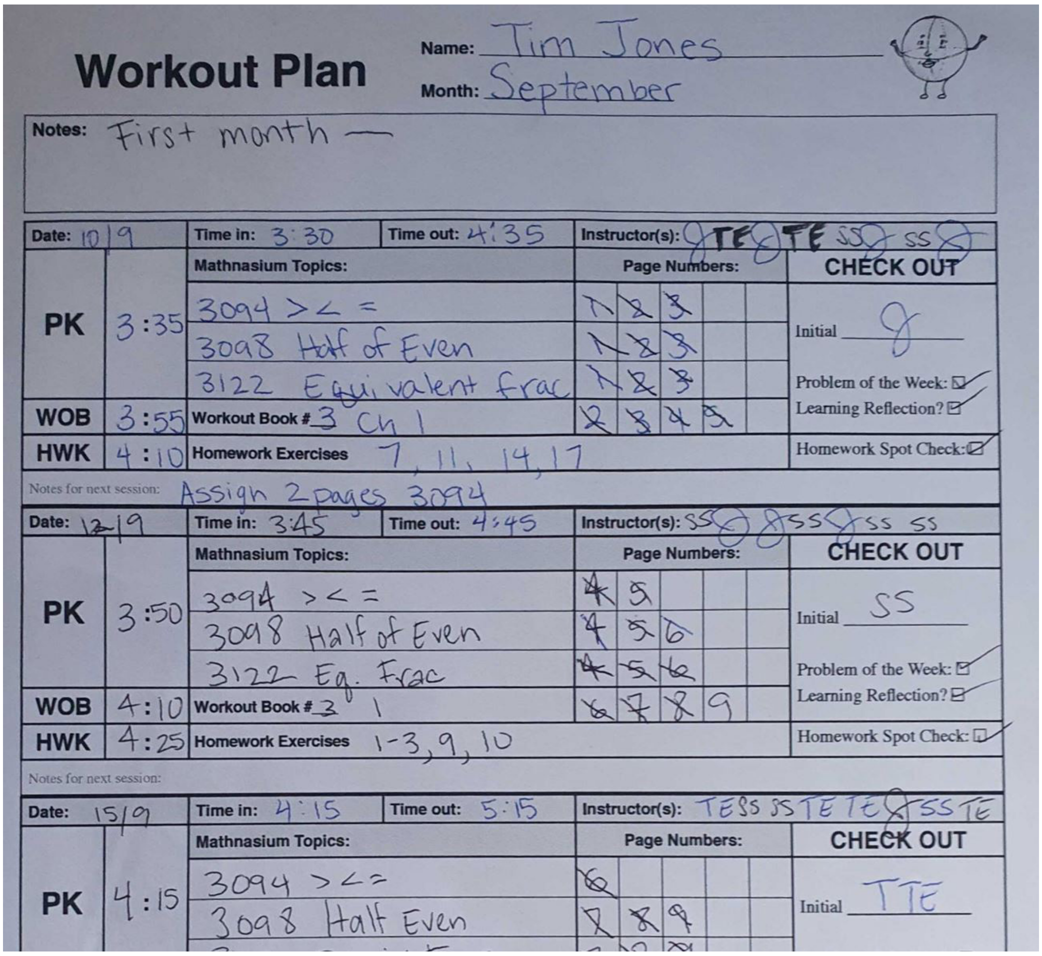
3) How long are students' sessions? Are secondary school students' sessions the same?

____

____

• Workout Plan •

Workout Plans are used to facilitate student progress, track sessions and communicate with other team members.



Workout Plan

Name: Tim Jones

Month: September

Notes: First month —

Date: 10/9		Time in: 3:30	Time out: 4:35	Instructor(s): TE TE SS SS					
PK	3:35	Mathnasium Topics:		Page Numbers:					CHECK OUT
		3094 > < =		1	2	3			Initial
		3098 Half of Even		1	2	3			
		3122 Equivalent frac		1	2	3			Problem of the Week: ☑
WOB	3:55	Workout Book # 3 Ch 1		2	3	4	5		Learning Reflection? ☑
HWK	4:10	Homework Exercises 7, 11, 14, 17							Homework Spot Check: ☑
Notes for next session: Assign 2 pages 3094									

Date: 12/9		Time in: 3:45	Time out: 4:45	Instructor(s): SS SS SS SS					
PK	3:50	Mathnasium Topics:		Page Numbers:					CHECK OUT
		3094 > < =		4	5				Initial SS
		3098 Half of Even		4	5	6			
		3122 Eq. Frac		4	5	6			Problem of the Week: ☑
WOB	4:10	Workout Book # 3 1		6	7	8	9		Learning Reflection? ☑
HWK	4:25	Homework Exercises 1-3, 9, 10							Homework Spot Check: ☑
Notes for next session:									

Date: 15/9		Time in: 4:15	Time out: 5:15	Instructor(s): TE SS SS TE TE SS TE					
PK	4:15	Mathnasium Topics:		Page Numbers:					CHECK OUT
		3094 > < =		6					Initial TTE
		3098 Half Even		7	8	9			

Try these: **Answer the following questions based on the Workout Plan above.**

1) How many pages did the student complete on 10/9? ______________________

2) How much time did the Instructor allow for homework on 12/9? ___________

3) How many times did the Instructors interact with the student on 10/9?

____

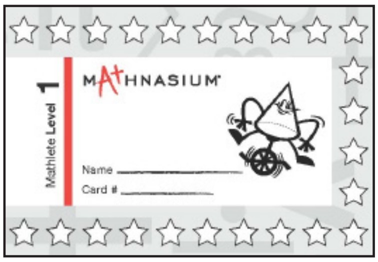
4) How do you know if the student's homework was spot-checked?

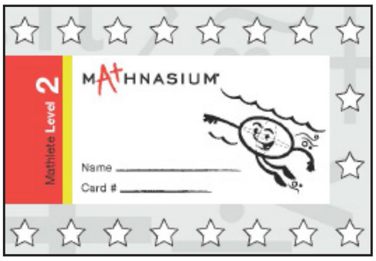
____

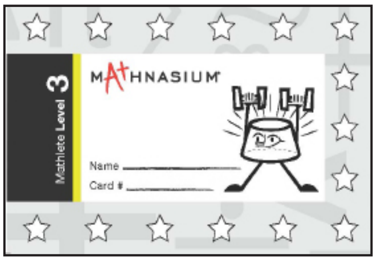
____

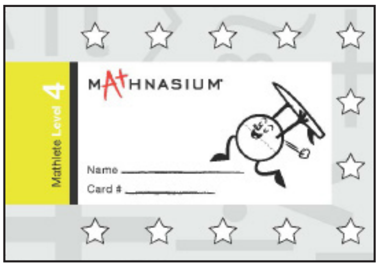
• Rewards Cards •

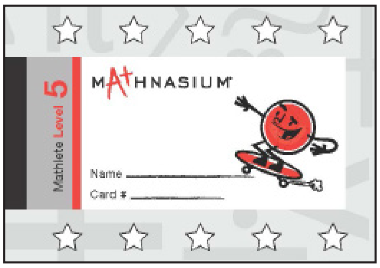
Your facilitator will discuss the Centre's Student Incentive Program during your Progress Check.











***Try these*:**

1) What happens as the Mathlete Levels increase?

____

____

2) How do students earns punches on their Rewards Cards?

____

____

3) How do students move up levels?

____

____

4) What is the process for trading in cards for items in the Rewards Cabinet?

____

____

Instructor Development

Number Sense Triangle

The Breakdown of the Prescriptive Activity

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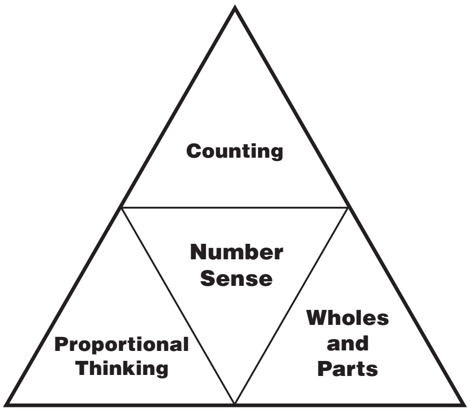
Number Sense Triangle

Identify the key concepts of the Mathnasium Curriculum

MATHNASIUM®

Key Concepts



Counting

[Counting *transforms* one number into another.]

Arithmetic is "the Art of Counting."

ari– (to join) + the + *metic* (measure)

Wholes & Parts

[whole = part + part + ... + part] ...

[part = whole – (part + part + ... + part)]

Wholes and *Parts* and the *relationship* between them, unite various aspect of mathematics.

Proportional Thinking

["The Ratio is Constant—the fractions are equal"]

Proportion means "according to amount."

pro– (according to) + *portion* (amount)

Quantity & Denomination

["how many" of "what" (by name–value)]

The **quantity** is the *amount* of something; the *number* of things.

The **denomination** is the *name–value* of that thing.

de– (according to) + *nominare* (name)

The Law of SAMEness

[apples + bananas don't make banapples]

We can only add and subtract things that have the *same name*, things that are of the *same denomination*.

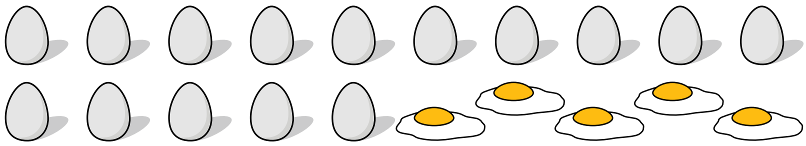
• Number Sense Triangle Examples •

These questions are examples of how the you will see the key concepts from the Number Triangle appear in the curriculum. Do NOT solve.

Counting

Each complete row has 10 eggs. Find the total number without counting them 1 by 1.

1) How many eggs have been cracked? ________________

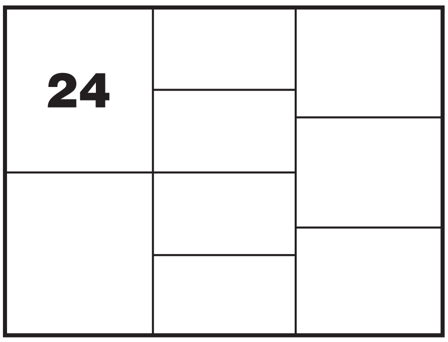


How many eggs are *not* cracked? ____

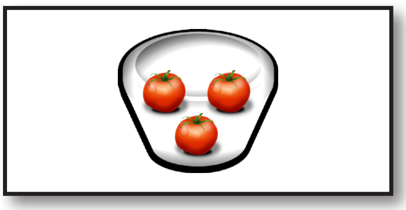
Wholes and Parts

How much is the whole box worth?

2) ___________



Proportional Thinking



It takes 3 tomatoes to make 1 glass of tomato juice.

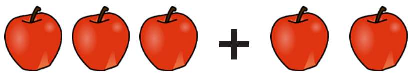
3) How many glasses of tomato juice can be made with 6 tomatoes?

____

4) How many tomatoes are needed to make 4 glasses of tomato juice?

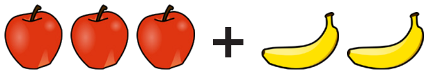
____

The Law of SAMEness



3 apples + 2 apples = 5 apples

3 bananas + 2 bananas = 5 bananas



3 apples + 2 bananas = 5____________ (*not* banapples!)

The Law of SAMEness

Quantity & Denomination

1) 1 apple + 2 apples = ____________

2) 2 hours + 3 hours = ____________

3) 1 half + 1 quarter = ____________

4) $3x^2 + 5x^2$ = ____________

5) $4x^3 + 6x^3$ = ____________

6) $2x^2 + 4x^3 + 3x^2 + x$ = ____________

Arrange the numbers in ascending order.

1) 1.25, 1.267, 1.2 ______________________

Breaking Down the Prescriptive

Recognise the structure of a Prescriptive

• The Breakdown of a Prescriptive •

Use the Prescriptive that starts on the following page to complete this activity. Spend a few minutes analysing the PK with the following questions in mind. Do NOT solve the exercises in the Prescriptive.

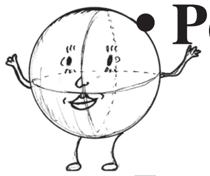
1) What is the general structure/form of the "review page?"

2) What are some things you see on the "instructional page?"

3) What is "scaffolding" and how does it appear on the "practice pages?"

4) What "Teaching Icons" do you see throughout this PK?

5) What is unique about the "Mastery Check" page?

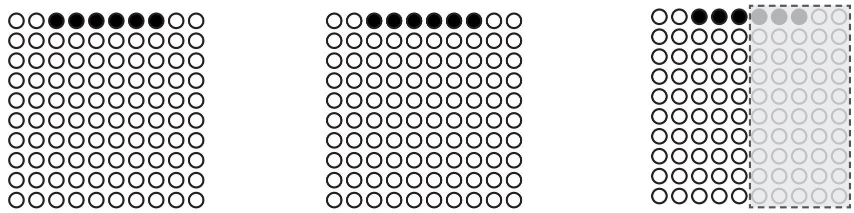


• Per Cent For Each 100 and Rounding *Review* •

Example: **Find 6% of 250.**

6% means "6 for each 100." **So count...**

6 for the *first* 100, **6** for the *second* 100 ***and*** **half of 6** for half of the *third* 100.



6 + 6 + 3 = 15

So **6% of 250 is 15**.

1) 8% of 200 = ______________ **2)** 8% of 50 = ______________

3) 16% of 150 = ______________ **4)** 10% of 250 = ______________

5) 6% of 350 = ______________ **6)** 6% of 150 = ______________

7) 20% of 250 = ______________ **8)** 12% of 300 = ______________

9) 3% of 550 = ______________ **10)** 11% of 450 = ______________

Round to the nearest multiple of 50.

11) 147 150 **12)** 503 500 **13)** 698 ________

14) 356 ________ **15)** 739 ________ **16)** 284 ________

17) 962 ________ **18)** 821 ________ **19)** 43 ________

20) 615 ________ **21)** 179 ________ **22)** 567 ________

• Estimating Percentages •

Mental Maths

When finding a percentage of a **NUMBER** that is not friendly for using **MENTAL MATHS**. it can still be helpful to use **MENTAL MATHS.** to find a *reasonable estimate* before finding the *actual answer*. To find the **ESTIMATE**, we can round the **NUMBER** to the ***nearest multiple of* 50** and then find the total amount ***for each* 100**.

EXAMPLE 1: 12% of 210 = ?

12 % of 200 = **24**

For Each 100 — 210 ROUNDED TO THE NEAREST 50 — ESTIMATE

12% of 210 is ***greater than*** / ***less than*** the **ESTIMATE.**

EXAMPLE 2: 6% of 145 = ?

6 % of 150 = **9**

For Each 100 — 145 ROUNDED TO THE NEAREST 50 — ESTIMATE

6% of 145 is ***greater than*** / ***less than*** the **ESTIMATE.**

Try these:

1) 15% of 387 = ?

____ % of ____ = $\boxed{}$

For Each 100 — 387 ROUNDED TO THE NEAREST 50 — ESTIMATE

15% of 387 is ***greater than / less than*** the **ESTIMATE.**

---

2) 7% of 266 = ?

____ % of ____ = $\boxed{}$

For Each 100 — 266 ROUNDED TO THE NEAREST 50 — ESTIMATE

7% of 266 is ***greater than / less than*** the **ESTIMATE.**

PK 3178 Direct_Percentages–2

• Estimating Percentages •

Mental Maths

99 + 99

In each exercise, find the ESTIMATE by rounding the NUMBER to the *nearest multiple of* 50 and then finding the total amount *for each* 100.

1) 8% of 432 = ?

8 % of 450 = ☐

For Each 100 — 432 rounded to the nearest 50 — Estimate

8% of 432 is ***greater than / less than*** the Estimate.

---

2) 11% of 589 = ?

____ % of ____ = ☐

For Each 100 — 589 rounded to the nearest 50 — Estimate

11% of 589 is ***greater than / less than*** the Estimate.

---

3) 14% of 330 = ?

____ % of ____ = ☐

For Each 100 — 330 rounded to the nearest 50 — Estimate

14% of 330 is ***greater than / less than*** the Estimate.

---

4) 9% of 62 = ?

____ % of ____ = ☐

For Each 100 — 62 rounded to the nearest 50 — Estimate

9% of 62 is ***greater than / less than*** the Estimate.

PK 3178 Direct_Percentages–3

• Converting Percentages to Decimals •

Direct Teaching

EXAMPLE: Convert 7% to a decimal.

Since 7% means 7 for each 100, 7 *out of* 100 or $\frac{7}{100}$...

We can *divide* 7 by 100 to convert 7% to a decimal.

The LONG WAY to Divide by 100	The SHORTCUT
$\begin{array}{r} .07 \\ 100\overline{)7.00} \\ -\underline{\quad 0} \\ 700 \\ -\underline{700} \\ 0 \end{array}$	Dividing by 100 moves the *decimal point* two places to the left. **7% → .07. → 0.07** 7% = **0.07**

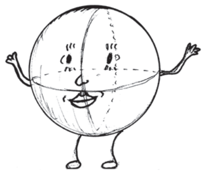
Try these: **Use the SHORTCUT to convert each percentage to a decimal.**

1) 6% = ______ **2)** 40% = ______ **3)** 73% = ______

4) 18% = ______ **5)** 9% = ______ **6)** 65% = ______

7) 54% = ______ **8)** 29% = ______ **9)** 37% = ______

10) 92% = ______ **11)** 1% = ______ **12)** 80% = ______



• Direct Percentages •

Direct Teaching

Example: What is 4% of 302?

To find the percentage of any number, we can use the following steps:

Steps to Solve:

Step 1:

Estimate your answer by rounding the NUMBER to the *nearest multiple of* **50.**

4% of 302 = ?

$\underset{\text{For Each 100}}{\underline{4}}$ % of $\underset{\text{302 rounded to the nearest 50}}{\underline{300}}$ = $\boxed{12}$ Estimate

Step 2:

Convert the percent to a decimal.

4% → .04. → **0.04**

Step 3:

Multiply the NUMBER by the decimal.

$$\begin{array}{r} 302 \\ \times \quad 0.04 \\ \hline 12.08 \end{array}$$

Step 4:

Check for reasonableness. The estimate was 12 so 12.08 is very reasonable.

Estimate = 12

Actual Answer = **12.08**

Try these: **In each exercise, estimate the answer. Then find the actual answer.**

1) 9% of 496 = ?

_____ % of _____ = $\boxed{}$

For Each 100 | 496 rounded to the nearest 50 | Estimate

____
Actual Answer

2) 12% of 242 = ?

_____ % of _____ = $\boxed{}$

For Each 100 | 242 rounded to the nearest 50 | Estimate

____
Actual Answer

• Direct Percentages •

In each exercise, estimate the answer. Then find the actual answer.

1) 4% of 393 = **?**

_____ % of _____ = ☐

For Each 100 — 393 rounded to the nearest 50 — Estimate

_____
Actual Answer

2) 15% of 509 = **?**

_____ % of _____ = ☐

For Each 100 — 509 rounded to the nearest 50 — Estimate

_____
Actual Answer

3) 22% of 161 = **?**

_____ % of _____ = ☐

For Each 100 — 161 rounded to the nearest 50 — Estimate

_____
Actual Answer

4) 7% of 687 = **?**

_____ % of _____ = ☐

For Each 100 — 687 rounded to the nearest 50 — Estimate

_____
Actual Answer

5) 31% of 215 = **?**

_____ % of _____ = ☐

For Each 100 — 215 rounded to the nearest 50 — Estimate

_____
Actual Answer

6) 6% of 939 = **?**

_____ % of _____ = ☐

For Each 100 — 939 rounded to the nearest 50 — Estimate

_____
Actual Answer

PK 3178 Direct_Percentages–6

• Direct Percentages •

In each exercise, estimate the answer. Then find the actual answer.

1) 5% of 497 = **?**

_____% of _____ = $\boxed{}$

FOR EACH 100 — 497 ROUNDED TO THE NEAREST 50 — ESTIMATE

_____
ACTUAL ANSWER

2) 20% of 648 = **?**

_____% of _____ = $\boxed{}$

FOR EACH 100 — 648 ROUNDED TO THE NEAREST 50 — ESTIMATE

_____
ACTUAL ANSWER

3) 13% of 981 = __________

4) 24% of 256 = __________

5) 9% of 77 = __________

6) 11% of 896 = __________

7) 16% of 360 = __________

8) 4% of 191 = __________

PK 3178 Direct_Percentages–7

• Direct Percentages •

In each exercise, estimate the answer. Then find the actual answer.

1) 12% of 762 = ?

_____% of _____ = [ ]

For Each 100 — 762 rounded to the nearest 50 — Estimate

_____ Actual Answer

2) 8% of 290 = ?

_____% of _____ = [ ]

For Each 100 — 290 rounded to the nearest 50 — Estimate

_____ Actual Answer

3) 7% of 815 = _________

4) 13% of 37 = _________

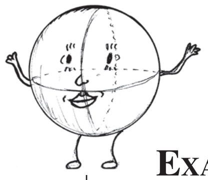
5) 28% of 655 = _________

6) 49% of 511 = _________

7) 18% of 348 = _________

8) 6% of 189 = _________

PK 3178 Direct_Percentages–8



• *Tricks* for Finding 10% and 1% •

EXAMPLES:

What is 10% of 143?	What is 1% of 143?
$\begin{array}{r} 143 \\ \times \quad 0.1 \\ \hline 14.3 \end{array}$	$\begin{array}{r} 143 \\ \times \quad 0.01 \\ \hline 1.43 \end{array}$
Multiplying by 10% (0.1) moves the decimal point ***one place to the left*** in the original number (143).	**Multiplying by 1%** (0.01) moves the decimal point ***two places to the left*** in the original number (143).
10% of 143 → 14.3. → **14.3**	1% of 143 → 1.43. → **1.43**

Try these:

1) What is 10% of 263? __________

2) What is 1% of 263? __________

3) What is 10% of 521? __________

4) What is 1% of 521? __________

5) What is 1% of 349? __________

6) What is 10% of 349? __________

7) What is 10% of 87? __________

8) What is 1% of 87? __________

9) What is 1% of 255? __________

10) What is 10% of 255? __________

• *Tricks* for Finding 10% and 1% •

EXAMPLES:

What is 10% of 45?	What is 1% of 45?
Move the decimal point *one place* to the left.	Move the decimal point *two places* to the left.
10% of 45 → 4.5. → **4.5**	1% of 45 → .45. → **0.45**

1) What is 10% of 69? __________ **2)** What is 1% of 69? __________

3) What is 1% of 436? __________ **4)** What is 10% of 436? __________

5) What is 10% of 507? __________ **6)** What is 1% of 507? __________

×2 Doubling

7) 1% of 130 is __________. So, 2% of 130 is __________.

8) 10% of 65 is __________. So, 20% of 65 is __________.

9) 10% of 310 is __________. So, 20% of 310 is __________.

10) 1% of 24 is __________. So, 2% of 24 is __________.

Extending Knowledge

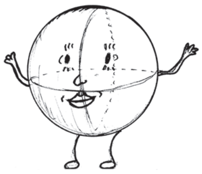
11) 1% of 200 is __________. So, 99% of 200 is __________.

12) 10% of 90 is __________. So, 90% of 90 is __________.

13) 2% of 150 is __________. So, 98% of 150 is __________.

14) 20% of 35 is __________. So, 80% of 35 is __________.

PK 3178 Direct_Percentages–10



• Direct Percentages •

1) 7% of 428 = __________

2) 9% of 318 = __________

3) 14% of 827 = __________

4) 10% of 409 = __________

5) 63% of 148 = __________

6) 12% of 524 = __________

7) A credit card company charges 10% interest on a balance owed of $869. How much is the interest charge?

____

8) Micah wants to leave a 20% tip for a meal that costs $43. How much money should he leave as a tip?

____

9) Is it reasonable to estimate that 54% of 653 is 512? Why or why not?

____

____

• Direct Percentages •

1) 18% of 348 = __________

2) 23% of 189 = __________

3) 99% of 700 = __________

4) 46% of 37 = __________

5) 33% of 655 = __________

6) 2% of 1,100 = __________

7) Barbara wants to leave a 20% tip for a meal that costs $85. How much money should she leave as a tip?

____

8) Buddy wants to buy a computer that is on sale for 10% off the original price of $810. What is the discounted price of the computer?

____

9) Is it reasonable to estimate that 45% of 152 is 68.4. Why or why not?

____

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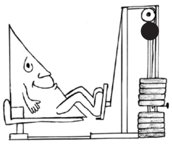
PK 3178 Direct_Percentages–12



Mastery Check: Direct Percentages •

1) 12% of 130 = __________

2) 7% of 192 = __________

3) 8% of 246 = __________

4) 10% of 463 = __________

5) It takes Larissa 363 total minutes to get from her house to her grandmother's house. If 87% of this time is spent on an airplane, how many minutes does she spend on an airplane?

____

6) Is it reasonable to estimate that 23% of 496 is 114.08? Why or why not?

____

____

Challenge:

7) 98% of 5,163 = ______________

PK 3178 Direct_Percentages–13

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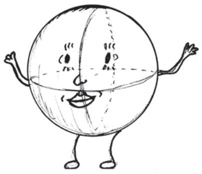

Observation
&
Progress Check

• Observation •

Observe an Instructor. Identify the processes, systems and instruction that are used on the floor.

Objectives:

1. Observe a student's Workout Plan being filled out at the beginning and end of their session.
2. Recognise how Instructors communicate with the Centre Director and each other.
3. Recognise Proactive Engagement and Disengagement.
4. Describe the how you saw the student reward process implemented.

Share details from your observation as they relate to the above objectives.

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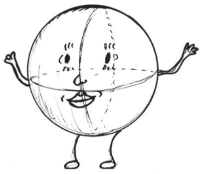

• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

Orientation

I have a good understanding of:					
1.	where to find Instructor resources; e.g. supplies, manipulatives, DeskTools and Answer Keys.	**0**	**1**	**2**	**3**
2.	the responsibilities of a Mathnasium Instructor.	**0**	**1**	**2**	**3**
3.	the criteria by which I will be evaluated as an Instructor.	**0**	**1**	**2**	**3**
4.	Mathnasium's dress code.	**0**	**1**	**2**	**3**
5.	how to communicate professionally with students and parents.	**0**	**1**	**2**	**3**
6.	student safety and how to hand off students to their parents/guardians.	**0**	**1**	**2**	**3**
7.	my responsibilities for keeping the Centre clean.	**0**	**1**	**2**	**3**

Learning Center Basics

I have a good understanding of:					
1.	how a student's binder is set up and organised.	**0**	**1**	**2**	**3**
2.	how a student's Learning Plan is related to their binder and learning goals.	**0**	**1**	**2**	**3**
3.	how Workout Plans are used to facilitate student progress, track sessions and communicate with other Team members.	**0**	**1**	**2**	**3**
4.	the Mathnasium Rewards system and how it is used to motivate students.	**0**	**1**	**2**	**3**
5.	what distributed practise means at Mathnasium.	**0**	**1**	**2**	**3**
6.	how to handle student homework in the Centre; e.g, time allotted and checking homework.	**0**	**1**	**2**	**3**

Continues on the next page →

• Progress Check •

Observation

I saw examples of:		YES	NO
1.	a student's Workout Plan being filled out at the beginning of their session.		
2.	a student's Workout Plan being filled out during their session.		
3.	a student's Workout Plan being filled out at the end of their session.		
4.	Instructor communication on the floor.		
5.	Instructors proactively engaging with students.		
6.	Instructors disengaging from students and leaving them with a meaningful task.		
7.	the student Rewards System; e.g, getting punches on their cards and/ or trading the cards in for something in the Rewards Cabinet.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

Learning Center Basics Course (section 2, p. 3)
Number Sense Triangle (section 2, p. 11)
Breaking Down the Prescriptive (section 2, p. 15)
Observation Experience (section 3)

NOTES:

____

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____

Share the observation portion of this Progress Check with the Instructor you are shadowing on your next work day. This will help make sure you get experience with all the elements of Team Teaching.

Team Teaching Course
&
Instructor Development

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Team Teaching Course

• Team Teaching •

Watch the <u>Right Instruction - Teach the Mathnasium Way: Team Teaching</u> training video. Then, read and complete the following pages:

The 10 Mathnasium Rules of Engagement
Team Teaching Checklist
Notes to Centre Director
The Mathnasium Hour
The Workout Plan

The Team Teaching course focuses on Mathnasium's unique approach to instruction, called Team Teaching and the tools designed to facilitate effective Team Teaching in your Centre.

Objectives:

In this course, you will learn about:

1. What Team Teaching is.
2. The Team roles.
3. Rotating and Reading the Floor.
4. Appropriate student engagements.
5. Appropriate ways to disengage.
6. Team communication.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the "Training Videos" section, click on "Right Instruction - Teach the Mathnasium Way: Team Teaching".

Notes:

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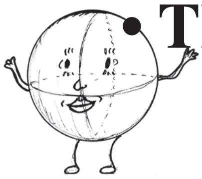


• The 10 Mathnasium Rules of Engagement •

1. Use the Mathnasium Teaching Constructs. Use the Teaching Icons as guides.

2. Do not repeat an explanation if it is not working. Try another approach. Do not force students to keep working on material they have already mastered.

3. Fall back on prerequisite knowledge when the student is having trouble. Extend knowledge when the student understands the concept.

4. Praise, encourage and constructively criticise when appropriate.

5. Use Socratic Questioning when it's appropriate. Use Direct Teaching when it's appropriate.

6. Use Mathnasium vocabulary because it makes sense. Avoid confusing nomenclature.

7. Use drawings, diagrams, manipulatives and DeskTools when appropriate to clarify and reinforce concepts visually.

8. Enable students to achieve metacognition, an awareness of one's own thinking process.

9. Require students to use mental maths. to enhance numerical fluency and limit reliance on pencil and paper.

10. Master Team Teaching.

***Try these*:**

1) What is Socratic Questioning and how does it differ from Direct Teaching?

____

____

____

2) How will you help students achieve metacognition?

____

____

____

• Team Teaching Checklist •

Use this detailed list of the elements of Team Teaching as a guide. Look for these elements when you are shadowing/instructing on the floor.

1) **Teaching the Mathnasium Way**
   - Engage student at strategic moment.
   - Interact based on student's needs.
   - Disengage and leave student with appropriate task.
   - Rotate and Read the Floor (walk around observing when not instructing).

2) **Team Roles and Tools**
   - Understand the role of the Team Leader.
   - Check students in using the Workout Plan.
   - Focus on quality instruction with the help of the Team Leader directing the flow of instruction.
   - Ensure that student works productively and balances session activities.
   - Use the Workout Plan to record student's session information and set session goals.

3) **Rotate and Read the Floor**
   - Provide individualised instruction to multiple students.
   - Move around the floor or zone observing multiple students.
   - Engage student responsively or seek opportunities for proactive engagement.
   - Read the floor for students asking for help, being distracted or working on a single page for an excessive amount of time.
   - Acknowledge other students in need of attention or send another Instructor.

4) **Team Communication**
   - Communicate when student does not understand Instructor's explanation.
   - Communicate concepts that student struggles with.
   - Communicate when work has been partially and completely corrected.
   - Communicate when there are sensitive needs for certain students.
   - Communicate when an unattended student needs help.
   - Communicate among other Instructors.

• Team Teaching Checklist •

5) **Communication with Centre Director**
- Communicate when student needs extra practice with a topic or skill.
- Communicate student performance in school/learning progress.
- Communicate when student has behavioural problems.
- Effectively use the Notes to Centre Director.

6) **Engage Students Appropriately**
- Greet each student.
- Use the Workout Plan to monitor and record each student's session goals and Learning Plan progress.
- Respond to students with raised hands promptly.
- Spot-check student work or follow up on assignments.
- Seek opportunities to help students achieve learning goals.
- Monitor students in an extended period of independent work.
- Follow up on previously assigned tasks.
- Correct errors before they become habits.
- Intervene to correct unproductive behaviour.
- Indicate to student that a nearby Instructor will help as soon as possible when other Instructors are occupied.

7) **Interact with Students Effectively**
- Deliver clear, concise explanation so that student gains understanding of current topic.
- Correct worksheets as they are completed and give instruction on incorrect exercises for student to reattempt.
- Use Mastery Checks to verify student's mastery of a concept.
- Maintain student's productivity by transitioning them through the session, reengaging them if necessary.

8) **Guidelines for Instruction Delivery**
- Sit across from student during instruction.
- Use supportive, patient and encouraging tone of voice.
- Maintain concise explanations.

• Team Teaching Checklist •

9) **Student Interactions**
- Determine teaching method (Socratic Questioning, Direct Teaching or a combination of both) when engaging student.

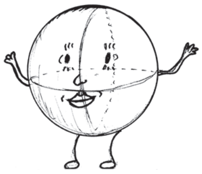
10) **Prepare for Independent Work**
- Encourage resistant student to attempt exercise independently.

11) **Correcting Student Work**
- Correct student work in presence of student as soon as possible.
- Ask student to explain their answer — correct or incorrect.
- Spot-check written work while rotating around the floor.
- Circle the exercise number of any errors.
- Return page to student to reattempt incorrect answers.
- Place start punch on page and Reward Cards.

12) **Mastery Checks**
- Correct after student completes the page without assistance.
- Review and return to student with incorrect answers circled.
- Communicate struggle with Mastery Check to Centre Director.

13) **Transitions Between Activities**
- Monitor activities using Workout Plan.
- Confirm student is working on correct activity.

14) **Disengage and Communicate**
- Disengage from student to allow the student time and space to work independently and develop mastery.
- Assign appropriate short-term task to student.
- Record Learning Plan progress and notes for other Instructors on student's Workout Plan.
- Relay significant behavioural issues to Centre Director verbally.



• Notes to Centre Director •

AAnswer the questions below based on the information from the course and what you have discussed with your facilitator.

Notes to Centre Director

Student Name	Item Number	Date Completed	Instructor Initials	Mastery Check Comments Further Practice for:	Curriculum Needed (Circle)	Completed By
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	
		/		Understanding / Accuracy	PK / WOB	

***Try these*:**

1) Where are the Notes to Centre Director located in the centre?

____

2) What information is communicated to the Centre Director in this document?

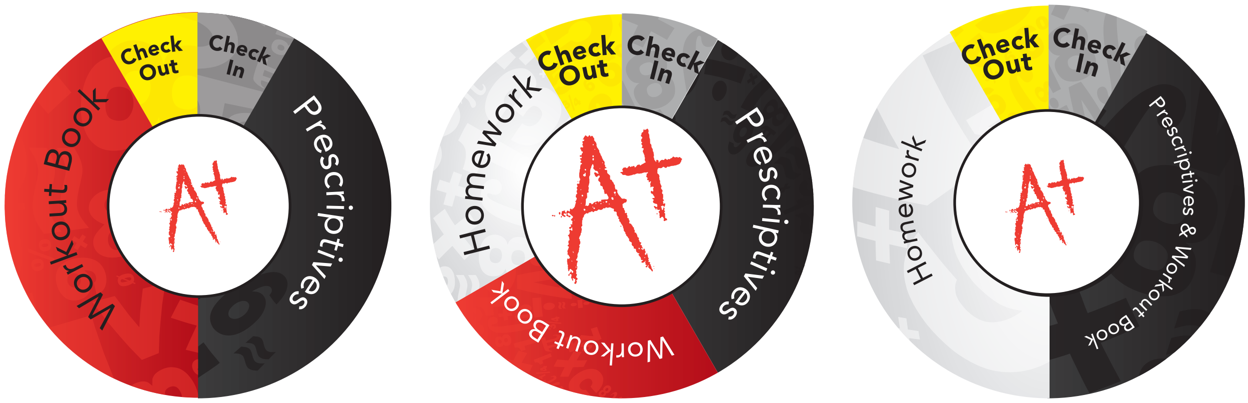
____

____

____

• The Mathnasium Hour •

Answer the questions below based on the information from the course and what you have discussed with your facilitator.



***Try these*:**

1) What is part of every Mathnasium Hour?

____

2) How does homework affect the Mathnasium Hour?

____

____

3) What happens when a student is getting checked in/out?

____

____

4) Describe three situations that would each require a different version of the Mathnasium Hour as seen above.

____

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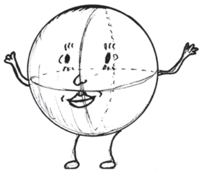

• The Workout Plan •

Answer the questions below based on the information from the course and what you have discussed with your facilitator.

Workout Plan

Name: ______________________________

Month: ______________

Notes:

Date:		**Time in:**	**Time out:**	**Instructor(s):**	
PK	**:**	**Mathnasium Topics:**		**Page Numbers:**	**CHECK OUT**
					Initial ___________
					Problem of the Week: ☐
WOB	**:**	Workout Book #___			Learning Reflection? ☐
HWK	**:**	Started? ☐ Selected Exercises:			Homework Spot Check: ☐
Notes for next session:					
Date:		**Time in:**	**Time out:**	**Instructor(s):**	
PK	**:**	**Mathnasium Topics:**		**Page Numbers:**	**CHECK OUT**

***Try these*:**

1) What are some things you may see or write in the "Notes" section at the top? What are some things to avoid?

____

____

2) What does "Selected Homework Exercises" mean?

____

____

3) What might you see or write in the "Notes for next session" section?

____

____

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Instructor Development

Centre Management – Student Behaviour

Curriculum and Instruction Icons

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Centre Management
Student Behaviour

Analyse behaviour issues

• Student Behaviour •

Read the following and explain how you would approach each situation.

1. Peter, a Year 5 student, checks in and quickly gets to his Mathnasium pages. A few minutes later, Peter's good friend Shant arrives for his session and grabs the seat next to Peter. Another Instructor notices the boys have been talking about yesterday's football match and asks them to focus back on their work. As you are rotating the floor, you notice the boys are again talking about the football match and not doing any maths. work.

   How would you approach this situation?

   ____

   ____

   ____

   ____

2. Jennifer comes in for her session right after school. When asked how her day was she says, "Busy! I had 3 quizzes and one of them was in maths. class on adding and subtracting fractions." When you stop and check-in with her about 20 minutes into her session, you notice she is still on the first page of adding and subtracting fractions. You begin to help her work through one of the problems but Jennifer is looking off into the distance.

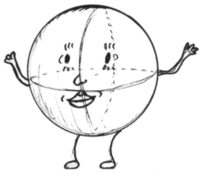
   What are some things you can you do to make the remaining part of Jennifer's session more productive?

   ____

   ____

   ____

   ____



• Student Behaviour •

Read the following and explain how you would approach each situation.

3. John is a Year 3 student new to the centre. His Mathnasium work is primarily Numerical Fluency material so he can improve his addition and subtraction techniques. As you are walking by, you notice John has a few pages piled up in front of him so you stop to correct them. While you are checking his completed pages, you see John using his fingers under the table to complete the addition page.

What type of student is John and what would you do in this situation?

____

____

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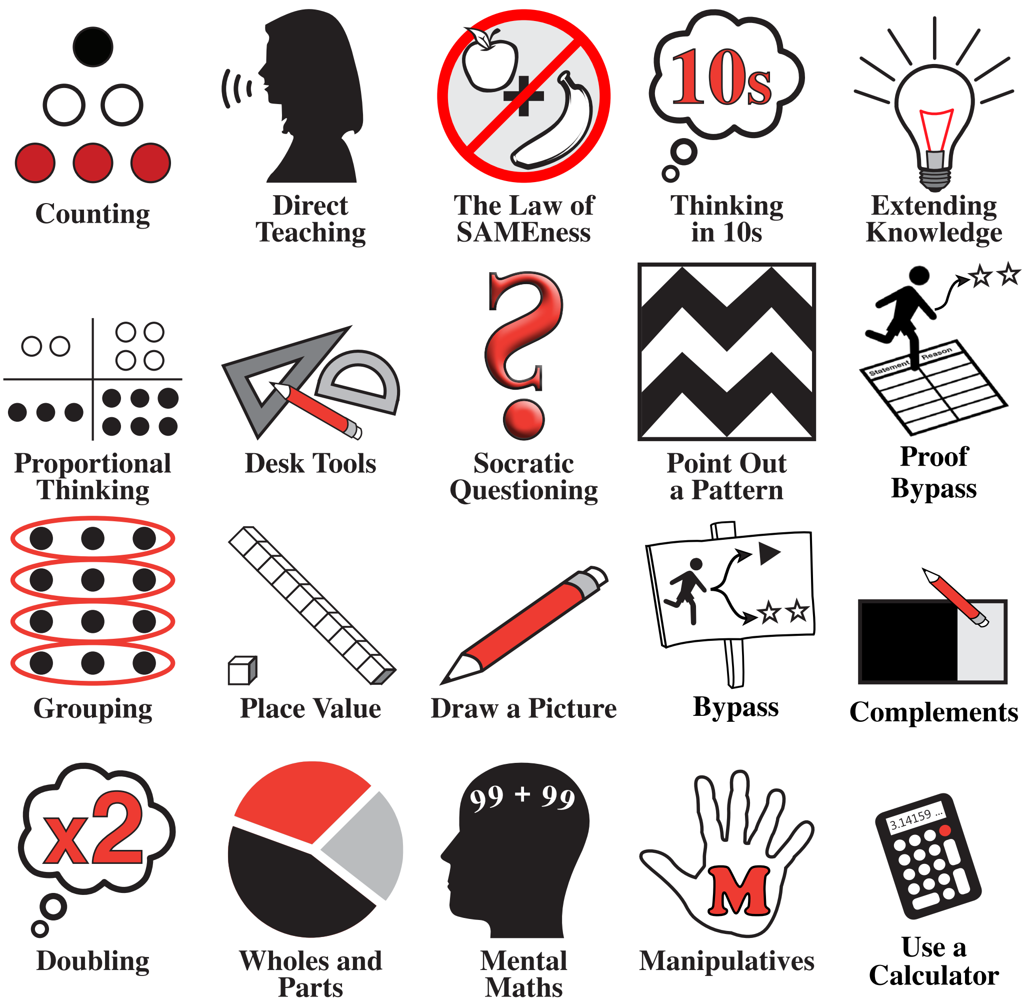
MATHNASIUM®

Curriculum and Instruction Icons

The Mathnasium Curriculum Icons are a visual prompt to use Mathnasium teaching techniques on that page. Familiarise yourself with the icons and what they mean. Descriptions about each icon are on the following pages along with some samples of Mathnasium Curriculum Pages where you might see them appear. You do **NOT** need to complete the worksheets.



• All About Icons •

Quantity, Denomination and the Law of SAMEness:
This icon can show up anywhere from fractions, to money concepts, to algebra. It signals the Instructor to use the ideas of ***Quantity***, ***Denomination*** and the ***Law of SAMEness*** when addressing the student.

Manipulatives:
When this icon appears the use of ***manipulatives*** might help the student to understand the problems at hand. Some forms of manipulatives include money, cards, dice and dominoes.

Bypass Page:
This icon signals the Instructor to make a decision. If the student shows really good understanding of the material, have the student skip to the Mastery Check; if not, the student continues on to the next page.

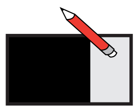
Proof Bypass Page:
If the student is in a class that includes proofs or if the Instructor thinks it would be good practice they should try the proof; if not, they can skip it and go to the Mastery Check.

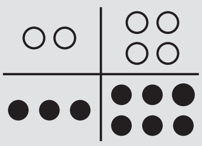
Thinking in 10s:
This icon will appear when ***thinking in 10s*** is a good strategy for solving the problems at hand.

Draw a Picture:
This icon signals when it is a good time for a student or Instructor to ***draw a picture*** to help solve a problem.

Wholes and Parts:
This icon will appear when some aspect of ***wholes and parts*** needs to be emphasised.

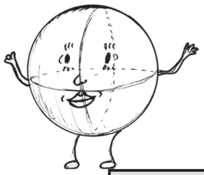
Complements:
This icon appears when ***whole minus part(s)*** can be used as an efficient method to solve the problem.





Proportional Thinking:
This icon will appear when ***proportional thinking*** should be employed in doing the current work.

Direct Teaching:
This icon suggests when a mini–lecture from the Instructor or an in–depth conversation with the student is appropriate.



• All About Icons •

Point Out a Pattern:

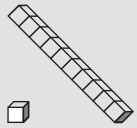
When this icon appears there are ***patterns*** that may not be immediately obvious to the student.

Extending Knowledge:

When this icon appears there are opportunities within the material to ***extend knowledge***.

Place Value:

This icon signals the Instructor to be thinking of ***place value*** when teaching the student.

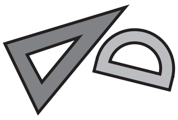


Doubling:

This icon will appear when ***doubling*** can be used as an efficient tool for doing the work at hand.

Mental Maths:

This icon marks problems that can be solved with ***mental maths***. If a student cannot solve the problem mentally, mental maths techniques should be taught by the Instructor.

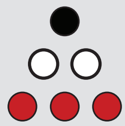


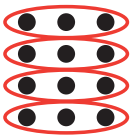
DeskTools:

This icon signals when the use of a ***desk tool*** can help the student understand the particular subject matter in a more visual manner.

Counting:

This icon signals an unexpected opportunity to use ***counting*** to gain a deeper understanding of the subject at hand.





Grouping:

This icon will appear when ***grouping*** can be used as an efficient tool for doing the work at hand.

Use a Calculator:

This icon signals a calculator ***may be*** used to help with the computational part of the question. This is a rare occurrence!

Socratic Questioning:

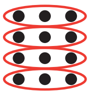
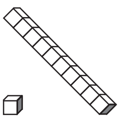
This icon suggests that Socratic Questioning is a better choice than Direct Teaching for a given topic.

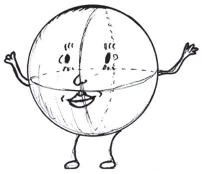
Let's Talk About It:

This icon acts as a reminder to the Instructor of the necessity of having a good maths conversation with the student.

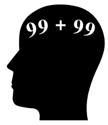
• All About Icons •

Look for the icons on the following curriculum pages. Explain what each of the icons signal an Instructor to do or think as it relates to the question(s) on the page.

 Mental Maths	
 Thinking in 10s	
 Grouping  Desk Tools	
 Socratic Questioning	
 Manipulatives	
 Doubling	
 Draw a Picture	
 Place Value	



• All About Icons •



Mental Maths

Thinking in 10s

1) 20 + 12 = ________

2) 10 + 20 + 35 = ________

3) 80 + 23 = ________

4) 100 + 70 + 35 = _______

5) 30 – 12 = ________

6) 50 – 23 = ________

7) 100 – 12 = ________

8) 100 – 51 = ________

9) 48 + _____ = 60

10) 78 + _____ = 100

11) 75 + _____ = 150

12) 99 + _____ = 200

13) 40 – _____ = 29

14) 100 – _____ = 85

15) 150 – _____ = 85

16) 200 – _____ = 99

---

17) **Count in 9s**: 0, 9, 18, ____, ____, ____, ____, ____, ____, ____, ____, ____, 108

18) **Count in 11s**: 0, 11, 22, ____, ____, ____, ____, ____, ____, ____, ____, ____, 132

19) half of 10 = ________

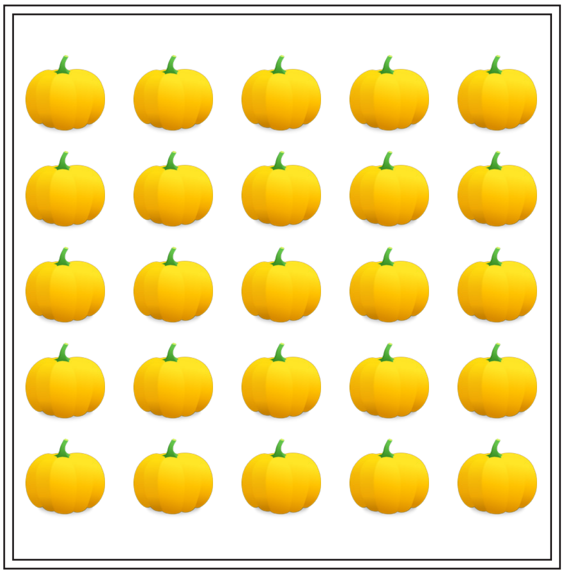
20) half of ______ = 10

21) 10 weeks = _______days

22) 10 metres = _______ centimetres

23) 10 hours + 10 days + 10 weeks = ________ hours

• All About Icons •



1) How many pumpkins are there in the picture?

____

2) If each pumpkin weighs three kilograms, how much do the pumpkins weigh altogether?

____

3) If each pumpkin costs \$2.50, how much do the pumpkins cost altogether?

____

Grouping

Desk Tools

---

4) Circle the fraction with the smallest missing piece to make one whole (1).

$\frac{4}{5}$ $\frac{9}{10}$ $\frac{7}{8}$ $\frac{11}{12}$

5) Circle the fraction with the largest missing piece to make one whole (1).

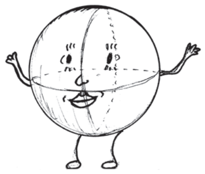
$\frac{4}{5}$ $\frac{9}{10}$ $\frac{7}{8}$ $\frac{11}{12}$

6) Circle the fraction whose value is closest to one whole (1).

$\frac{4}{5}$ $\frac{9}{10}$ $\frac{7}{8}$ $\frac{11}{12}$

7) Circle the fraction whose value is closest to zero (0).

$\frac{4}{5}$ $\frac{9}{10}$ $\frac{7}{8}$ $\frac{11}{12}$



• All About Icons •

Socratic Questioning

Show your work.

1) 1 day = __________ minutes

2) 1 metre = __________ centimetres

3) 3 weeks = __________ hours

4) 1 week = __________ minutes

5) 2 decades = __________ months

6) 2 kilograms 5 grams = __________ grams

7) 4 litres 2 millilitres = __________ millilitres

8) 2 days 3 hours = __________ minutes

9) 3 centuries 4 decades = __________ months

---

10) Six girls each have six bracelets. Each bracelet has five charms. How many charms are there altogether?

____

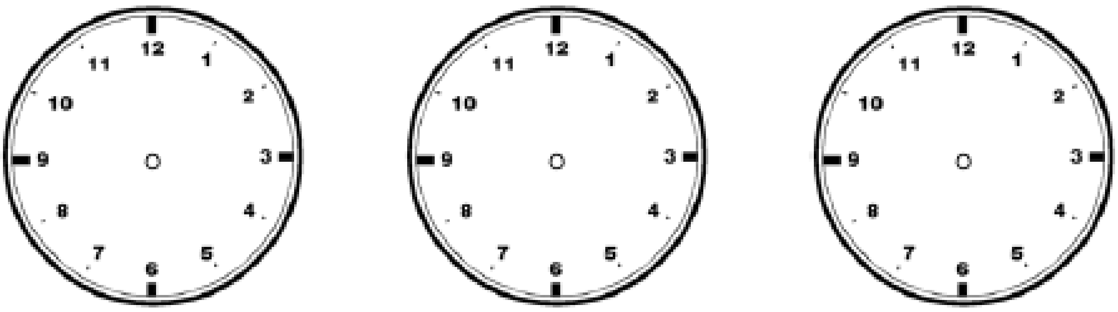
11) How many 10–gram bags can be made from one tonne of sand?

____

12) A family plans to drive 500 kilometres on a trip. Their car gets 20 kilomtres per litre of petrol. If petrol costs $2.35 per litre, how much will petrol cost for the trip?

____

• All About Icons •



1) Rosa went to the print shop to make copies. If she left at 10:15AM and returned at 10:55 AM, how long was she gone?

____

2) Rosa left work at 3:30 PM. It took her two hours to get home because of traffic. What time did she get home?

____

3) Rosa starting cooking dinner at 6:15 PM. If it took 20 minutes to cook, what time was dinner ready?

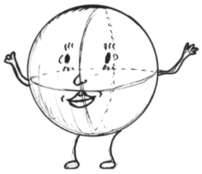
____

4) Rosa was sick for 3 days. If she got sick on Monday, when did she go back to work?

____

5) It is Sunday and Rosa will not go to work until Thursday. How long was her time off?

____



• All About Icons •

Doubling

1) 3, two times is ___; so, 3, four times is ___.

2) 5, two times is ___; so, 5, four times is ___.

3) 7, two times is ___; so, 7, four times is ___.

4) 6, two times is ___; so, 6, four times is ___.

5) 4, two times is ___; so, 4, four times is ___.

6) 9, two times is ___; so, 9, four times is ___.

7) 8×2 is ___; so, 8×4 is ___.

8) 10×2 is ___; so, 10×4 is ___.

9) 7×2 is ___; so, 7×4 is ___.

10) 11×2 is ___; so, 11×4 is ___.

11) 6×2 is ___; so, 6×4 is ___.

12) 12×2 is ___; so, 12×4 is ___.

• All About Icons •

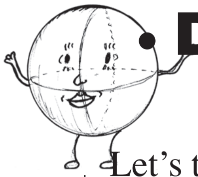
Draw a Picture

1) A pie is cut into 8 pieces. A kid ate a quarter $\left(\frac{1}{4}\right)$ of the pie. How many pieces are left? ___________

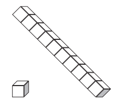
2) Which would you rather have: one piece of a chocolate bar cut in 7 pieces or one piece of the same sized chocolate bar cut into 3 pieces? Why?

CIRCLE YOUR ANSWER: **1 piece out of 7** **1 piece out of 3**

3) Half way through the second quarter, how much of the game is left? ___________



Decimal Place Value – Identify the Digits •



Let's take a look at the **values** of the first two *places* to the **right** of the decimal point. The *first place* to the **right** of the decimal point is the *tenths* place. The *second place* to the **right** of the decimal point is the *hundredths* place.

100s	10s	1s		$\frac{1}{10}$s	$\frac{1}{100}$s
______ HUNDREDS	______ TENS	______ ONES	•	______ TENTHS	______ HUNDREDTHS

WHOLE NUMBER PART   FRACTIONAL PART

GENERAL RULE: 10 of one place value makes 1 of the next largest place value.

(EXAMPLES: 10 tenths = 1 one. 10 hundredths = 1 tenth.)

EXAMPLE: Which digit is in the ***tenths*** place in the number **149.62**?

100s	10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s
1	4	9	.	6	2

The first digit to the right of the decimal point is **6**.

So, **6** is in the *tenths* place.

Try these:

1) Which digit is in the *hundredths* place? **214.97** ____________

2) Which digit is in the *tenths* place? **75.01** ____________

3) Which digit is in the *tens* place? **628.13** ____________

4) Which digit is in the *hundreds* place? **543.86** ____________

---

5) Write a number with a **9** in the *hundredths* place. ____________

6) Write a number with a **5** in the *tenths* place. ____________

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Observation
&
Progress Check

• Observation •

Observe an Instructor. Identify the processes, systems and instruction that is used on the floor. Share your previous Progress Check observation with the Instructor you are shadowing.

OBJECTIVES:

1. Observe a student's Workout Plan being filled out at the beginning and end of their session.
2. Recognise how Instructors communicate with the Centre Director and each other.
3. Recognise Proactive Engagement and Disengagement.
4. Describe how you saw the student reward process implemented.

Share details from your observation as they relate to the above objectives.

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

Team Teaching

I have a good understanding of:					
1.	the elements of Team Teaching and what I will be evaluated on.	**0**	**1**	**2**	**3**
2.	the Notes to Centre Director.	**0**	**1**	**2**	**3**
3.	the Mathnasium Hour and how it can vary from student to student.	**0**	**1**	**2**	**3**
4.	how to mark student work.	**0**	**1**	**2**	**3**

Instructor Development

I have a good understanding of:					
1.	how to address negative student behaviour in the Centre.	**0**	**1**	**2**	**3**
2.	what each Teaching Icon means.	**0**	**1**	**2**	**3**
3.	some ways I might see the icons appear in the curriculum.	**0**	**1**	**2**	**3**

Observation

I saw examples of:		YES	NO
1.	a student's Workout Plan being filled out during their session.		
2.	Instructor communication on the floor.		
3.	Instructors proactively engaging with students.		
4.	Instructors disengaging from students and leaving them with a meaningful task.		
5.	the student Rewards System; e.g. getting punches on their cards and/ or trading the cards in for something in the Rewards Cabinet.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

Team Teaching Course & Resources (section 4, p. 3 - 11)
Centre Management – Student Behaviour (section 4, p.15)
Curriculum and Instruction Icons (section 4, p.19)
Observation Experience (section 5)

NOTES:

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Instructor Development
&
A First Look (Part 1)

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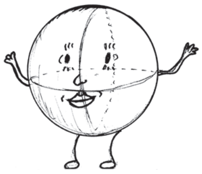

Instructor Development

Modes of Teaching

The Mathnasium Learning Principle

Mathnasium Teaching Constructs
(Counting, Wholes & Parts)

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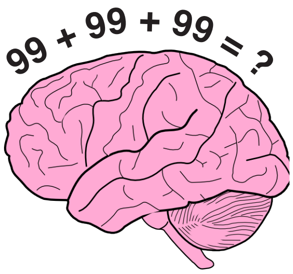
Modes of Teaching & The Mathnasium Learning Principle

Demonstrate solving questions using various methods
Differentiate between awareness, competency and mastery

• Modes of Teaching •



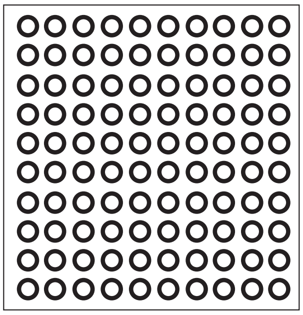
Mental

99 + 99 + 99 =

100 + 100 + 100 – 1 – 1 – 1 =

300 – 3 =

= 297



Visual

If each circle weighs 1 gram, how much do all the circles in the picture weigh?

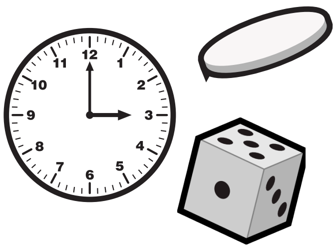
If each circle weighs 10 grams, how much do all the circles in the picture weigh?

If each circle weighs 25 grams, how much do all the circles in the picture weigh?

Verbal

Per Cent means "for each 100." "7% of 300" means count "7 for the first 100, 7 for the second 100 and 7 for the third 100."

So, 7% of 300 = 7 + 7 + 7 = 21.

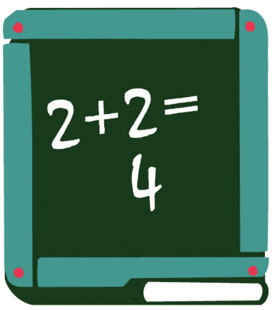


Tactile

Counting chips & Playing Cards for +, –, ×, ÷.

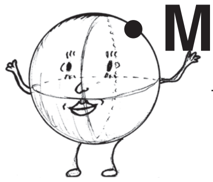
Dice for probability.

Learning about time with analogue clocks.



Written

$$\begin{array}{r} 1{,}236 \\ 234 \\ +\quad 28 \\ \hline \end{array} \qquad \begin{array}{r} 2{,}000 \\ -\quad 123 \\ \hline \end{array} \qquad \begin{array}{r} 259 \\ \times \quad 7 \\ \hline \end{array} \qquad 6\overline{)341}$$



• Mental–Visual–Verbal–Tactile–Written •

The MATHNASIUM® **METHOD** employs five modes in delivering our program.

Mental

Questions like "99 + 99 + 99," "$6\frac{1}{2}$% of 200," and "$\frac{3}{4}$ of 20" are best done mentally. Strong mental maths. skills help students develop confidence which is at the heart of both self–esteem and a willingness to explore maths. in addition to being a much more efficient way to handle many questions.

Visual

A significant number of MATHNASIUM® worksheets contain pictures that help student to focus on the critical attribute(s) of the question at hand.

Verbal

Direct Teaching and Socratic Questioning are verbal modes of delivery. Also, asking students to explain how they got their answers is a verbal experience.

Tactile

When appropriate, our Instructors use manipulatives (coins, dice, cards, scales, clocks, fraction circles...) to guide and reinforce students' thinking.

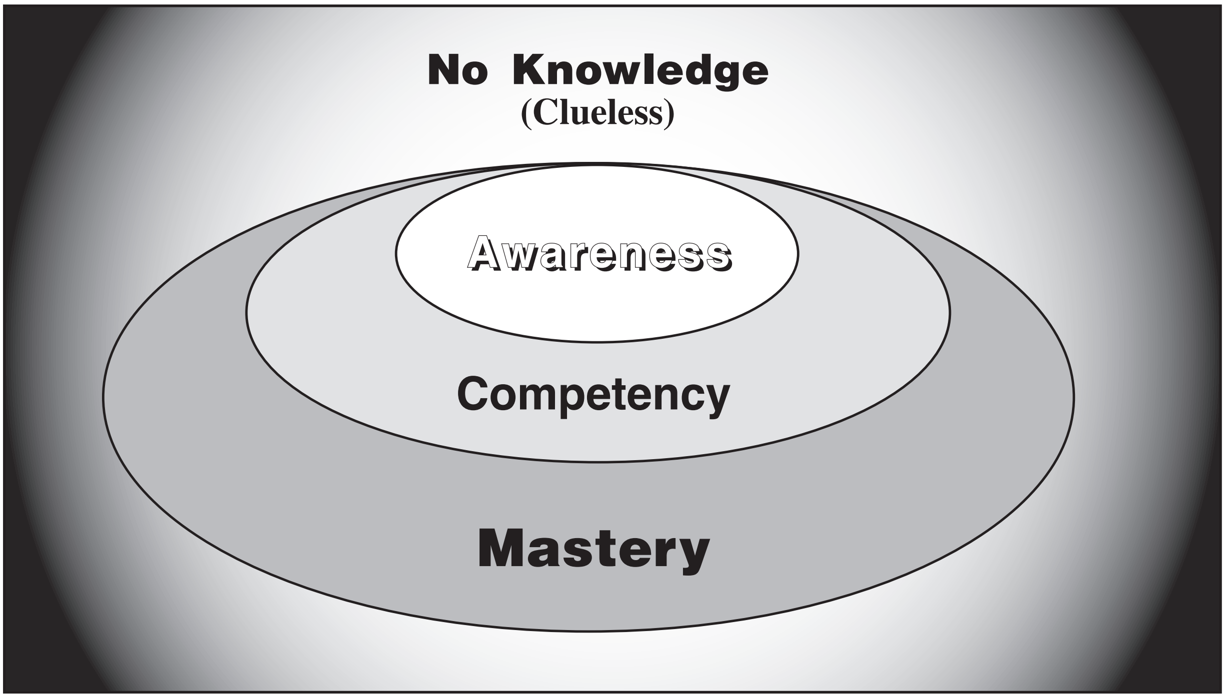
Written

MATHNASIUM® teaches all of the standard algorithms for addition, subtraction, multiplication and division, as well as *when* written procedures are preferable to mental ones and vice versa.

Most lessons involve a combination of several of these modes.

Being able to think like a child with an adult's knowledge is the key to transferring the information on a level that makes sense to the student.

The Three Stages of Learning



The MATHNASIUM® Learning Principle:
***Awareness* precedes *Skills*.**

Topic: Introduction to half ($\frac{1}{2}$)

Awareness

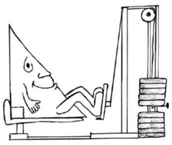
Any number can be cut in half.

Competency

$\frac{1}{2}$ of $6 =$ _____ • $\frac{1}{2}$ of $7 =$ _____ • $\frac{1}{2}$ of $\frac{1}{2} =$ _____ • $\frac{1}{2}$ of $6\frac{1}{2} =$ _____

Mastery

$\frac{1}{2}$ of $9\frac{1}{2} =$ _____ • $\frac{1}{2}$ of ____ $= 12$ • $\frac{1}{2}$ of ____ $= 5\frac{1}{4}$ • $\frac{1}{2}$ of ____ $= 49\frac{3}{4}$



• Reflective and Critical Questions •

Complete the following after reading the Modes of Teaching and Mathnasium Learning Principle pages.

1) Explain how you would solve $\frac{3}{4} \times 240\frac{20}{667}$ mentally.

2) Demonstrate a visual approach to solving this question.
An 6-meter lamppost casts an 8-meter shadow. How tall is a building that casts a 20-meter shadow?

3) A sculpture is made of 10 blocks. Each large block weighs 10 kilograms and each small block weighs 5 kilograms. If the sculpture weighs 85 kilograms in total, how many of each size block were used?

4) Determine which statements relate to a student stage of learning Absolute Value. Write Awareness, Competency and Mastery next to the appropriate statements.

a) The student can solve questions like:

$$3x^2 + 4x + 5 = 0.$$ ____________________

b) The student can solve questions like:

$$x^2 = 16.$$ ____________________

c) The student can graph a Quadratic Equation on the coordinate plane. ____________________

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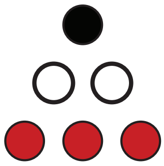
Mathnasium Teaching Construct: Counting



Counting is the key to unlocking addition and subtraction in early maths. development. At Mathnasium, our goal with Counting is to have a student become comfortable with counting by any number, starting at any number, forward and backward. Once counting up or down in 1s is mastered, students can further develop their understanding of arithmetic by counting by larger numbers and using fractional numbers. They will also apply their reasoning to understand counting by an irregular pattern.

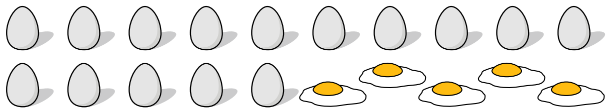
As you work through the exercises on the following pages, think about how you would instruct a student without defaulting to your traditional algorithms. You will have a chance to “teach your facilitator” during your next Progress Check.

• Counting by •

Counting

Example 1: How many eggs have been cracked? 5

Counting by 1s, we find there are 5 cracked eggs.



There are 10 eggs in each row. Five of the eggs in the second row are cracked so 5 eggs are *not* cracked.

Altogether, there are 10 + 5 = 15 eggs not cracked.

How many eggs are *not* cracked? 15

Example 2: Count in **12s** starting at **25**.

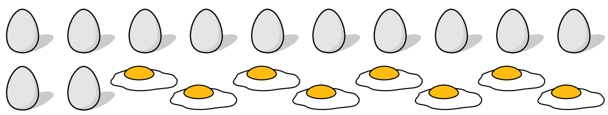
To count by **12s**, we can **add 10** ***first*** and *then* add **2**.

25, 37, 49, 61, 73, 85

+10, +2	+10, +2	+10, +2	+10, +2	+10, +2

Try these:

1) How many eggs have been cracked? 15 *not* cracked? _______

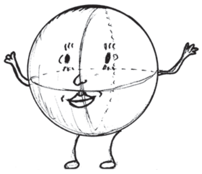


2) Count *forwards* in $\frac{3}{4}$s, starting at **0** :

_______, _______, _______, _______, _______, _______, _______, _______

3) Count *backwards* in **15**s, starting at **123**:

_______, _______, _______, _______, _______, _______, _______, _______



• Smart Counting •

Counting

Let's take a look at *smart ways* to add numbers using grouping.

Example: Add 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

Let's make groups of 11.

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$ 55

There are 5 groups of 11 so the sum is 55.

Try these: **Find a smart way to group the numbers to help you add. Show your grouping marks.**

1) $1 + 9 + 3 + 7 + 4 + 6 + 5 + 5 + 8 =$ __________

2) $3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 =$ __________

3) $10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90 + 100 =$ __________

4) $20 + 20 + 20 + 20 - 19 - 18 - 17 - 16 =$ __________

5) $1 + 2 + 3 + \ldots + 98 + 99 + 100 =$ __________

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Mathnasium Teaching Construct: Wholes & Parts

Understanding **Wholes & Parts** is critical to the development of a student's number sense. As the student begins to truly understand the relationship between a whole and the parts that compose it, a whole world of mathematical concepts and exercises can be explored. A strong understanding of Wholes and Parts allows students to approach seemingly complex exercises with accuracy and efficiency.

As you work through the exercises on the following pages, think about how you would instruct a student without defaulting to your traditional algorithms. You will have a chance to "teach your facilitator" during your next Progress Check.

• The Laws of **Wholes & Parts** •

Wholes and Parts

• The Law of the Whole •

The **WHOLE** is equal to the *sum* of its **PARTS**.

In symbols:

$$W = \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3 + \ldots + \mathbf{P}_i + \ldots + \mathbf{P}_n$$

Example:

A bag contains 3 red marbles, 4 blue marbles and 5 green marbles.

The total number of marbles (the *whole*) equals the number of red marbles, *plus* the number of blue marbles, *plus* the number of green marbles (the *parts*).

total number of marbles = 3 + 4 + 5 = 12

• The Law of Parts •

Each part equals the **WHOLE** *minus* the *sum* of all the other **PARTS**.

Complements

In symbols:

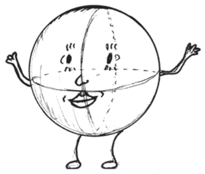
$$\mathbf{P}_1 = W - (\mathbf{P}_2 + \mathbf{P}_3 + \ldots + \mathbf{P}_{i-1} + \mathbf{P}_i + \mathbf{P}_{i+1} + \ldots + \mathbf{P}_n)$$

Example:

A bag contains 25 marbles.
Five of the marbles are red, 6 are blue, 4 are green,
and the rest are yellow.

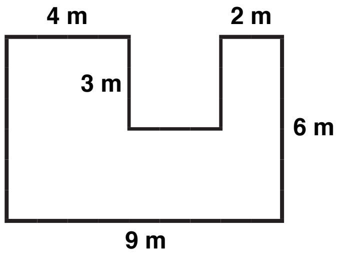
The number of yellow marbles (one of the *parts*) equals
the total number of marbles (the *whole*) *minus*
the combined total of red, blue and green marbles (the other *parts*).

number of marbles = 25 – (5 + 6 + 4) = 10



• Wholes & Parts •

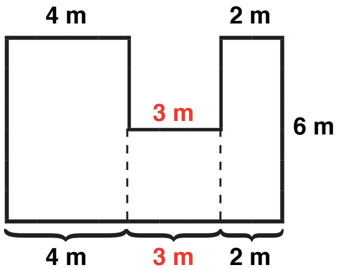
To find the area of this shape, we can approach the problem from two different perspectives.



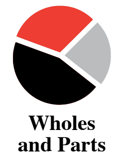
We can either break the whole into a series of parts:

$4 \times 6 = 24$
$3 \times 3 = 9$
$6 \times 2 = 12$

$24 + 9 + 12 = 45\ m^2$



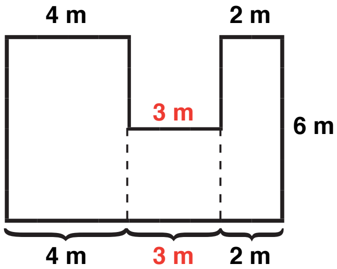
$W = P_1 + P_2 + P_3$
The WHOLE equals the *sum* of its PARTS.



or we can find the area of the "whole" rectangle and then subtract "the part we don't want"—the COMPLEMENT:

$9 \times 6 = 54$
$3 \times 3 = 9$

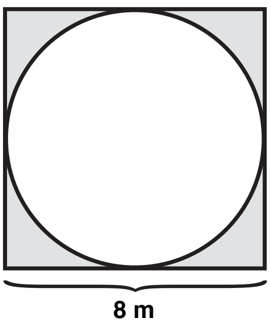
$54 - 9 = 45\ m^2$



$P_1 = W - P_2$
The PART equals the WHOLE *minus* the other PART(s).

Try this: **Find the area of the shaded region. Leave answer in terms of** π.

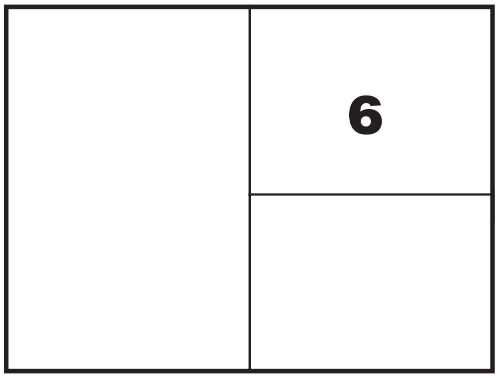
1)



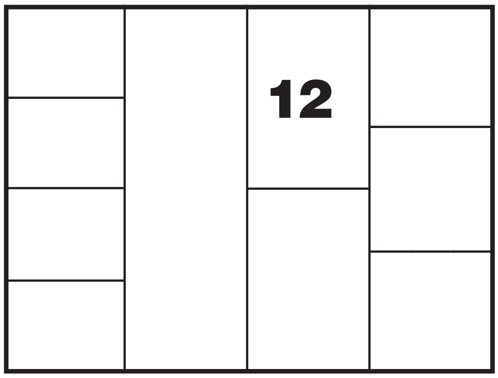
AREA = ______________

How much is the whole box worth?

1) ____________



2) ____________



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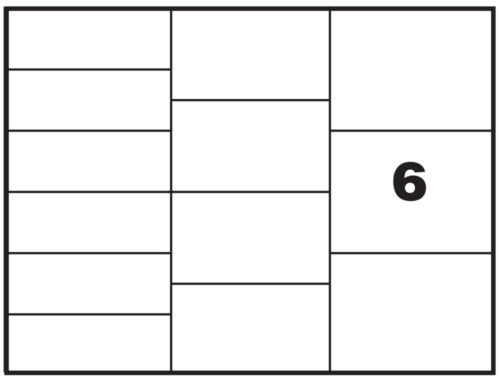
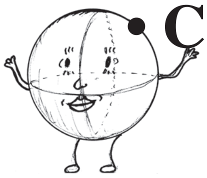




A First Look at Important Elements of the MATHNASIUM® Curriculum

Part 1

* These pages have ***Try These*** questions to complete

Number Sense

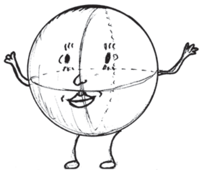
The ability to appreciate the *size* and *scale* of numbers, in the context of the question at hand.

SIZE establishes *magnitude*.

SCALE establishes *relationship*.

NUMBER SENSE includes:

- the ability to count *from* any number *to* any number *by* any number, forwards and backwards,
- the ability to compute (mentally and on paper) using single–digit and some double– and triple–digit numbers,
- understanding how 10s work,
- understanding the nature of fractions, decimal and percentages,
- understanding and applying measurement,
- being able to compare numbers by SUBTRACTION (*difference*) and by DIVISION (*ratio*),
- developing a "sense of proportion," a "sense of scale" AND
- being able to explain in words, pictures and symbols one's mathmatical thinking.



MATHNASIUM®

Key Concepts

Counting

[Counting *transforms* one number into another.]

Arithmetic is "the Art of Counting."

ari– (to join) + the + *metic* (measure)

Wholes and Parts

[whole = part + part + ... + part]

W*holes* and *Parts* and the *relationship* between them, unite various aspects of maths.

Proportional Thinking

["The Ratio is Constant—the fractions are equal."]

Proportion means "according to amount."

pro– (according to) + *portion* (amount)

Quantity & Denomination

["how many" of "what" (by name)]

The **denomination** of something is its *name–amount*.

de– (according to) + *nominare* (name)

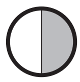
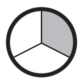
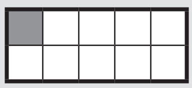
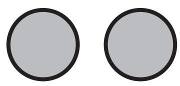
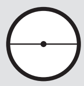
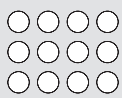
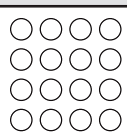
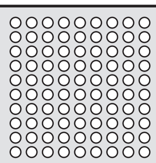
The **quantity** is the *amount* of that something; the *number* of things.

The Law of SAMEness

[apples + bananas don't make banapples]

We can only add and subtract things that have the *same name*, things that are of the *same denomination*.

• Benchmark Numbers •

Name:	Image:	Symbol:	Meaning: (Critical Attribute)
Zero		0, 0%, 0.0	None of it; no units; that which has no parts.
Whole, One, Unity		1, 100%, 1.0	All of it; one unit; unity.
Half		$\frac{1}{2}$, 50%, 0.5	Two parts the same; one of two equal parts.
Fourth, Quarter		$\frac{1}{4}$, 25%, 0.25	Half–of–a–half; Four parts the same; one of four parts.
Third		$\frac{1}{3}$, $33\frac{1}{3}$%, $0.\overline{3}$	Three parts the same; one of three equal parts.
Tenth		$\frac{1}{10}$, 10%, 0.1	Ten parts the same; one of ten equal parts.
Two		2, 200%, 2.0	Two wholes; all of it, twice.
Pi		$\pi = 3.14159...$	The distance around a circle divided by the distance across; the ratio of the circumference to the diameter.
Ten		10	The "base" of our number system.
Twelve		12	The "most divisible" small number; {1, 2, 3, 4, 6, 12} are factors of 12.
Sixteen		16	A "doubling" number; a "halves" number; {1, 2, 4, 8, 16} are powers of 2.
One Hundred		100	Ten 10s; the "per cent" number.

• Tips on Teaching Number Facts •

Development vs. Memorising

Number Facts include all addition, subtraction, multiplication and division problems resulting in single-digit and some double-digit, number answers. For example, consider the following four problems:

$3 + 7 = 10$ $9 - 6 = 3$ $5 \times 9 = 45$ $20 \div 4 = 5$

These examples are all **facts**. It is an easily demonstrated fact that 3 plus 7 does indeed equal 10, that 9 minus 6 equals 3 and so on.

Students must know the basic facts of each operation (the times table, for instance, is the list of basic facts for multiplication) before they can be successful with the more advanced computational algorithms (multi-column addition, subtraction and multiplication, as well as long division).

What is the best way for a child to acquire these facts?

We must teach children to create mental structures, "frameworks for learning," to facilitate the acquisition of material we now try to have them memorise.

In 1987, the *Fourth Mathematics Assessment of Educational Progress* reported:

> "Students may have trouble with two-step (word) problems for many reasons… many students have difficulty when they attend to two or more pieces of information… Many students do not know basic number facts."

These students lack the mental structures, the techniques for "figuring it out," that are necessary for success with complex tasks.

> The root of this problem lies in the way in which number facts are taught in the early year groups. Great emphasis is put on memorising addition, subtraction, multiplication and division facts through to Year 4. Not only is this a painful process for everyone involved (kids, teachers and parents) but in many cases it also does not result in the desired outcome.

Instead of attempting to have students *memorise* the number facts, I suggest that students be taught to *develop* them. Real mathematical growth takes place when students are led through a process of building the concepts themselves, from the inside out. In addition to learning the number facts, the students will also acquire the basic structures of multi-step problem solving.

Here's how it's done.

Counting how many altogether:
The process of forming a whole.

Setting the Stage: "Breaking down" numbers in addition

As children learn to count by 1s, they should be asked questions like:

- What number is 1 bigger than 5?…2 bigger than 9?
- What number is 1 smaller than 5?…3 smaller than 12?

Over a period of weeks and months, the child should learn to give all the answers to questions like:

- Name two numbers that add up to 5.
- 3 and what number add up to 5?

In preparation for more advanced work, add questions like:

- 5 and how much more make 7?
- 8 is how much more than 5?

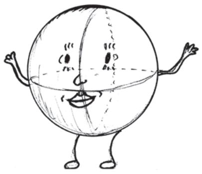
Initially, all of these questions should be limited to problems in which the answer is less than 10.

Each child must be led to develop the skill of "breaking down" a number into various combinations of two smaller numbers. This breaking down process is an important tool in the child's internal development of addition and subtraction facts. This will be shown in the following pages.

The Power of 10

Central to learning number facts is the number 10. "10" is the base number, the "name" of our decimal (*deci*—a Latin prefix meaning "ten") number system. As such, it has some very special properties. For instance, remarkable changes take place when you count up to multiples of 10: single-digit "9" changes to double-digit "10," the initial number changes when you count from "19" to "20," double-digit "99" becomes triple-digit "100." Most of the techniques and tricks that follow rely on one or more of the special properties of 10.

Some children figure out these tricks for themselves. But many don't.



Combinations of 10 (Complements of 10)

The basic form of these questions is: **The given number and *how much more* make 10?**

After students are able to count to 100, both forward and backward, they should be ready for questions on combinations of 10:

Set 1: 9 and how much more make 10?
8 and how much more make 10?
7 and how much more make 10?
6 and how much more make 10?
5 and how much more make 10?
4 and how much more make 10?
3 and how much more make 10?
2 and how much more make 10?
1 and how much more make 10?
0 and how much more make 10?
10 and how much more make 10?

These questions can be asked in order or at random and should be used repeatedly over time to reinforce the concept in students.

Over 10

Once students are comfortable with combinations of 10, they should be ready to practice addition with 10.

Set 2a:		**Set 2b:**	
	10 and how much more make 11?		10 and 1 make how much?
	10 and how much more make 12?		10 and 2 make how much?
	10 and how much more make 13?		10 and 3 make how much?
	10 and how much more make 14?		10 and 4 make how much?
	10 and how much more make 15?		10 and 5 make how much?
	10 and how much more make 16?		10 and 6 make how much?
	10 and how much more make 17?		10 and 7 make how much?
	10 and how much more make 18?		10 and 8 make how much?
	10 and how much more make 19?		10 and 9 make how much?
	10 and how much more make 20?		10 and 10 make how much?
	10 and how much more make 10?		10 and 0 make how much?

It is useful to point out to students the fact that the "teen" numbers can easily be broken down into 10 and a digit (e.g., fourteen is *four* plus *ten*).

Doubles

If students are comfortable with the questions in Set 1 and Set 2, they are ready for questions on doubles facts.

Set 3: How much is 1 + 1?
How much is 2 + 2?
How much is 3 + 3?
…
How much is 12 + 12?

The reverse question for doubles involves finding half of the sum. Questions like "How much is half of 6?…10?…20?…50?" should be used to reinforce the teaching of doubles, as well as to introduce fractions to young children. This process can take several months to complete.

How Much More Than 10

Once students are comfortable with combinations of 10, addition with 10 and doubles, they should be ready for variations on the questions in Set 2a and Set 2b.

Set 4: 11 is how much more than 10?
12 is how much more than 10?
13 is how much more than 10?
…
20 is how much more than 10?

Again, it is useful to point out to students the fact that the "teen" numbers can easily be broken down into 10 and a digit (e.g., fourteen is *four* plus *ten*).

After successfully practicing the questions in Sets 1–4, students are ready to transition from single-digit to double-digit numbers.

Set 5 11 is how much more than 10?…than 9?…than 8?…than 7?…than 3?…than 0?
12 is how much more than 11?…than 10?…than 9?…than 4?…than 0?
…
20 is how much more than 19?…than 12?…than 5?…than 0?

Using these questions through verbal, manipulative and written practice, along with the students' individual learning plans, will help them internalise these addition facts.

Techniques for Developing Addition Facts

The basic technique of adding two single-digit numbers (the numbers from 0 to **9**) is:

For two numbers that are less than 10, with a sum *less* than 10, **count up** from the larger number using the smaller number

- 4 + 5: "Start at five, then six, seven, eight, nine. 4 + 5 = 9."

For two numbers that are less than 10, with a sum *greater* than 10:

- Start with the greater number ("…put the big number in your mind"),
- Add enough of the smaller number to make 10 (by breaking the smaller number down into a "complement of 10" and another number) and then
- Add that other number to the 10 (using the "over 10" technique).

For example, children should be led to see the problem 7 + 8 as:

$$\boxed{7 + 8 = 8 + (2 + \mathbf{5}) = (8 + \mathbf{2}) + 5 = 10 + 5 = 15.}$$

Help them to see that 8 is 2 less than 10 (as shown in Set 1), that 7 breaks down to (2 + **5**) and that 10 and 5 (the rest of the **7**) make 15 (as in Set 2b).

It's much easier to internalise this process by saying the problem aloud to yourself than it is to learn it through reading a description of the technique. Try it yourself and see how much less complicated one is than the other.

Instructors: Please avoid the use of terms like "commutative property" and "associative property" until well *after* the student has mastered the number facts.

Doubles Plus/Minus 1

Another child (or the same child at a different time) might see the question as a use of doubles:

$$\boxed{\begin{array}{c} 7 + 8 = 7 + (7 + \mathbf{1}) = (7 + \mathbf{7}) + 1 = 14 + 1 = 15, \\ \text{or} \\ 7 + 8 = (8 - \mathbf{1}) + 8 = 8 + (8 - \mathbf{1}) = (8 + \mathbf{8}) - 1 = 16 - 1 = 15. \end{array}}$$

Still another approach some kids use is:

7 + 8 = 7 + 10…oops, 2 too many...back up by 2… = 15

Children tend to develop these strategies themselves *if* they are provided the opportunity to do so.

After doing some of these tricks a few dozen times, the child will develop a set of personalised techniques enabling number facts to appear almost instantly upon request, as though they had been memorised. Although the external result seems the same, the interior structure of the child's mind is now better prepared than any amount of memorising could accomplish.

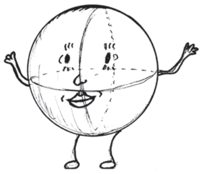
The child now has a structure in place that will come in handy for solving problems like "32 – 27":

from 27 to 30 is 3, from 30 to 32 is 2 and 3 and 2 make 5 so 32 – 27 = 5.

This type of quick mental calculation should be learned before pencil-and-paper work involving "carrying" and "borrowing" ("regrouping") is introduced.

As mentioned earlier some children figure out these tricks for themselves. Unfortunately, many do not. It is therefore vitally important that instructors know and understand these techniques and use them as required to help each student self-develop the number facts.

This self-development process teaches children to reason well, including those times when they attend to two or more pieces of information. Its focus on structure instills a sense of order in a child's thinking and thus aides that child in learning problem solving. This structure provides a swift, sure method of attack, a way to quickly "figure it out" in those moments when one doesn't remember the answer.



• Tips for Teaching **Addition** •

Counting on (start at *x* and count up by *y*)

1) 7 + 2 = ________

2) 8 + 3 = ________

Doubles

3) 5 + 5 = ________

4) 9 + 9 = ________

Doubles plus 1

5) 5 + 6 = 5 + 5 + 1 = ________

Doubles minus 1

6) 8 + 7 = 8 + 8 – 1 = ________

Breaking down numbers

7) 6 + _____ = 9

8) _____ + 7 = 11

“How far apart are two numbers?”

9) How far apart are 6 and 10? ____

“How far is it from x up to y?”

10) How far is it from 9 up to 12? ____

Combinations that make 10

11) 8 + 2 = ________

12) 6 + 4 = ________

“10 plus a number”

13) 10 + **7** = 17

14) 10 + **9** = 19

“10 plus what number?”

15) 10 + _____ = 16

16) 10 + _____ = 19

Putting it all together:

17) “**8** + **6** = ” **8** plus how much makes **10**? [**2**]
Since **6** – **2** = **4**, **10** plus the leftover (**4**) equals **14**. [**10** + **4** = **14**]

18) “**9** + **7** = ” **9** plus how much makes **10**? [**1**]
Since **7** – **1** = **6**, **10** plus the leftover (**6**) equals **16**. [**10** + **6** = **16**]

• Subtraction Tips •

Counting how far apart and how much is left:
The process of removing a part from a whole.

Subtraction has two aspects:

1) the notion of "how much is **left**?" and
2) the idea of "how **far apart** are the two numbers (how far is it from the smaller number up to the bigger number)?".

Use "how much is left?" when the numbers are *fairly far apart* and ***count down***.

EXAMPLE: "12 – 3" is best thought of as "counting *down* from 12 by 3," and

"11 – 1" is best thought of as "counting *down* from 11 by 1."

On the other hand, use "how far apart are the two numbers?" when the numbers are *fairly close to each other* and ***count up***.

EXAMPLE: "12 – 9" is best thought of as, "How far is it from 9 *up to* 12?"

"How far is it from 9 up to 10 (**1**) and how far is it from 10 up to 12 (**2**) and put the answers together (**1** + **2** = **3**).

The question "How much is 15 take away 9?" can be solved by asking any of the following questions:

- 9 and how much more make 15? 9 is how much less then 15?
- How far apart are 9 and 15? How far is it from 9 to 15?
- How much is left if I take 9 away from 15?

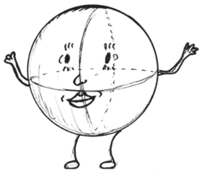
Try these:

1) Which method would you use for "100 – 98"? (CIRCLE ONE)
   HOW FAR APART   HOW MUCH IS LEFT

2) Which method would you use for "100 – 3"? (CIRCLE ONE)
   HOW FAR APART   HOW MUCH IS LEFT

3) Which method would you use for "100 – 87"? (CIRCLE ONE)
   HOW FAR APART   HOW MUCH IS LEFT

4) Which method would you use for "100 – 15"? (CIRCLE ONE)
   HOW FAR APART   HOW MUCH IS LEFT



• Multiplication Tips •

Counting equal groups:
The process of forming a whole from equal parts.

One of the greatest concerns in early mathematics education is how to have children effectively learn their times tables. Year after year, decade after decade, memorisation has always been mandated. But memorising has proven to be successful only with a small number of students.

Here are two tips which do not rely on memorising. Instead, they focus on teaching children structures they can use to quickly figure things out in moments when they've forgotten the answer.

Tip #1: In the beginning of primary school, most children learn to count in 1s, 2s, 5s and 10s. Starting a year or two later, just before the child is to learn the times tables verbally counting in 3s, 4s, 6s, 7s, 8s, 9s, 11s and 12s is added.

- First, during these learning periods, point out to students that when they are *counting in* 2s (or 5s or 10s), they are *adding* 2 (or 5 or **10)** to each previously counted number.

  It is important to point this out because, while most children easily learn the rhythm of counting by 2s, 5s and 10s, many of them do not learn the mathematical sense of what they are doing. Without this understanding, subsequent progress, particularly in the comprehension of the process of multiplication, is greatly impaired.

- Then add counting by 3s up to at least 15 (or as much farther as the child wants to go) and by 4s up to about 20.

  After doing these exercises a few times a week for 6 to 8 weeks, extend the 3s to 36 (and beyond) and 4s to at least 48. Again, make sure that your students understand that counting by 3s (and 4s) means adding 3 (or **4)** to each previous answer.

- Next, introduce counting in 11s. Point out the rhythm of *twenty-two, thirty-three, forty-four*, etc. Most children get into this right away.

  After each student can reliably count to 99 this way, point out that counting in 11s is the same as adding 10 and then adding 1. That is: 11 + (10 + **1)** = 22, 22 + (10 + **1)** = 33, 33 + (10 + **1)** = 44 and so on. This will make the jump from 99 to 110 (and up) easy and will set the stage for counting by 9s and 12s.

- Now, counting in 12s can be understood as adding 10 and then adding 2, thus: 12 + (10 + **2)** = 24, 24 + (10 + **2)** = 36, etc.

  For many children, the jump from 96 to 108 will be no big deal because they now have both a method and the experience to make it easy.

- Counting in 9s can be seen as adding 10, then subtracting 1: 9 + (10 – **1**) = 18, 18 + (10 – **1**) = 27, etc.

- After these have been mastered (and this usually takes 3 to 4 months of practice), introduce counting by 6s, 7s and 8s. Lead the children through the discovery of their own tricks to accomplish these tasks.

I strongly suggest instructors point out to students that the Multiplication Tables chart that adorns most classroom walls is also a "counting by" chart.

Tip #2: The second tip on taking times tables out of the realm of memorisation deals with syntax; that is, our choice and arrangement of the words we use when talking about multiplication.

Instead of asking, "What is 5 times 4?" which calls for a memorised response, a much better question form is: "How much is 5, four times?"

Most children say that the expression, "5, four times" brings to mind an image of (5 + 5 + 5 + **5**), whereas "5 times 4" carries no image at all.

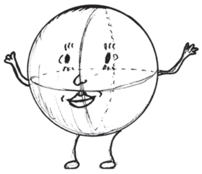
Consider this example: 7 × 8 = ☐

Children (and adults) who hear this question as "7, eight times" are able to use previously acquired knowledge (such as the fact that "7, seven times = 49") to say that "7, eight times = 49 + 7 = 56" (7, *seven* times + 7, *one* time = 7, *eight* times).

Children who have been brought up on the syntax of "How much is 7 times 8?" often try intuitively to use these helpful computational devices but, as a result of that syntax, frequently wind up going wrong: "7 × 8 = (7 × **7**) + *8* = 57." What's happening here?

> The child sees the problem clearly but when trying to solve it, reads the 8 as *both* multiplier and addend, mostly because 8 was the last number mentioned. When the problem is expressed as "7, eight times," however, word order makes it clearer to the child that there are eight things to deal with and all eight of them are 7s.

Of course, after children have developed their facility with numbers, they will be able to hear the question either way and still give the right answer. But, in the early months of learning multiplication tables, I strongly suggest using the "***x***, ***y*** times" syntax instead of the "***x*** times ***y***" approach.



• Division Tips •

Counting how many of "these" are there in "that":
The process of separating a whole.

Division is repeated subtraction and answers the question, "How many of *these* are there in *that?*"

The problem "12 divided by 3" poses the following questions:

> How many groups of 3 are there in 12?
> How many times can 3 be taken out of 12?
> How many times can 3 be subtracted from 12 before the answer is less than 3?

12 – 3 – 3 – 3 – 3 = 0 shows that there are four 3s inside of 12. So "12 divided by 3" is 4, with nothing (0) left over.

This conceptual approach to division is much different from telling elementary students that division is "the opposite of multiplication," and applying multiplication tables in reverse to an endless series of written problems.

The written process of division should not be started until the student can reliably v*erbally* answer questions like these:

> How many 10s are there in 30?
> How many stacks of 5 can you make out of 20 pennies?
> How many quarters are there in $3.00? …in $4.50?
> How many 20s are there in 100?
> How many 125s are there in 1,000?
> How many 15s are there in 60?

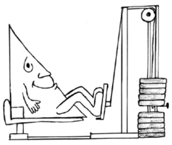
At this point, questions like $5\overline{)15}$, 15 ÷ 5 and $\frac{15}{5}$ should be read verbally by the student as

> "'15 divided by 5' means 'How many 5s are there in 15?'."

Here is a chance for students to use the counting skills they developed when learning multiplication facts.

> Counting by 5s, "5, 10, 15" (and counting on fingers is OK), it is easy to see that there are *three* 5s in 15.

Although the answer can easily be obtained by "reverse multiplication" ("Remember your multiplication facts: what number times 5 equals 15?"), you must resist the temptation to follow this old, established route. To properly place a usable, expandable concept of division in the child's mind, you must stress the "How many of these are there in that?" syntax.



• Reflective and Critical Questions •

Complete the following:

1) Complete all the "***Try these***" questions on the following pages:
Page 29: Tips for Teaching Addition
Page 30: Subtraction Tips

2) The following are some of the benchmark numbers. Give an example of how you could verbally explain the meaning of each number to a student.

EXAMPLE: Zero means none of it.

a) Whole, One or Unity: ______________________________

b) Half: ______________________________

c) Fourth or Quarter: ______________________________

d) Third: ______________________________

e) Tenth: ______________________________

f) Two: ______________________________

3) Why is it important for children to develop the skill of "breaking down" a number?

____

____

____

4) How can you reinforce a student's understanding of double facts?

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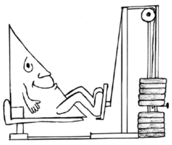

• Reflective and Critical Questions •

5) Review the various addition question types on the Tips for Teaching Addition page (p. 29), then explain what *language* you would use to help a student find the sum of:

$$9 + 8 + 7$$

6) What is the significance of asking a student, "What is 5, four times?" vs "What is 5 times 4?"

7) The question, "How many of these are in that?" should be used when teaching

____________________________.

8) True or False: When counting by a number it is OK for a child to use their fingers?.Why or why not?

Ex: Count in 5s, "5, 10, 15…"

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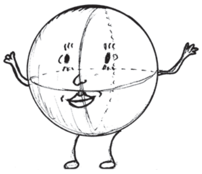

Shadowing & Progress Check

• Shadowing •

Shadow an Instructor. Identify the processes, systems and instruction that is used on the floor. Share your previous Progress Check observation with the Instructor you are shadowing.

OBJECTIVES:

1. Observe a student's Workout Plan being filled out at the beginning and end of their session.
2. Recognise how Instructors communicate on the instruction floor.
3. Recognise Proactive Engagement and Disengagement.
4. Describe how you saw the student reward process implemented.
5. Identify various modes of teaching (mental, visual, verbal, tactile, written) used by Instructors.

Explain the modes of teaching you saw other Instructors using on the floor. Be specific as to some of the language you heard.

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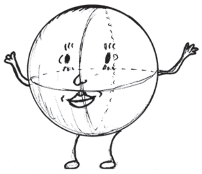

• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

Instructor Development/First Look

I have a good understanding of:					
1.	why Mathnasium Instructors use various methods to deliver instruction.	**0**	**1**	**2**	**3**
2.	the difference between No Knowledge, Awareness, Competency and Mastery.	**0**	**1**	**2**	**3**
3.	the language and methods used to teach students number facts.	**0**	**1**	**2**	**3**

Shadowing

I saw examples of:		**YES**	**NO**
1.	a student's Workout Plan being filled out during their session.		
2.	Instructor communication on the floor.		
3.	Instructors proactively engaging with students.		
4.	Instructors disengaging from students and leaving them with a meaningful task.		
5.	the student Rewards System; e.g. getting punches on their cards and/ or trading the cards in for something in the Rewards Cabinet.		
6.	Instructors using various teaching methods, i.e, mental, verbal, visual, tactile and/or written.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

Modes of Teaching (section 6, p. 5)
The Mathnasium Learning Principle (section 6, p. 8)
Mathnasium Constructs (Counting, Wholes & Parts) (section 6, p. 11)
Shadowing Experience (section 7)

Notes:

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Instructor Development
&
A First Look (Part 2)

Instructor Development

Proactive Teaching

Mathnasium Teaching Constructs
(Proportional Thinking, Quantity and Denomination, Law of SAMEness)

Proactive Teaching

Recognise the elements of proactive teaching

Proactive Teaching is the art and science of *anticipating* the needs of students and *preemptively* preparing them for success.

Here are some ideas that will help you become a better PROACTIVE Instructor. Also, please refer to the ***101⁺ Misconceptions About Math and How to Avoid Them.***

Encounter and Interaction

In 1985, the *California Mathematics Framework* said:

> "Students should encounter all types of numbers [and operations] long before they are ready to perform [paper and pencil] computations with them."

This statement is as true today as it was then. This is the major goal of the Mathnasium® curriculum: to provide all students with meaningful **encounters** and **interactions** with all of the elements of mathematics in a friendly, interesting, low–pressure and non–threatening environment. This environment respects each student's individual "learning style" and preserves their individual "sense of dignity."

If the learning process is to take place, it is necessary that students **encounter** and **interact**, that is, come face–to–face with and deal with concepts or skills in terms that *make sense* to *them*.

Encounter and **interaction** can take the form of *mental*, *visual, verbal*, *tactile* and/or *written* experiences. See the chart on page 22 of the previous section.

Engagement

Once students have **encountered** (become aware of the existence of) and **interacted** (got their mind into the work) with the material, they are ready to **engage** themselves in the learning process—to become students who have taken ownership of their own education rather than being the passive recipients of the words and actions of others.

For Instructors, **engagement**—holding the students' mental focus for an extended period of time as they interact with the) often means they must undertake the time–consuming task of employing *Direct Teaching* socratic *Questioning* and *Discovery* to make the concepts and skills at hand "second nature" to the learner.

Transfer of Knowledge

TRANSFER OF KNOWLEDGE is the process of the student seeing the connection between the current content and previously learned material.

Often, students see the connection themselves. They might say something like, “Hey, these are like the equations we learned last week, only a little harder.”

Other times, the Instructor must facilitate the process by a comment socratic Questioning and/or Direct Teaching. “Adding proper fractions is almost exactly the same as subtracting proper fractions. The only difference is...”

Feedback

The complete learning cycle requires a high degree of METACOGNITION, that is, the understanding of one’s own thought process. Students must acknowledge, in a meaningful way, that the content of the lesson has indeed *made sense* to them.

Without this FEEDBACK, it is possible that the Instructor may have succeeded in presenting the material to the student but may or may not have *taught* the student the desired content.

There is a need for a “sense check”
at each step in the learning process.
Otherwise:

Instructor says: “The *whole* is equal to the *sum* of its parts and is greater than *any one*.”

Student hears: “The *hole* is equal to the *sum* of its parts and is greater than *anyone*.”

Instructor says: “Two cans are worth \$1.”

Student hears: “Toucans are worth \$1.

(That’s awfully cheap for such a pretty bird.)

Intervention

Intervention is the process of observing students at work and "inserting" yourself into their world the moment they need help and guidance.

It is often useful to "intervene" at appropriate times when kids are doing their work, either to extend a question and develop further understanding or to involve students in deeper thinking as to what they are doing. It is always a good idea to ask students how they obtained the answers they are getting.

> For example, if a student is working on multiplying decimals and gets "87.5" for the answer to "2.5 × 3.5," it is appropriate for an Instructor to intervene and say, "Is it reasonable for a number less than 3 times a number less than 4 to equal 87 point something?"

This really engages students and makes your teaching time in front of them much more productive. Since you only have a limited amount time to work with each student, make each moment of your teaching time as valuable as possible.

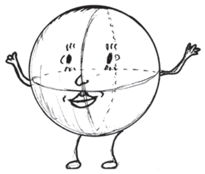
If you judge incorrectly and intervene at the "wrong" time, it is okay to bail and make a graceful exit.

> For example, a student is working on ordering common fractions and they write than $\frac{4}{7}$ is less than $\frac{1}{2}$. If an Instructor intervenes and says, "You better check that last answer" and the student replies, "I was, using mental maths. and I realised the mistake I made, before you said anything." Gracefully saying, "Looks like you've got this under control," and moving on to another student is a good way to handle a situation like this.

Team Teaching

Team Teaching is the process of helping students get engaged with their work and "move on" to another student, allowing the first student the time and space to think and work independently with a meaningful task, while still under your "very watchful eye." This helps many students break the cycle of "learned helplessness," the sad state of affairs where students have, over time, become psychologically dependent on thinking that they need help with every problem, new and old.

Team Teaching involves the "Chess Master" model, where you work with a student, rotate to another student and cycle back to previous students in a timely manner, without losing track of what's going on.



Heads–Up Teaching

Heads–up Teaching is the practice of “reading the floor” as you are working with a student to observe the needs of the rest of the students in the room.

> Think of how ticked off you get in a restaurant when your coffee cup is empty, and you can’t get anyone’s attention.

Being able to work closely with an individual student while remaining aware of the rest of the room is a skill that will enable you to provide the excellent instructional experience that makes Mathnasium® special.

Direct Teaching and Socratic Questioning

Direct Teaching is the traditional way maths. instruction is delivered, wherein the Instructor imparts the material in a lecture–type format.

Socratic Questioning is the process in which the Instructor asks the student pointed questions aimed at having the student “dig deeper”to respond and ultimately understand the material at a higher level.

The key to distinguishing Socratic Questioning from questioning per se is that Socratic Questioning is systematic, disciplined, deep and usually focuses on fundamental concepts, principles, theories, issues or problems. [From *Wikipedia]*

The MATHNASIUM® Teaching Principle:

K.I.S.S.

Kept It Short and Simple

[Not too many words and not too few, either.]

Short

Half means "the division of a whole into two equal parts."

Half means "two parts the same."

Simple

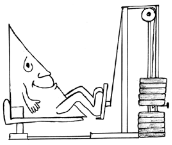
To give students an introductory explanation of "what is π, where did it come from" rather than saying:

"π is ratio of the circumference of a circle to its diameter."

A simpler introductory explanation, with more words but less jargon, is:

"the distance around a circle divided by the distance across the circle always comes out the same number, namely, 3.141592... an English mathematician gave this number the name π."

[Sometimes *simpler* is *longer*.]



• Reflective and Critical Questions •

Complete the following after reading the Proactive Teaching document.

1) Why is it important to use both Direct Teaching and Socratic Questioning?

____

____

____

2) How might a student make a connection between previously learned material and current content?

____

____

____

3) What does it mean to have a student engaged? What are some methods an Instructor can use to engage a student?

____

____

____

4) Give an example of a reason it would be appropriate for an Instructor to intervene while a student is working?

____

____

____

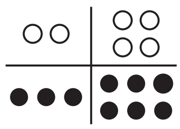
5) Write the following statement in simpler form:

The commutative property of addition says changing the order of the addends does not change the sum. Ex: “3 + 4” and “4 + 3.”

____

____

Mathnasium Teaching Construct: Proportional Thinking

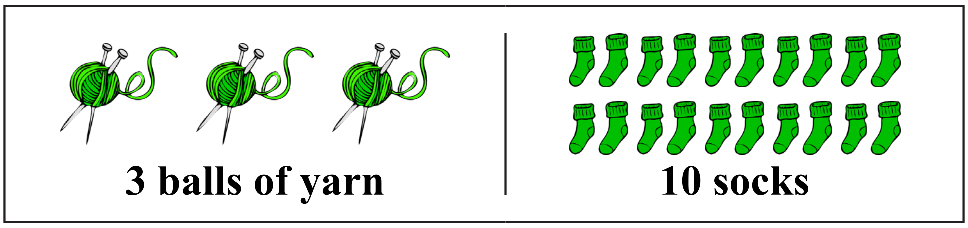


Proportional Thinking is introduced to students early in the curriculum, establishing a fundamental concept that will eventually lead to a stronger understanding of critical concepts like ratios, proportions, direct and indirect variation and algebraic reasoning. Students will develop Proportional Thinking through the practice of "reasoning in groups," where students will examine ratios of physical objects or quantities, like the ratio of lemons to glasses of lemonade. This practice is aided by visual representations of the objects and quantities and Instructors should be ready to use drawings and/or manipulatives to further illustrate the types of quantitative relationships that require Proportional Thinking.

As you work through the exercises on the following pages, think about how you would instruct a student without defaulting to traditional algorithms. You will have a chance to "teach your facilitator" during your next Progress Check.

Proportional Thinking

If it takes 3 balls of yarn to knit 10 socks...



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1) How many socks can be knitted with 12 balls of yarn?

How many *groups* of **3** balls are there *inside of* **12** balls? [4]

10 socks, [4] times = ________
NUMBER OF SOCKS

---

2) How many balls of yarn are needed to knit 60 sweaters?

How many *groups* of **10** socks are there *inside of* **60** socks? [ ]

3 balls of yarn, [ ] times = ________
NUMBER OF BALLS OF YARN

---

3) How many socks can be knitted with 24 balls of yarn?

How many *groups* of **3** balls are there *inside of* **24** balls? [ ]

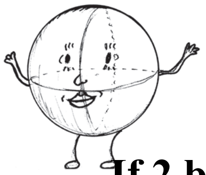
10 socks, [ ] times = ________
NUMBER OF SOCKS

---

4) How many balls of yarn are needed to knit 100 sweaters?

How many *groups* of **10** socks are there *inside of* **100** socks? [ ]

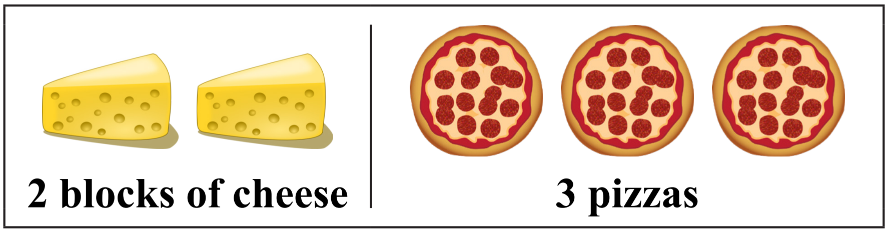
3 balls of yarn, [ ] times = ________
NUMBER OF BALLS OF YARN



• Proportional Reasoning •

Proportional Thinking

If 2 blocks of cheese are needed to make 3 pizzas...



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1) How many blocks of cheese do you need to make 18 pizzas?

How many *groups* of **3** pizzas are there *inside of* **18** pizzas? ☐

2 blocks of cheese, ☐ times = ________
NUMBER OF BLOCKS OF CHEESE

2) How many pizzas can you make with 16 blocks of cheese?

How many *groups* of **2** blocks are there *inside of* **16** blocks? ☐

3 pizzas, ☐ times = ________
NUMBER OF PIZZAS

3) How many blocks of cheese do you need to make 12 pizzas?

How many *groups* of **3** pizzas are there *inside of* **12** pizzas? ☐

2 blocks of cheese, ☐ times = ________
NUMBER OF BLOCKS OF CHEESE

4) How many pizzas can you make with 22 blocks of cheese?

How many *groups* of **2** blocks are there *inside of* **22** blocks? ☐

3 pizzas, ☐ times = ________
NUMBER OF PIZZAS

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Mathnasium Teaching Construct: Quantity & Denomination

The **Quantity & Denomination** construct examines two aspects of a numerical value. Quantity asks "How many" or "How much?" Denomination asks "of what?" These aspects are obvious when counting objects or measured quantities, like 4 students or 18.5 kilometres. Quantity and Denomination are also applied to composed and decomposed numbers whose digits refer to the quantity of the respective place value denomination. Quantity and Denomination also have important applications in fractions, measurements and counting.

Understanding Quantity and Denomination is an important step towards building number sense. By applying this construct, students will have a stronger understanding of addition and subtraction as well.

As you work through the exercises on the following pages, think about how you would instruct a student. You will have a chance to "teach your facilitator" during your next Progress Check.

• *On* Denomination, Quantity & Value • ☆

The Law of SAMEness

Quantity & Denomination

"How many, of what, worth how much."

There are several key concepts that unify the mathematics curriculum and are critical elements in ones ability to be a good problem solver.

DENOMINATION ("the name of"—"amount", "worth")

Every *thing* has a **name**. This **name** gives the DENOMINATION of the *thing*, making it unique within a larger group of things.

"Tuna, shark and salmon are DENOMINATIONS of fish."
"Halves, fourths (quarters) and thirds are DENOMINATIONS of fractions."
"Penny, nickel, dime and quarter are DENOMINATIONS of coins."
"1¢, 5¢, 10¢ are what pennies, nickels and dimes are worth."

QUANTITY ("the number of")

After knowing the **name** or **worth** of some *thing*, you should also know the QUANTITY of that *thing* in the given problem.

"3 cars"	QUANTITY = 3
"half–of–a–dozen"	QUANTITY = $\frac{1}{2}$ of 12
"2 hours in minutes"	QUANTITY = 120

VALUE ("how much each one is worth")

Many QUANTITIES have a VALUE associated with them.

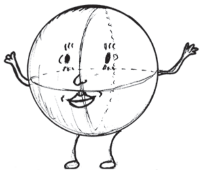
"Cost per kilogram"	"$3.69 per kilogram."
"Worth *of each*"	"the shirts are worth $20 *each*."
"Units of Measure"	"One kilometre equals 1,000 metres ."

Relating DENOMINATION, QUANTITY and VALUE

Many problems can be solved using the idea that:

Total = [QUANTITY times VALUE] of the first *thing* **+**
[QUANTITY times VALUE] of the second *thing* **+ ...**
[QUANTITY times VALUE] of the last *thing*.

When more than one *thing* is involved, **The Law of SAMEness** comes into play.



• Wholes & Parts of Mixtures •

The equation below represents what happens when we mix 2 litres of 10% juice with 6 litres of 30% juice.

2 LITRES • 10% JUICE + 6 LITRES • 30% JUICE = _____ LITRES • _____% JUICE

1) What are the "parts" in this equation? What is the "whole"?

Part: ____

Part: ____

Whole: ____

2) What is the amount of pure juice in 2 litres of 10% juice?

____

3) What is the amount of pure juice in 6 litres of 30% juice?

____

4) What is the total amount of pure juice in the whole mixture?

____

5) What percentage of the resulting mixture is pure juice?

____

• Wholes & Parts of Mixtures •

EXAMPLE: How much of a 50% salt solution should be mixed with 10 litres of a 10% salt solution in order to create a mixture that is 25% salt?

In this scenario, the amount of one part of the mixture is unknown. The total amount must therefore be the sum of the known amount and the unknown amount (*i.e.*, $10 + \boldsymbol{x}$).

$$10 \text{ litres} \bullet 10\% \text{ salt} + \boldsymbol{x} \text{ litres} \bullet 50\% \text{ salt} = (10 + \boldsymbol{x}) \text{ litres} \bullet 25\% \text{ salt}$$

$$10 \bullet 0.1 + \boldsymbol{x} \bullet 0.5 = (10 + \boldsymbol{x}) \bullet 0.25$$

$$1 + 0.5\boldsymbol{x} = 2.5 + 0.25\boldsymbol{x}$$

$$0.25\boldsymbol{x} = 1.5$$

$$\boldsymbol{x} = 6 \text{ litres}$$

Six litres of 50% salt solution must be added to 10 litres of 10% salt solution to create a mixture that is 25% salt.

Try this:

1) How many litres of 30% acid must be added to 2 litres of 15% acid to result in a mixture that is 25% acid.

____

Mathnasium Teaching Construct: The Law of SAMEness

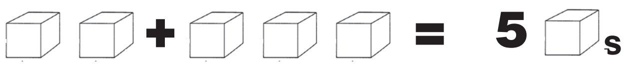
The Law of SAMEness is a concept that students naturally apply in their reasoning without being aware of it. Quantities of apples and oranges cannot be added together, unless first referred to as the same name, fruit. The number of boys and girls cannot be added together, unless first referred to them as children. The Law of SAMEness is a construct that students will apply in various mathematical contexts, including addition and subtraction with whole numbers, decimals and fractions, measurements and combining like terms in an algebraic expression.

As you work through the exercises on the following pages, think about how you would instruct a student without defaulting to traditional algorithms. You will have a chance to "teach your facilitator" during your next Progress Check.

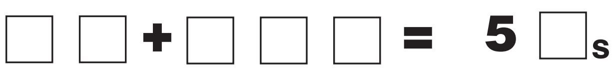
The Law of SAMEness

The Law of SAMEness

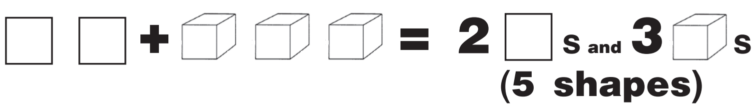
We can only add and subtract things that are of the *same denomination*, things that have the *same name*.



2 cubes + 3 cubes = 5 cubes



2 squares + 3 squares = 5 squares



2 squares + 3 cubes = 2 squares + 3 cubes = 5 shapes

Try these:

1) **3** centimetres + **4** millimetres = **34**________________

2) $2\frac{1}{2}$ hours – **30** minutes = **2**________________

3) **2** weeks + **5** days = **19**________________

4) **5** eighths – **1** half = **1**________________

5) Explain why **3** apples *plus* **2** bananas does **NOT** equal 5 ***banapples***.

____

____

____

6) Explain why **2** hours *minus* **30** minutes equals 90 minutes.

____

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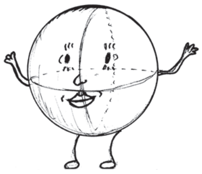

What is wrong with the problem below and how would you explain to a Year 7 student the correct way to solve it?

(This is a question appropriate for years 4, 5 or 6.)

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$$

What are "Common Denominators"

AND

Why Do We Need Them?

What are DENOMINATORS?

1) ____

2) ____

3) ____

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A First Look at
Important Elements of the MATHNASIUM® Curriculum

Part 2

* These pages have ***Try These*** questions to complete

• The Concept of Complements •

Complements are separate things which, when taken together, form a whole. One way to think of complements is "the complement is *the rest of it*."

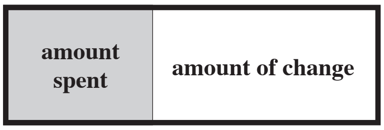
Two important properties of complements are:

- as one gets bigger, the other gets smaller ***by the same amount*** (see "Direct and Inverse Variation") and
- when one is zero, the other is the whole.

Example:

ten dollar note

- the amount of change you get from a ten dollar note is the complement of how much you spend (the amount of change + the amount spent = the whole ten dollars).



As the amount you spend increases, the amount of change you get decreases. When the amount of change is zero, then you have spent the whole ten dollars.

- If three fourths of a circle is shaded in, the complement, one fourth, is not shaded. (One fourth is the complement because three fourths + one fourth = four fourths, a whole.)

  As the amount of the circle that is shaded increases, the amount not shaded decreases. When all of the circle is shaded in, none of it is unshaded.

Many mathematical ideas in the areas of algebra and geometry require a solid understanding of complements. Teachers and parents should point out complements whenever possible in the early year groups.

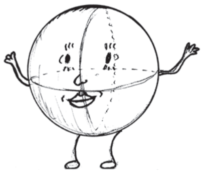
Try these: **Explain each of the following in terms of complements.**

1) The chance of rain vs. the chance of it not raining.

____

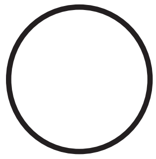
2) The number of cookies you eat out of a box vs. the number of cookies left in the box.

____



• The Concept of Fractions •

Here is a picture of a whole thing. In this case, it's a whole circle.



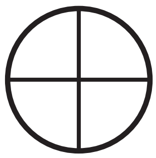
Fractions are created by dividing (cutting, breaking, "fracturing") a whole thing into *equal* parts.

Every fraction has two components:

$$\frac{\text{the } \textbf{numerator}}{\text{the } \textbf{denominator}}$$

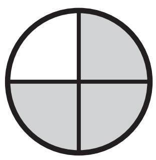
and

The **denominator** is "the one that names." It is determined by the total number of equal parts into which the whole is divided. This circle



has been divided into 4 equal parts so the denominator (the name) of the fraction represented is *fourths* (*quarters*).

The **numerator** is "the one that numbers." It tells us how many of the parts to use this time. Here, 3 parts have been shaded in so the numerator (the number of parts used) of the fraction represented here is *3*.

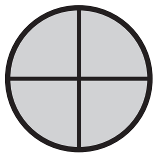


(Notice that the numerator tells us "how many," so it is an adjective: 3 apples, 3 cars, 3 fourths (quarters). The denominator is the "name" of the fraction so it's a noun.)

The fraction three fourths (quarters) is usually written $\frac{3}{4}$. Here we're using 3 of the 4 equal parts of the whole thing. We can say we have "3 out of 4 (equal) parts."

If you cut a whole into 4 equal parts and use all 4 of them (“4 out of 4”), then you have used the whole thing, *all* of it.

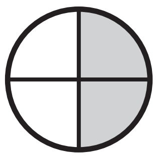
So, $\frac{4}{4}$ and 1 whole are two different names for the same thing.



$\frac{4}{4}$ = 1 whole

If you cut a whole into 4 equal parts and use 2 of them (“2 out of 4”), then you have used *half* of it, since 2 is half of 4.

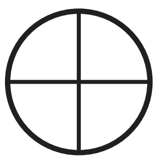
So, $\frac{2}{4}$ and 1 half ($\frac{1}{2}$) are two different names for the same thing.



$\frac{2}{4} = \frac{1}{2}$

If you cut a whole into 4 equal parts and use *none* of them (“0 out of 4”), then you have used *none* of it.

So, $\frac{0}{4}$ and none (0) are two different names for the same thing.

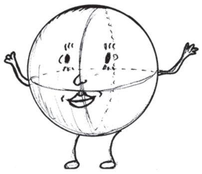


$\frac{0}{4} = 0$

Note:

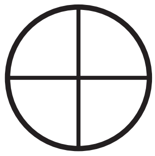
When the numerator is *less* than the denominator, as in $\frac{3}{4}$, the value of the fraction is less than 1. Why? Because if you have 3 out of 4 parts, you don’t have *all* of it yet. You are 1 part short.

(The same sort of thing is true for $\frac{2}{4}$ and $\frac{1}{4}$. With $\frac{2}{4}$, you are 2 parts short; with $\frac{1}{4}$, you are 3 parts short.)



What does 7 fourths ($\frac{7}{4}$) mean? Let's see…

The denominator (4) tells us that each whole has 4 equal parts.



The numerator (7), tells us we need 7 equal parts. Well, the circle above has 4 of the parts we need so let's draw another whole circle and cut it into 4 more equal parts

Now, let's shade in 4 fourths (quarters) in the first circle and 3 fourths (quarters) in the second circle: 7 fourths (quarters) altogether.

Ah-ha! We have shaded in 1 whole circle and 3 fourths of another circle. This is written $1\frac{3}{4}$. So, $\frac{7}{4}$ (7 fourths/quarters) is the same as $1\frac{3}{4}$ (1 whole and 3 fourths/quarters more).

Note: In later maths. courses the notation for a fraction ($\frac{a}{b}$) is also used to mean "***a*** divided by ***b***." This does not conflict with the concepts presented here because a fraction is one of the three ways of stating a division problem.

Try these:

1) Draw a picture of $2\frac{1}{4}$.

2) Give another name for $2\frac{1}{4}$ (in fourths/quaters). ______________________________

3) Draw a picture of 10 fourths (quarters).

4) Give two other names for 10 fourths (quarters). ____________________________

5) What is another name for 12 fourths (quarters)? ____________________________

Why? ____

• Simplifying Fractions •

Mechanically, simplifying fractions is very straightforward.

- Determine, usually by inspection, which number(s) divide evenly into both the numerator and the denominator. If possible, determine the largest number that will divide evenly(the Highest Common Factor, the HCF) and

- divide both the numerator and the denominator by that number. If the number used is not the HCF, repeat the process until there are no common factors in the numerator and the denominator.

For example, to simplify $\frac{6}{8}$,

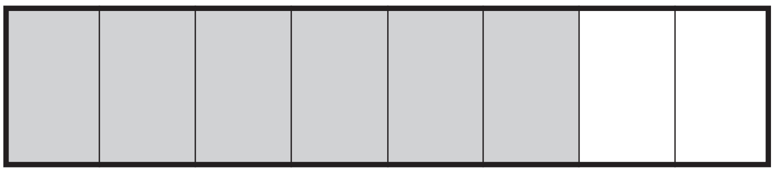
- first determine the largest number that will divide evenly into both 6 and 8 (in this case, 2) and

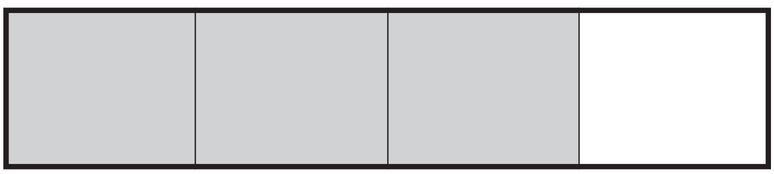
- divide 6 by 2 (which equals 3) and divide 8 by 2 (which equals 4), giving $\frac{3}{4}$.

Thus, $\frac{6}{8}$ simplifies to $\frac{3}{4}$, making them equivalent fractions.

> Note that dividing the numerator and the denominator by same number (2 in this case) is the same as dividing the fraction by $\frac{2}{2}$, which equals 1. And since dividing by 1 does not change the *value* of a number, we have created another fraction with the *same value* but with a ***different name***.

In addition to being able to simplify fractions using this method, students should also be able to "see" that $\frac{6}{8}$ and $\frac{3}{4}$ represent the same quantity, that they truly are *equivalent* fractions.

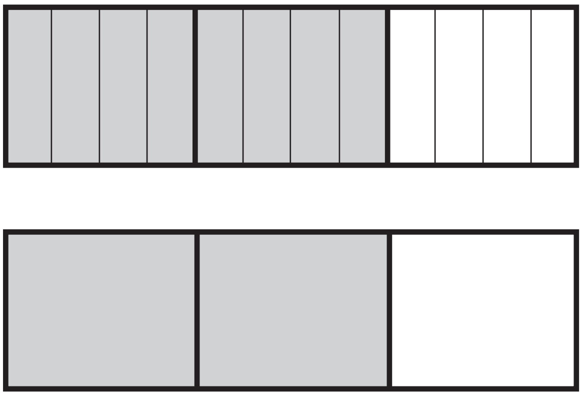




The important thing to understand here is that when a whole is cut into 8 equal parts and we take 6 of them, this is the same as cutting that same whole into 4 parts (half as many parts) and taking 3 of them (taking half as many).

As another example, let's simplify $\frac{8}{12}$ and "see" why our answer is correct.

The largest number that divides 8 and 12 is 4 so we divide 8 by 4 (which equals 2) and we divide 12 by 4 (which equals 3) so $\frac{8}{12}$ simplifies to $\frac{2}{3}$.



In this case, the important thing to "see" is that when a whole is cut into 12 parts and we take 8 of them, this is the same as cutting that same whole into 3 parts (one-fourth as many parts) and taking 2 of them (taking one-fourth as many).

The ability to use fractions in advanced work often depends on the student's ability to use equivalent forms of a fraction. Some students develop this ability to "see" equivalence without much help from the teacher. Most students, however, need specific guidance and deliberate instruction before they have the big "Ah-ha" regarding equivalent fractions.

There are a number of fraction kits and other teaching aids available from various manufacturers and distributors.

Try these:

1) Draw a picture of $\frac{4}{8}$ and draw a picture of $\frac{1}{2}$. Are these equivalent fractions? Explain why or why not.

2) Draw a picture of $\frac{2}{4}$ and draw a picture of $\frac{1}{3}$. Are these equivalent fractions? Explain why or why not.

3) Draw a picture of $\frac{4}{6}$ and draw a picture of $\frac{2}{3}$. Are these equivalent fractions? Explain why or why not.

• Tricks for Halving & Doubling •

Finding Half

When my son was six years old, I asked him, "How much is half of 25?" After thinking about it for about 20 minutes he said, "10, 2 and a half of 1."

> He did this problem by breaking 25 up into 20, 4 and 1 and then splitting each of these in half. When you put all these parts back together, the answer is 12 $\frac{1}{2}$.

This method of:

- breaking a number into easier parts,
- finding half of those parts and
- then putting the parts back together again,

is an important step in a student's mathematical development because many types of problems can be solved this way.

For example, to find half of 248,

- break 248 into 200, 40 and 8,
- find half 200 (100), half of 40 (20) and half of 8 (4) ["Find half of each part."] and
- add 100, 20 and 4. ["Combine (add up) the results."]

Thus, half of 248 is 124.

Doubling

Many students who have little trouble learning

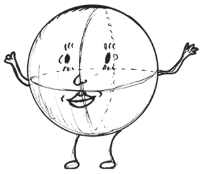
- 1 + 1 = 2, 2 + 2 = 4, 4 + 4 = 8, 8 + 8 = 16,
- 10 + 10 = 20, 15 + 15 = 30, 20 + 20 = 40, 25 + 25 = 50, 40 + 40 = 80 and
- 100 + 100 = 200, 200 + 200 = 400, etc.

have difficulty with

- 16 + 16, 32 + 32, 64 + 64, 128 + 128, 256 + 256, 512 + 512, etc.

Using the technique above (for finding half), these problems can be solved mentally as follows:

- since 16 = 15 + 1, 16 + 16 = (15 + 15) + (1 + 1) = 30 + 2 = 32,
  ["Double the 15 and double the 1 and combine the results."]
- since 32 = 30 + 2, 32 + 32 = (30 + 30) + (2 + 2) = 60 + 4 = 64,
  ["Double the 30 and double the 2 and combine the results."]
- since 64 = 60 + 4, 64 + 64 = (60 + 60) + (4 + 4) = 120 + 8 = 128,
  ["Double the 60 and double the 4 and combine the results."]



- since 128 = 125 + 3,   128 + 128 = (125 + 125) + (3 + 3) = 250 + 6 = 256,
  ["Double the 125 and double the 3 and combine the results."]
- since 512 = 500 + 12,   512 + 512 = (500 + 500) + (12 + 12) = 1,000 + 24 = 1,024, etc.
  ["Double the 500 and double the 12 and combine the results."]

Other Uses

Many problems can be solved using this basic technique.

EXAMPLES:

- **4 × 26**   26 = 25 + 1   so, 4 × 26 = 4 × 25 + 4 × 1 = 100 + 4 = 104
  ["Break 26 into 25 and 1 and multiply 4 times the 25 (100) and 4 times the 1 (4) and combine the results (104)."]

- **5 × 32**   32 = 30 + 2   so, 5 × 32 = 5 × 30 + 5 × 2 = 150 + 10 = 160
  ["Break 32 into 30 and 2 and multiply 5 times the 30 (150) and 5 times the 2 (10) and combine the results (160)."]

- **11 × 25**   11 = 10 + 1   so, 11 × 25 = 10 × 25 + 1 × 25 = 250 + 25 = 275
  ["Break 11 into 10 and 1 and multiply 10 times the 25 (250) and 1 times the 25 (25) and combine the results (275)."]

- **50% of 48**   48 = 40 + 8   so, 50% of 48 = 50% of 40 + 50% of 8 = 20 + 4 = 24
  ["Break 48 into 40 and 8 and find 50% (half) of the 40 (20) and find 50% (half) of the 8 (4) and combine the results (24)."]

Try these:

1) 3 × 23 = _______

2) 5 × 42 = _______

3) 5 × 123 = _______

4) 11 × 40 = _______

5) 25% of 88 = _______

• Comparing Numbers •

The solutions to many problems we study in maths. require us to *compare* numbers. There are two ways to do this:

1) the **Subtraction Method**: this tells us "how much more (or less) one number is than the other," and
2) the **Ratio Method**: this tells us "how many of *these* are there in *that*—how many times bigger/smaller one number is than another"

The Subtraction Method (Comparing by Difference)

As the name implies, the Subtraction Method of comparing numbers involves subtracting the two given numbers. The trick is to state the answer properly. Here's how.

Suppose you are asked to compare 12 to 4.

By subtracting these numbers, we find that 12 is 8 *more than* 4.

If we are asked to compare 4 to 12, again we subtract and we find that 4 is 8 *less* than 12.

Notice that the *order* in which the numbers are given *makes a difference* in the answer we get.

When we compare a larger number to a smaller number, we say the first number is more than the second number.

When we compare a smaller number to a larger number, we say that the first number is less than the second number.

Similarly, when we compare 8 and 6 by the Subtraction Method, we find that 8 is 2 more than 6 and when we compare 6 and 8, we find that 6 is 2 less than 8.

This is one way to compare numbers, by subtracting the numbers and seeing how much bigger or smaller one is than the other. This method is also called the **Difference Method**.

The Ratio Method (Comparing by Division)

The second way to compare numbers is the Ratio Method, "How many of *these* are there in *that*? How many times bigger/smaller is one number than another?"

Suppose you are again asked to compare 12 and 4 but this time you are asked to find out how the *size* of 12 compares to the *size* of 4.

To do this, form a fraction with the first number (12) as the upper part of the fraction (the numerator) and the second number (4) as the lower part of the fraction, like this:

$$\frac{12}{4}$$

Now, divide the bottom number (4) into the top number (12). This gives an answer of 3.

$$\frac{12}{4} = 3$$

This tells us that the size of 12 is *three times* the size of 4. Another way to say this is "the *ratio* of 12 to 4 is 3 to 1." This is often written as 3:1 (12:4 = 3:1).

Now let's compare 4 to 12.

Again, we form a fraction, this time with the first number (4) as the upper part and the second number (12) as the lower part:

$$\frac{4}{12}$$

Since the bottom number *will not* divide into the top number at least 1 time, all we have to do is to simplify this fraction.

So,

$$\frac{4}{12} = \frac{4}{12} \div \frac{4}{4} = \frac{1}{3}$$

shows that the *size* of 4 is *one-third* ($\frac{1}{3}$) the *size* of 12, which can also be said as, "the *ratio* of 4 to 12 is '1 to 3'." This is often written as 1:3.

Notice again that the *order* in which the numbers are given *makes a difference* in the answer we get.

As another example, let's compare 20 and 5 using the **Ratio Method**.

First, let's compare 20 to 5.

To do this, we form the fraction $\frac{20}{5}$. Since 5 goes into 20 exactly 4 times, we find that 20 is 4 times as big as 5.

In terms of ratios, we can say that the ratio of 20 to 5 is "4 to 1," or 4:1.

Now let's compare 5 to 20, again using the ratio method.

This time, we form the fraction $\frac{5}{20}$, which simplifies to $\frac{1}{4}$. So, 5 is one fourth ($\frac{1}{4}$) of 20.

In terms of ratios, we can say that the ratio of 5 to 20 is "1 to 4," or 1:4.

Now let's compare 6 and 8 using the **Ratio Method.**

Let's begin by comparing 6 to 8. We do this by forming the fraction $\frac{6}{8}$. Since 8 will not divide into 6 at least one time, we simply simplify $\frac{6}{8}$ to lowest terms, which gives us $\frac{3}{4}$.

$$\frac{6}{8} = \frac{3}{4}$$

This means that 6 is three fourths ($\frac{3}{4}$) of 8.

In terms of ratios, we say that the ratio of 6 to 8 is "3 to 4," which can be written as 3:4.

Next, let's compare 8 to 6.

This time we get the fraction $\frac{8}{6}$. Notice that 6 does not divide into 8 evenly (that is, with no remainder). In cases where the denominator does not divide evenly into the numerator, there are two things to consider:

First, simplify the fraction to lowest terms, if possible but *do not* change the improper fraction to a mixed number at this point.

In the case of $\frac{8}{6}$, we have $\frac{8}{6} = \frac{4}{3}$ so the ratio of 8 to 6 is "4 to 3," or 4:3.

Second, change the improper fraction to a mixed number. This gives us $\frac{4}{3} = 1\frac{1}{3}$ so we can say that 8 is one and one-third ($1\frac{1}{3}$) times as big as 6.

As a final example, let's compare 5 and 13.

First we will compare 5 to 13. Forming our required fraction, we get $\frac{5}{13}$.

Since $\frac{5}{13}$ cannot be simplified (it is already in lowest terms) this tells us that 5 is five thirteenths ($\frac{5}{13}$) of 13.

This means that the ratio of 5 to 13 is "5 to 13," or 5:13. (Yes, it really is that easy.)

Now let's compare 13 to 5.

The required fraction is $\frac{13}{5}$, which tells us that ratio of 13 to 5 is "13 to 5," or 13:5 and that 13 is two and three-fifths ($2\frac{3}{5}$) times as big as 5.

Try these:

1) Compare 9 and 12 by the Subtraction Method and the Ratio Method.

____

2) Compare 12 and 9 by the Subtraction Method and the Ratio Method.

____

3) Compare 4 and 13 by the Subtraction Method and the Ratio Method.

____

4) Compare 13 and 4 by the Subtraction Method and the Ratio Method.

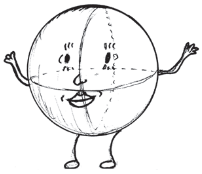
____

5) Compare 8 and 8 by the Subtraction Method and the Ratio Method.

____

6) Compare 100 and 200 by the Subtraction Method and the Ratio Method.

____



• Tips on Computations with Half •

Pencil-and-paper, symbolic computations with fractions should wait until a student has a firm grasp of the concepts involved from a verbal reasoning point of view. The following exercises should be done verbally with children. Notice that I have written out the numbers instead of using symbols (for example, “one half” instead of “$\frac{1}{2}$”).

Addition with half

“One half plus one half equals a whole” is the basic notion required for adding fractions.

Then, “two and one half plus three and one half” can be thought of as:

> “two plus three is five” (**add the whole numbers**),
> “one half plus one half equals a whole” (**add the fractions**) and
> “five and one are six” (**combine the results**).

Note: Make sure that your students hear “two *and* one half.” Many students, especially young ones, mistakenly hear “two halves.”

Subtraction with half

“Seven take away two and one half” is often answered by students as “five and one half” because they subtract the whole numbers and just “bring down” the half. They don’t realise that they have to “break in” to the whole to get the answer. Here’s how to show them the right way to think about this question.

- Hold up seven fingers, five on the left hand and two on the right and say, “Here are seven.”
- Next, put down the two fingers on the right hand and say, “I’ve taken away two so how many are left?” (Five.)
- Then say, “I have five left. Now I’ll take away the half so how much is left?” Now, fold down the first joint of the thumb of the left hand. (Four and one half.)

This demonstration will help students realise that they must “break in” to the whole to get the answer.

Multiplying *by* Half

“Two times three and one half” means “three and one half, two times.” Students frequently do this type of problem by saying “two times three is six and bring over the half so it’s six and one half.”

What they need to have pointed out is that “three and one half, two times” means “three, two times *and* one half, two times.” Saying “$3\frac{1}{2}$, two times” does this; saying “2 times $3\frac{1}{2}$” doesn’t.

Dividing *by* Half

“Three divided by one half” can be thought of as “How many halves are there in three wholes?” or “How many half sandwiches can you make out of three whole sandwiches?”. When phrased this way, really young students in Year 2 or 3 can divide “easy” fractions.

• Order: More Than Half—Less Than Half •

One day I was explaining to my six-year-old how to do something. He said, "Dad, tell me the first thing and then the next thing and then the next thing and then the last thing."

The model he suggested, "first thing…next thing…next thing…last thing," is a good way to learn many types of things.

For example, learning:

red, then
red orange, yellow, then
red orange, yellow, green, blue and finally
red orange, yellow, green, blue, indigo, violet,

in stages over a period of time is a good way to learn the basic order of the colors of the rainbow.

I have added other models to the one above:

- first thing..last thing
  (**First-Last analysis: the Big Picture Model**)

- first thing.................middle thing.................last thing (adding some detail)

- first thing...less than half...middle thing...more than half...last thing (more detail)

The general model is:

first thing.........thing.........thing.......thing..........*last* thing
primary – intermediate – middle – intermediate– final

Example: Arrange in order from *smallest* to *largest*: 2, 1, $3\frac{1}{2}$, 0, 3, $1\frac{1}{2}$, 4:

- First establish the smallest and the largest numbers in the group, the endpoints. (0 and 4)

- Next establish what number is in the middle*, the midpoint.

  (In this example, the midpoint is 2, which happens to be in the group to be ordered. While this will not always be the case, it is always important to identify the middle.)

- For the remaining numbers (1, $3\frac{1}{2}$, 3, $1\frac{1}{2}$) ask, "Is $1\frac{1}{2}$ between 0 and 2 or it is between 2 and 4? How about $3\frac{1}{2}$?"

Thus, the proper order is: 0, 1, $1\frac{1}{2}$, 2, 3, $3\frac{1}{2}$, 4.

* To find the midpoint between two numbers, add the numbers together and divide by 2. In this example, we want the middle between the smallest number (0) and the largest (4) so $(0 + 4) \div 2 = 2$.

- Another good way to address a question like this is to draw on the student's life experiences:

  "0 is when you were born. 4 is when you were 4 years old. First you were 0, then you were 1…how old were next?…next?…next?…"

Example: Arrange in order from smallest to largest: $1, \frac{1}{2}, \frac{5}{6}, \frac{3}{8}, 0$

- (Proper) Fractions are numbers that are greater than 0 and less than 1 so 0 is the smallest and 1 is the largest of this group. (The midpoint in this example is $\frac{1}{2}$ because $(0 + 1) \div 2 = \frac{1}{2}$.)

- $\frac{5}{6}$ is greater than $\frac{1}{2}$ because:

  Half of 6 is 3 so $\frac{3}{6}$ is another name for $\frac{1}{2}$.

  Since 5 sixths is greater than 3 sixths, $\frac{5}{6}$ is greater than $\frac{1}{2}$.

- Similarly, $\frac{3}{8}$ is less than $\frac{1}{2}$ because:

  Half of 8 is 4 so $\frac{4}{8}$ is another name for $\frac{1}{2}$.

  Since 3 eighths is less than 4 eighths, $\frac{3}{8}$ is less than $\frac{1}{2}$.

So, the proper order for these fractions is: $0, \frac{3}{8}, \frac{1}{2}, \frac{5}{6}, 1$.

More advanced methods for comparing and ordering can be developed by building on the foundation and framework established here.

Try these:

1) Arrange in order from *smallest* to *largest*: $0, 1, \frac{7}{10}, \frac{7}{16}, \frac{1}{2}$.

____

2) Arrange in order from *smallest* to *largest*: $0, 1, \frac{3}{11}, \frac{7}{8}, \frac{1}{2}$.

____

3) Arrange in order from *smallest* to *largest*: $0, 1, \frac{5}{9}, \frac{5}{11}, \frac{1}{2}$.

____

4) Arrange in order from *smallest* to *largest*: $0, 1, \frac{18}{19}, \frac{1}{11}, \frac{1}{2}, \frac{6}{13}, \frac{17}{18}$.

____

• How to Name Common Fractions •

Whole Number:

A **WHOLE NUMBER** is a number without a *fractional part*—*no* COMMON or DECIMAL fraction.

EXAMPLES: **1, 12, 667, 999**

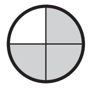
Notice that *whole numbers* come from counting by 1s.

Fraction:

A **FRACTION** is a *part* of a *whole*.

The *denomination* (the **NAME**) of a fraction is the *number* of **EQUAL** parts in the whole.

EXAMPLE: If a whole is broken into four equal parts, the *denomination* (the **NAME**) of the fraction is FOURTHS. If we count *three* of these parts, then we have created the fraction *three*–FOURTHS $\left(\frac{3}{4}\right)$.



Unity:

A **FRACTION** whose value is *equal* to **one whole** (1)..

EXAMPLES: $\frac{2}{2}$, $\frac{4}{4}$, $\frac{9}{9}$, **1**

Notice that the *numerator* (the top number) is *equal* to the *denominator* (the bottom number).

Proper Fraction:

A **FRACTION** whose absolute value is *greater than* or *equal* to **zero** (0) and is *less* than **one whole** (1).

EXAMPLES: $\frac{1}{2}$, $\frac{5}{9}$, $\frac{3}{4}$, $\frac{19}{20}$

Notice that the *numerator* (the top number) is *less* than the *denominator* (the bottom number).

Improper Fraction:

A **FRACTION** whose absolute value is *greater* than **one whole** (1).

EXAMPLES: $\frac{3}{2}$, $\frac{9}{4}$, $\frac{23}{12}$, $\frac{17}{9}$

Notice that the *numerator* (the top number) is *greater* than the *denominator* (the bottom number).

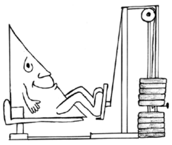
Mixed Number:

A number that is a *mixture* of a **whole number** and a **fraction**.

A **MIXED NUMBER** is another name for an **improper fraction**.

EXAMPLES: $3\frac{1}{2}$, $2\frac{3}{4}$, $5\frac{7}{9}$, $1\frac{3}{11}$

Notice that the value of a *mixed number* is always *greater* than 1.



• Reflective and Critical Questions •

Complete the following.

1) Complete all the "***Try these***" questions on the following pages:
Page 26: The Concept of Complements
Page 29: The Concept of Fractions
Page 31: Simplifying Fractions
Page 33: Tricks for Halving & Doubling
Page 36: How to Compare Numbers
Page 39: Order: More Than Half — Less Than Half

2) A fraction whose value is greater than 0 but less than one whole (1) is a

__________________ fraction.

3) How would you explain what a numerator and denominator are to a child? How should you **NOT** explain it?

____

____

____

4) Describe two different number sense approaches to find a quarter of 632.

____

____

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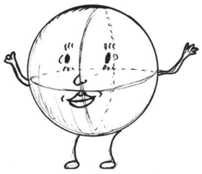


Shadowing
&
Progress Check

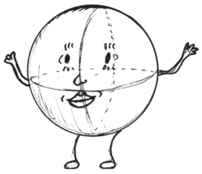
• Shadowing •

Shadow an Instructor. Identify the processes, systems and instruction that is used on the floor.

Objectives:

1. Observe a student's Workout Plan being filled out at the beginning and end of their session.
2. Recognise how Instructors communicate on the instruction floor.
3. Recognise Proactive Engagement and Disengagement.
4. Describe how you saw the student reward process implemented.
5. Identify various modes of teaching (mental, visual, verbal, tactile, written) used by Instructors.

If you observed an Instructor using one of the Mathnasium Teaching Constructs, describe that experience. What construct was it? What language did you hear?



• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

Instructor Development/First Look

	I have a good understanding of:				
1.	why the construct language is so important.	**0**	**1**	**2**	**3**
2.	methods used to keep a student actively engaged.	**0**	**1**	**2**	**3**
3.	when it is appropriate to intervene while a student is working.	**0**	**1**	**2**	**3**
4.	comparing numbers by the subtraction method and ratio method.	**0**	**1**	**2**	**3**
5.	why we don't call the numerator the "top number" and the denominator the "bottom number".	**0**	**1**	**2**	**3**
6.	how to order fractions by comparing them to a half.	**0**	**1**	**2**	**3**

Shadowing

	I saw examples of:	YES	NO
1.	a student's Workout Plan being filled out.		
2.	Instructor communication on the floor.		
3.	Instructors proactively engaging with students.		
4.	Instructors disengaging from students and leaving them with a meaningful task.		
5.	the student Rewards System; e.g, getting punches on their cards and/ or trading the cards in for something in the Rewards Cabinet.		
6.	Instructors using the Mathnasium Teaching Constructs.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

Proactive Teaching (section 8, p. 5)
Mathnasium Constructs (Proportional Thinking, Quantity & Denomination, The Law of SAMEness) (section 8, p. 13)
Shadowing Experience (section 9)

Notes:

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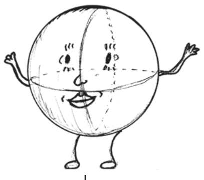
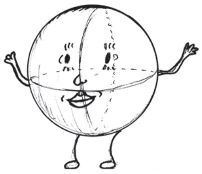


Instructor Development
&
A First Look (Part 3 & 4)



Instructor Development

Larry's Do's and Don'ts

Summarise Larry's tips and tricks on Instruction

•Larry's "Do's" and "Don'ts" for Instructors•

Do's:

1) **Do** leave "blank spots" in your conversations with students so they can "fill in the blank."

When doing "7 + 9" by using the *Up to 10, Over 10* method:

> "So Josh, let's see, 7 plus 3 equals ______...(Josh responds "10")... and 10 plus 6 equals ______ (Josh responds "16")."

instead of

> "So Josh, let's see, 7 plus 3 equals 10, right ... and 10 plus 6 equals 16."

If Josh can't handle "7 + 3" on his own, then he is not ready for "7 + 9."

2) **Do** give students a few seconds to self correct when they give an incorrect answer. Prompt when you think it is appropriate.

When doing "24 + 23" mentally:

> the student might say "74" instead of "47."

Prompt the student by saying:

> "Think about that answer. Is the answer greater than or less than 50?"

Most of the time the student will self correct.

3) **Do** "Extend Knowledge" whenever possible.

50% of 20 = ___, 25% of 20 = _____, 20% of 20 = _____. (Y^{ea}r 5 and up)

Many students know that 50% means "**half** and that you divide by 2 to get the answer," and that 25% means "a **fourth** and that you divide by 4 to get the answer," but have no idea how and why they know these facts. As a result, they don't how to do "20% of 20 = _____."

> Ask, "How many times does 50 go into 100 and how many times does 25 go into 100?"

Most students then realise that this is why divide by 2 for 50% and by 4 for 25%. Now ask, "How many times does 20 go into 100?" A typical response is, "Oh so I divide by 5 to get 20%!"

Finally, ask, "How much is 10% of 20... how much is 5% of 20?"

Dont's:

1) **Don't** use mathematical language and symbols and expect that they will immediately have "inherent and obvious" meaning to the student.

   Before talking to a student about **HIGHEST COMMON FACTOR**, make sure the student knows what a **FACTOR** is.

   In Algebra, using "$f(x)$" instead of "y" confuses many students. The phrase "is a function of" needs to be thoroughly explained.

   Since we work one–on–one with students, we have the luxury of time to explain things in terms students understand.

2) **Don't** assume that correct handling of manipulatives automatically guarantees that the student understands the underlying maths.

3) **Don't** use verbal forms that are ambiguous or tentative.

   Better than
   "Turn to Section 3, okay?"
   is
   "Turn to Section 3, please."

   It isn't like the student has a choice. Ask them politely to comply with your request. **NOTE: if your tone of voice is "just right," you can get away with using "okay" and not have it sound like a question.**

   Better than
   "We are just going to do some long division problems."
   is
   "It's time to do some division."

   "Just" has no place in this sentence. The teacher is trying to ease the pain of doing division but "just" doesn't make the work any easier.

   Better than
   "Do you think you can tell me what half of 11 is?"
   is
   "How much is half of 11?"

   Phrases like "Do you think you can do..." and "Can you tell me what..." are mentally distracting to the student and are "too many words."

• Larry's "Do's" and "Don'ts" Questions•

Complete the following after reading Larry's Do's and Don'ts for Instructors.

1) Which of Larry's Do's and Don'ts do you think you need the most development? Why?

____

____

____

____

____

2) The words "just", "easy" and "simple" are commonly used. Why could these words be harmful when working with a student?

____

____

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3) Describe a situation where a student may be able to use manipulatives correctly but does not understand the underlying concept.

____

____

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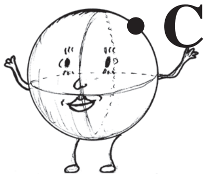


A First Look at Important Elements of the MATHNASIUM® Curriculum

Part 3

* These pages have ***Try These*** questions to complete

• Place Value: Whole Numbers & Decimals •

Our digits—0, 1, 2, 3, 4, 5, 6, 7, 8, 9—are called Hindu-Arabic numerals, in honor of the Hindus who devised them and the Arabs who put them into widespread use.

These ten digits are the only symbols used in our Decimal Number System (the Base 10 system) and can be used to represent any whole number or fractional quantity.

Whole Numbers

Whole numbers are formed by counting in 1s,

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

and writing each digit in the 1s place. When we run out of digits (when we have used all ten digits in order), we introduce a new place, the 10s place, put a 1 in it (standing for "one 10") and put a 0 in the 1s place, producing the new symbol 10—"1 ten and 0 ones."

Now, we continue to count in 1s in the 1s place,

10, 11, 12, 13, 14, 15, 16, 17, 18, 19,

and, when we again run out of digits, we increase the 10s place by 1, reset the the 1s place to 0 and continue to count by 1s:

20, 21, 22, 23, 24, 25, 26, 27, 28, 29.

This pattern repeats:

30, 31, 32, 33, 34, 35, 36, 37, 38, 39,
40, 41, 42, 43, 44, 45, 46, 47, 48, 49,
50, 51, 52, 53, 54, 55, 56, 57, 58, 59,
60, 61, 62, 63, 64, 65, 66, 67, 68, 69,
70, 71, 72, 73, 74, 75, 76, 77, 78, 79,
80, 81, 82, 83, 84, 85, 86, 87, 88, 89
90, 91, 92, 93, 94, 95, 96, 97, 98,

until we get to 99. Now we introduce the symbol 100 (one hundred, "1 hundred, 0 tens and 0 ones,") and continue to count by 1s. (Notice that every time we count 10 something new happens—we enter the 20s, 30s…the 100s…200s…)

Notice the pattern of 1, 10, 100:

- 1, 10, 100 (**ones**: one, ten [ones], hundred [ones]),
- 1,000, 10,000, 100,000 (**thousands**: one thousand, ten thousand, hundred thousand),
- 1,000 000, 10,000,000, 100,000,000 (**millions**: one million, ten million, hundred million).

This pattern is repeated over and over, producing each whole number.

The Decimal Point and Decimal Fractions

Now we introduce the **decimal point**, a marker that separates the whole numbers (on the left) from the fractions (on the right):

____ , ____ ____ ____ . ____ ____ ____

whole numbers **fractions**

The place before the decimal point is the ones (1s) place, $\frac{1}{1}$.

____ , ____ ____ 1s . ____ ____ ____

whole numbers **fractions**

The first place after the decimal point is the tenths ($\frac{1}{10}$s) place. The second place is the hundredths ($\frac{1}{100}$s) place.

Again, the pattern of 1, 10, 100 occurs, this time in fraction form: $\frac{1}{1}$, $\frac{1}{10}$, $\frac{1}{100}$.

Next comes $\frac{1}{1{,}000}$, $\frac{1}{10{,}000}$, $\frac{1}{100{,}000}$, then $\frac{1}{1{,}000{,}000}$, $\frac{1}{10{,}000{,}000}$, $\frac{1}{100{,}000{,}000}$, etc.

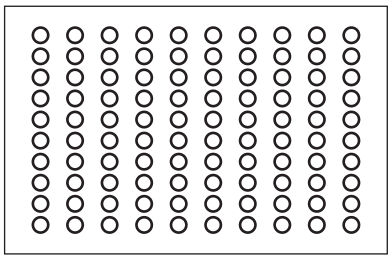
It is critically important that the student understands the meaning of fractions (a whole "fractured" into equal parts) and the concepts of **proper** and **improper** fractions and **unity**, before being introduced to the decimal representation of fractions.

Being able to identify 0.3 as $\frac{3}{10}$ without knowing what $\frac{3}{10}$ *is*, is a tragedy. Don't let this happen to your students.

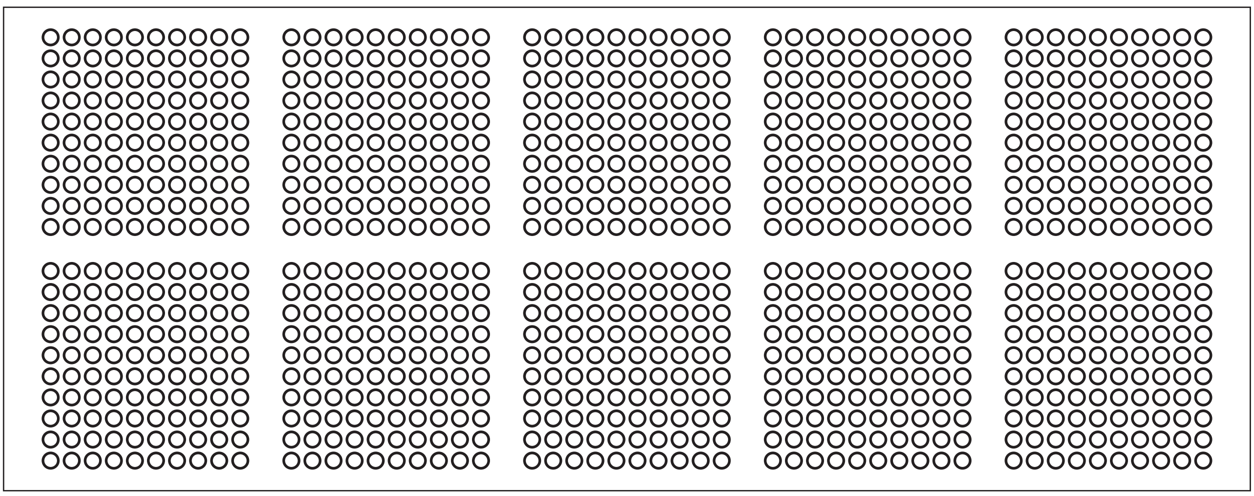
• Thinking in **1s, 10s, 100s & 1,000s** •

1) How many **1s** make 10? (Circle groups of 1 below.) _____

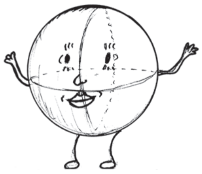
2) How many **10s** make **100**? (Circle groups of 10 below.) _____



3) How many **100s** make **1,000**? (Circle groups of 100 below.) _____



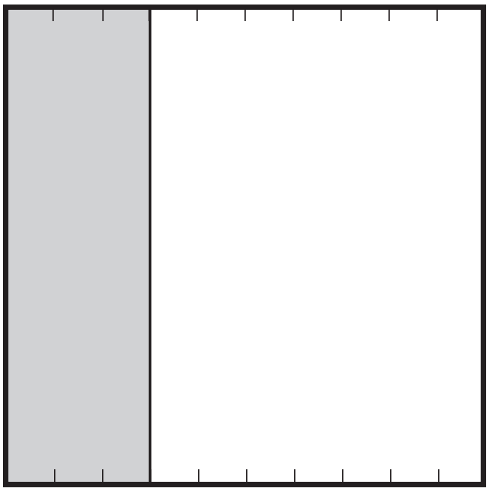
4) How many **10s** make **1,000**? _____



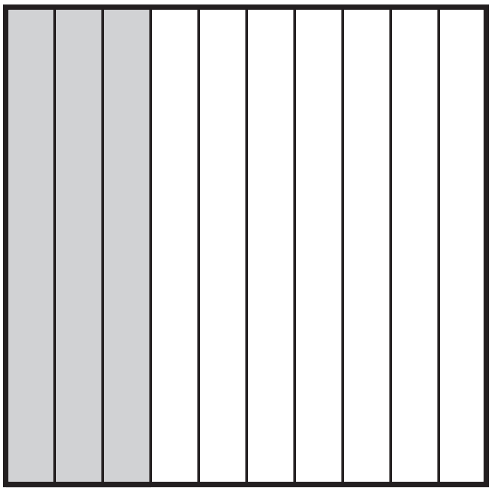
• Naming **Decimal Fractions** •

EXAMPLE 1:

How much of the square is shaded?



Divide the square into 10 equal parts.

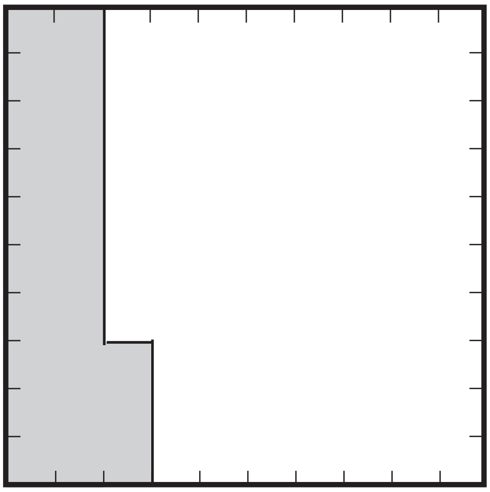


Since 3 out of 10 equal parts are shaded, **3 tenths** of the square is shaded.

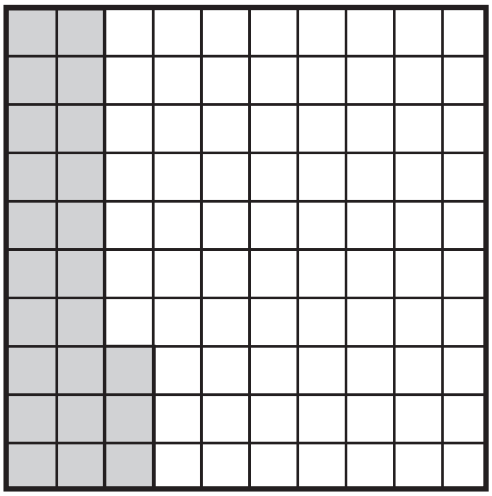
This can be written as the **common fraction** $\frac{3}{10}$ and as the **decimal fraction 0.3.**

EXAMPLE 2:

How much of the square is shaded?



Divide the square into 100 equal parts.



Since 23 out of 100 equal parts are shaded, **23 hundredths** of the square is shaded.

This can be written as the **common fraction** $\frac{23}{100}$ and as the **decimal fraction 0.23.**

• The Decimal Number System •

___	,	___	___	___	,	___	___	___	,	___	___	___	.	___	___	___
1		100	10	1		100	10	1		100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
billions			**millions**				**thousands**				**ones**			**tenths**	**hundredths**	**thousandths**

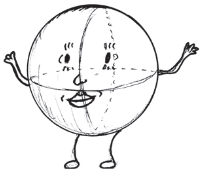
10 ones = 10
10 tens = 100
10 hundreds = 1,000
(1,000 ones = 1 thousand)

10 thousands = 10,000
10 ten thousands = 100,000
10 hundred thousands = 1,000,000
(1,000 thousands = 1 million)

10 millions = 10,000,000
10 ten millions = 100,000,000
10 hundred millions = 1,000,000,000
(1,000 millions = 1 billion)

10 thousandths ($\frac{10}{1,000}$) = $\frac{1}{100}$
10 hundredths ($\frac{10}{100}$) = $\frac{1}{10}$
10 tenths ($\frac{10}{10}$) = 1 whole
100 hundredths = 1 whole
1,000 thousandths = 1 whole

General Rule: 10 of a smaller unit makes the next biggest unit.



• "Per Cent" •

Per Cent is an appropriate topic for 2^{nd} and Y^{ea}r 3 students if it is presented as an outgrowth of Counting, Wholes and Parts and Proportional Thinking.

Per Cent means "for each 100, " "parts per hundred," "how many for each hundred":

per (for each) + *cent* (hundred, as in *cent*ury, *cent*s...).

Year 2 and Year 3 students quickly learn that 100% is the "whole thing," "all of it."

- "100% orange juice" means "made *only* of oranges."
- "100% right" means "*all* right, *none* wrong."

Kids who have learned to mentally compute half of various quantities have no trouble learning that 50% means "half of it." After all, 50 is half of 100!

- "50% off" means "*half* price."
- "50% right" means "*half* right, *half* wrong."

Similarly, 25% means "a quarter," since it is "half of a half," and 25 is half of 50.

Year 3 and Year 4 students who have learned to count by groups and who know that percentage means "how many for each 100" can apply that knowledge to solve problems like:

- "How much is 15% of 300?" (Count 15 for each hundred…there are 3 hundreds…15 + 15 + 15 = 45.)

- "How much is 5% of 600?" (Count 5 for each hundred…there are 6 hundreds…5 + 5 + 5 + 5 + 5 + 5 = 30.)

- "How much is 12% of 250?" (Count 12 for each hundred…there 2 whole hundreds and half of a hundred (200 + 50)…12 + 12 + 6 = 30.)

Notice that all of these questions can be done verbally, without ever using pencil and paper.

Thoughts on the Numbers 0 and 1

The number 0 (zero) counts "none." When we speak of 0% chance, we mean, "not a chance, *no* way."

"There is a 0% chance that pigs will fly on their own."

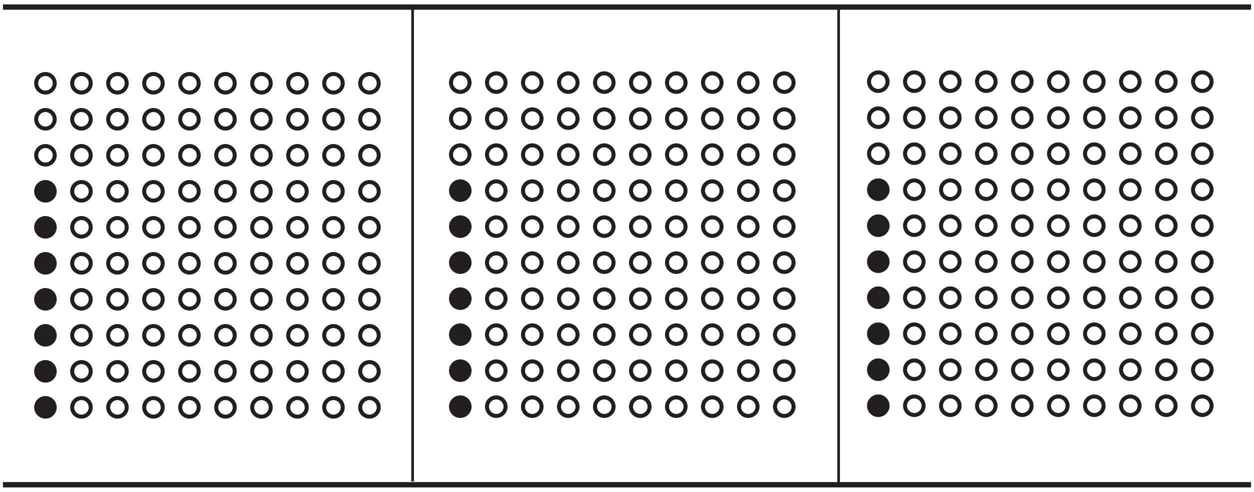
The number 1 (one) represents the whole (all) of a thing. When we refer to 100% (all of it, 1 whole), we are saying, "absolutely certain, can't be any other way."

"There is a 100% probability that every living thing will someday die."

Thus 0 and 1 represent the extremes, "none" and "all."

• The Meaning of **Per Cent** •

Per Cent means "**for each**"-"**100**"

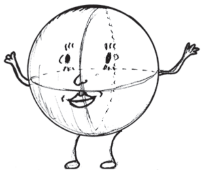


To find **7% of 300**, count
7 for each **100**.

Since **300** is made up
of **3** ***hundreds***,
count

7 for the *first* hundred,
7 for the *second* hundred and
7 for the *third* hundred.

7 + 7 + 7 = 21.
So, 7% of 300 = **21**.



Reverse Questions

Reverse questions involving per cent take the form, “10% of what number is 40?”

To answer this question, remember that per cent means “for each 100,” so 10% means “count 10 for each 100.” Let’s see, 10 for the first 100, 10 for the second 100 (20 so far), 10 for the third 100 (30 so far), 10 for the fourth 100 (ah-ha, 40). To get 40, we had to count “10 for each 100” *four* times so the number we are looking for is 400.

So, 10% of *400* is 40.

Let’s do “12% of what number is 30?”

Let’s see, 12 for the first 100, 12 for the second 100 (24 so far), 12 for the third 100 (36, oops, too much). From 24, we only need 6 more and since 6 is half of 12, we don’t take a whole 100, we only take half of 100, 50. So, to get 30 we need the first 100, the second 100 and half of the third 100, making a total of 250.

Thus, 12% of *250* is 30.

When students have mastered this technique and have a basic understanding of fractional parts, they are ready for the usual pencil-and-paper work on percentages.

Try these:

1) Find 8% of 300. __________

2) Find 8% of 250. __________

3) Find 30% of 400. __________

4) Find 30% of 50. __________

5) Find $\frac{1}{2}$% of 300. __________

6) 10% of what number is 20? __________

7) 5% of what number is 20? __________

8) 7% of what number is 35? __________

9) 20% of what number is 70? __________

10) 9% of what number is $22\frac{1}{2}$? __________

• Fractional Parts •

After a student can name fractions and is able to verbally answer questions like,

> "How much is half of 10?…5?…19?…36?…49?…99?…246?…999?…1,000?," and
> "How can you share 12 donuts evenly with 3 kids?…6 kids?…4 kids?",

fractional parts can be introduced.

What are fractional parts?

A whole divided into equal parts produces **fractional parts.**

6 is a **fractional part** (one half) of 12.

How do you find fractional parts?

One fourth ($\frac{1}{4}$) of 12 equals 3, because when 12 is divided into 4 equal parts, each part equals 3. In this case, 3 is a fractional part ($\frac{1}{4}$) of 12. Notice we divide by the *denominator*.

Since *one* fourth of 12 equals 3, *two* fourths of 12 equals one fourth, twice. Thus, $\frac{2}{4}$ of $12 = 3 + 3 = 6$.

Then, *three* fourths of 12 equals *one* fourth, three times. Thus, $\frac{3}{4}$ of $12 = 3 + 3 + 3 = 9$.

> **Finding Fractional Parts**
>
> - find one part (by dividing the whole into parts) and
> - use that part the required number of times.

Example: Find $\frac{2}{3}$ of 12.

First, think of 12 divided into 3 equal groups, "3 parts the same". ○ ○ ○ ○ | ○ ○ ○ ○ | ○ ○ ○ ○
(4 in each group)

Then, count how many there are in 2 groups. ● ● ● ● | ● ● ● ● | ○ ○ ○ ○
(2 groups of 4 = 8)

Example: How many hours are there in $\frac{3}{4}$ of a day?

> Since a whole day is 24 hours, one fourth ($\frac{1}{4}$) of a day equals 6 hours ($24 \div 4$).
> Then, three fourths of 24 equals 6, three times—18. So, $\frac{3}{4}$ of a day equals 18 hours.

Try these:

1) $\frac{2}{3}$ of 24 = ____ + ____ = ____

3) $\frac{3}{8}$ of 16 = ____ + ____ + ____ = ____

2) $\frac{3}{4}$ of 20 = ____ + ____ + ____ = ____

4) $\frac{3}{3}$ of 60 = ____ + ____ + ____ = ____

• A Sense of Scale •

It takes many years of training for most students to develop a “sense of scale.” What is “scale”?

The word “scale” has to do with the *relationship* between things. For example, the ***m*** and the ***s*** in the word “mathematics” below are out of scale (that is, their sizes are inconsistent with the uniform size of the rest of the letters).

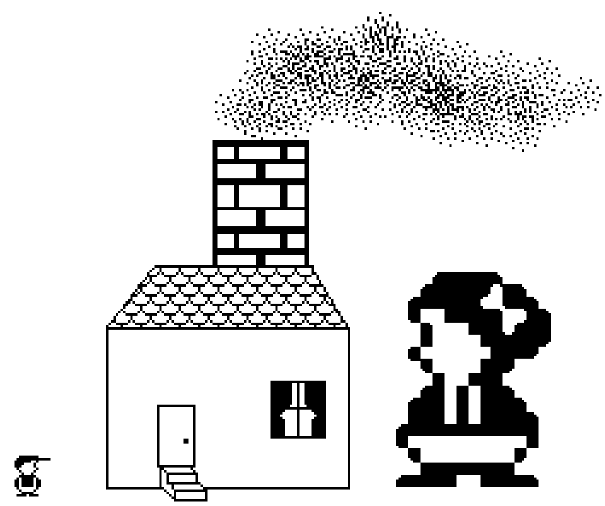
mathematics

Most children have an intuitive sense of scale, as is shown when they do “What’s wrong with this picture?”-type activities. Development can be quickened by asking questions like:

- If the size of your house was simplified to one foot, how tall would you be?
- If the world was the size of a basketball, how big would you be?

A more formal and precise understanding of scale can be developed from this intuitive understanding by teaching the child about ratio and proportion.

In the following picture, there are two things that are grossly out of scale. See if you can find them.



by Nick Martinek, © 1991

Find two things that are “out of scale.”

• Ratio and Proportion •

Ratio

In mathematics, we use **ratio** to mean "a comparison between two numbers *by division*." (See "Comparing Numbers," page 34 from the previous section.)

Generally, there are two ways we compare things:

1) part to part
2) part to whole.

If a team won 8 games and lost 4 games (12 games total), we can compare this information as follows:

First, a "part to part" comparison tells us they won twice as many games as they lost (8 to 4 $= \frac{8}{4} = \frac{2}{1} = 2$) and we can say they lost half as often as they won (4 to 8 $= \frac{4}{8} = \frac{1}{2}$) or that they won twice as many games as they lost.

Now, a "part to whole" comparison tells us they won two thirds of the 12 games they played (8 out of 12 $= \frac{8}{12} = \frac{2}{3}$) and we can say they lost one third of the 12 games they played (4 out of 12 $= \frac{4}{12} = \frac{1}{3}$).

If a recipe calls for 4 eggs and 6 cups of milk and will make a dish that serves 8 people, then we can form the following ratios:

eggs to milk = 4 to 6 $= \frac{4}{6} = \frac{2}{3}$ (part to part),

milk to eggs = 6 to 4 $= \frac{6}{4} = \frac{3}{2}$ (part to part),

eggs to people = 4 to 8 $= \frac{4}{8} = \frac{1}{2}$ (part to whole) and

milk to people = 6 to 8 $= \frac{6}{8} = \frac{3}{4}$ (part to whole).

Try these:

A box contains 2 red marbles, 4 white marbles and 6 blue marbles. In each exercise, form the ratio. Tell whether each ratio is *part* to *part* or *part* to *whole*. Interpret each answer in words.

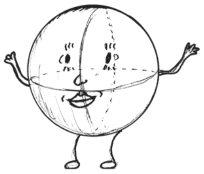
1) red to white ______________________________

2) blue to total ______________________________

3) blue to white ______________________________

4) red to blue ______________________________

5) red to total ______________________________



Proportion

When things are in **proportion**, the relationship between the parts stays the same even though one or more of the parts changes in value. Allow me to explain, again by example.

> If a recipe calls for 4 eggs and 500 millilitres of milk and will make a dish that serves 8 people, then for half as many people (4), you will need half as many eggs and half as much milk. For twice as many people, you will need twice as much in the way of ingredients.

The key feature here is that the *relationships* between the ingredients and the number of people and between the amounts of the ingredients themselves, *stay the same*.

Notice that: 4 eggs for 8 people (4 for 8 = $\frac{4}{8} = \frac{1}{2}$),
2 eggs for 4 people (2 for 4 = $\frac{2}{4} = \frac{1}{2}$) and
8 eggs for 16 people (8 for 16 = $\frac{8}{16} = \frac{1}{2}$),

all produce the same fraction, $\frac{1}{2}$ and the same relationship, 1 unit of eggs for every 2 people.

Similarly, 500 millilitres of milk for 8 people (500 for 8 = $\frac{500}{8} = \frac{125}{2}$),
250 millilitres of milk for 4 people (250 for 4 = $\frac{250}{5} = \frac{125}{2}$) and
1,000 millilitres (or 1 litre) of milk for 16 people (1000 for 16 = $\frac{1000}{16} = \frac{125}{2}$),

all produce the same fraction, $\frac{125}{2}$ (125 units of milk for every 2 people).

This is what **proportion** means: as the amount of one thing changes, both the relationship between the object and the whole and the relationship between the object and the other parts, stay the same.

Example: For a team that wins two thirds of its games, if the proportion of wins and losses stays the same, they will win 20 out of 30, 40 out of 60, 120 out of 180, because

$$\frac{20}{30} = \frac{40}{60} = \frac{120}{180} = \frac{2}{3}.$$

Try these: **Solve these exercises using Mental Math.**

1) Three cups of rice will serve 5 people. How much rice is needed to serve 10 people?

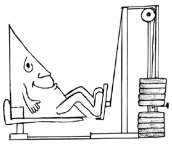
____

2) On a map, 4 centimetres represent 100 kilometres. How many kilometres does one meter represent?

____

3) A certain recipe calls for 4 eggs and 500 millilitres of milk to make a dish for 8 people. How many eggs and how much milk are needed for only 2 people?

____



• Reflective and Critical Questions •

Complete the following.

1) Complete all the "***Try these***" questions on the following pages:

Page 15: Per Cent
Page 16: Fractional Parts
Page 18: Ratio
Page 19: Proportion

2) Explain how you found the answer to the following ***per cent*** questions:

a) Find 8% of 250. __________

____

____

____

b) 7% of what number is 35? __________

____

____

____

3) Pick one of the three questions on the previous page (p. 19) and explain how you used mental maths. to solve it.

____

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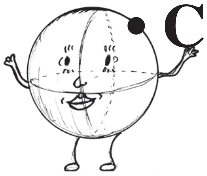


A First Look at Important Elements of the Mathnasium® Curriculum

Part 4

* These pages have ***Try These*** questions to complete

• Three Ways to Write a Number •

Every whole number and every fraction can be written three ways:

• as a *common fraction*, • as a *decimal fraction*, • as a *percentage*.

For example, **one half** can be written:

- $\frac{1}{2}$ (the common fraction),
- 0.50 (the decimal fraction),
- 50% (the percentage).

Why are these the same? Let's see.

100% means "all of it," so 50% is "half of it." Thus, 50% = $\frac{1}{2}$.
Now, 50% also means "50 hundredths," and the decimal for $\frac{50}{100}$ is 0.50.
Also, $\frac{50}{100}$ simplifies to $\frac{1}{2}$ so $\frac{1}{2} = 0.50 = 50\%$.

How about **one fourth** (a **quarter**)?

Well, a quarter ($\frac{1}{4}$) is half of $\frac{1}{2}$ so $\frac{1}{4}$ = half of 50% = 25% and half of 0.50 = 0.25.
So, $\frac{1}{4} = 0.25 = 25\%$.

Three fourths is one fourth, 3 times.

$\frac{3}{4} = 3 \times 25\% = 75\%$ and
$= 3 \times 0.25 = 0.75$.

So, $\frac{3}{4} = 0.75 = 75\%$.

When first explaining these things to students, do it *verbally*.

When a student can *verbally* explain the three representations of one half, one fourth and three fourths, the student is ready for more formal, *written* work.

Try these:

1) Write $\frac{1}{10}$ as a *decimal* and as a *percentage*. ______________________

2) Write 20% as a *decimal* and as a *fraction*. ______________________

3) Write .45 as a *percentage* and as a *fraction* ______________________

• Rate •

Phrases like "50 kilometres per hour," "21 kilometres per litre," and "3 for a quarter" are common in everyday life. Each of these involves a *rate*. The underlying concept of rate is that

> something happens **for each** or **in each** ("**per**") occurrence of something else.

"50 kilometres per hour" means "50 kilometres are travelled *in each* hour."
"21 kilometres per litre" means "21 kilometres can be driven *for each* litre."
"3 for a $1" means "3 things can be purchased *for each* dollar."

It is often assumed that students intuitively understand the meaning of rate. Many don't. Ask them.

Verbal questions like:

"At 50 kilometres per hour, how far will you go in 3 hours?"
"A car gets 21 kilometres per litre. How far can it go on 10 litres of petrol?" and
"If 3 candies cost a $1, how many candies can you buy for $10?"

will solidify the notion of rate in your students' minds.

Try these: **Rephrase each of the following using "for each" or "in each":**

1) "55 kilometres per hour" 55 kilometres are travelled in each hour.

2) "5 for a dollar" ______________________________

3) "30 kilometres per litre" ______________________________

4) "$12.00 per hour" ______________________________

• Relationships: Direct & Inverse •

When two quantities form a **whole**, one of two relationships will hold:

1) **Direct Relationship**:

As one quantity gets larger, the other quantity gets larger by *the same amount*.

As one quantity gets smaller, the other quantity gets smaller by *the same amount*.

EXAMPLES: The *more* steps you take, the *further* you have walked.

The *less* water you pour into a cup, the *emptier the* cup is.

2) **Indirect (Inverse) Relationship**:

As one quantity gets larger, the other quantity gets smaller by *the same amount*.

As one quantity gets smaller, the other quantity gets larger by *the same amount*.

EXAMPLES: The *more* candy you buy, the *less* change you will get from a $10 note.

The *smaller* the steps you take, the *more* steps it will take to cross a room.

Compare with "The Concept of Complements" on page 26 (previous section) and "Variation: Direct & Inverse" on page 25.

Try these: **State whether each of the following are *directly* or *indirectly (inversely)* related.**

1) The relationship between the amount of cereal eaten and the amount of cereal left in the box.

____

2) The relationship between number of kilometres driven and number of kilometres from the starting point.

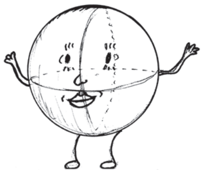
____

3) The relationship between number of kilometres driven and number of kilometres left on your trip.

____

4) The relationship between amount you spent and amount of change received from a $10 note.

____



• Variation: Direct & Inverse •

When two related quantities change, one of two relationships will hold:

Direct Variation:

The ratio of the changing quantities remains constant. **Direct variation** occurs for changing quantities x and y when $\frac{y}{x} = k$, for a constant value k. **Direct variation** between two quantities x and y can also be expressed as $y = kx$, for some constant value k.

Examples: Suppose you buy x number of chocolate bars and each chocolate bar costs $1.50

The total cost of the chocolate bars y cand be written using a **direct variation** equation $y = (1.50)x$.

The *more* candy bars you buy, the *more* it costs.

The *fewer* you buy, the *less* it costs.

Compare with "The Concept of Complements" and "Relationships: Direct & Indirect".

Try these: **State whether each of the following are *directly* or *indirectly (inversely)* related.**

1) The higher the interest rate, the more money you can earn on a fixed amount of principal.

2) The higher the interest rate, the less time it will take to earn a given amount on a fixed amount of principal.

3) The cheaper the price, the more chocolate bars you can buy with a fixed amount of money.

4) For a rectangle with a fixed length, the shorter the width of the rectangle, the smaller the area.

• Zero and Negative Numbers •

Zero: nothing to it.

Zero is the first number in our counting system. What does it count?

> Hold up three fingers. Now make a fist. The first time you displayed the number 3. The second time you displayed the number 0. Zero counts *none*.

Often thought of as only a “placeholder,” zero has many other roles in the real world:

> When you read a ruler, the first position on the ruler is not 1, it is 0. Zero is the *beginning*: it indicates your starting point.
>
> If you overdraw your checking account, you pass through the gateway of zero into the infinite void of negative numbers (see below). Zero is the *centre*: from there you can go in one direction (positive) or the other (negative).
>
> Countdown to blast off: …3, 2, 1, 0. Zero is the *end*: it indicates when you’re done.

Some people say that zero is “not a number.” It is indeed a number, the number that counts **none** and the number that gives you a sense of direction on the number line.

Negative Numbers

Negative numbers are the numbers that are **less than 0**.

Beginning in the first year of school, students should be aware of the existence of negative numbers.

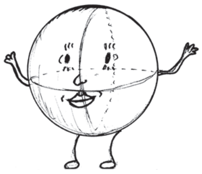
Counting can be extended to …3, 2, 1, 0, $^{-}1$, $^{-}2$, $^{-}3$… with stories like:

> “Imagine you are on a diving board. When you jump off you are 3 metres above the water, then 2 metres , 1 meter, splash, 0 and then you go underwater, negative 1, negative 2, negative 3…”

As they learn subtraction, students can handle

> “3 take away 5: 3, 2, 1, 0, $^{-}1$, $^{-}2$; you have to go into minus (negative) numbers by 2,” and “When you subtract a bigger number from a smaller number, the answer is negative.”

Being comfortable with the existence of negative numbers sets the stage for later skill development.



• Units of Measure •

Young children learn about units of measure the same way they learn about the meaning of any word: by hearing the word repeated several times as a meaningful image is presented simultaneously.

The trick to teaching children about measurement and units of measure is to provide them with meaningful experiences that highlight measurement and to repeat the experiences until all aspects of measurement become second nature. This is easiest to do when it is a continuing, long-term project. Some suggestions:

- When you tell a child that there are 100 centimetres in a meter have the child measure several things that are:

    - about a meter long,
    - about half a meter long and
    - about 2 metres long;

  For example, measure a table, a chair and a door. This provides the child with an image of the idea of a meter, its parts and its expansions.

- Have kids hold their hands or fingers 1 centimetre apart and then 1 meter apart.

- When you talk to children about the fact that there are 60 seconds in a minute, count to 60 (at the rate of 1 count per second) so that the child can get a feeling of the length of both a second and a minute.

- Find as many opportunities as possible to tell children the basic measurement facts:

    - 1 day = 24 hours
    - 1 week = 7 days
    - 1 year = 12 months
    - 1 year = 52 weeks
    - 1 hour = 60 minutes
    - 1 meter = 100 centimetres
    - 1 kilometre = 1000 meters

- Ask students questions like:

    - "Would we measure the distance across this room in kilometres? Why not? What could we measure this room in?"
    - "How long does it take to get from your house to school? A few days, a few hours, a few minutes, a few weeks? How about from here to Paris? Or to the moon?"
    - "Can we measure your height in grams? Why not?"
    - "Can we measure your weight in seconds? Why not?"

Expansion

After the child learns basic measurement facts, ask the child questions like:

- "How many centimetres are there in 2 metres ?"
- "Which is longer, a centimetre or a meter?"
- "Is a meter part of a centimetre or is a centimetre part of a meter?"
- "How many minutes are there in a half an hour?"
- "How many centimetres are there in a half a meter?"
- "How many minutes are there in one and a half hours?"
- "How many centimetres are there in two and a half metres ?"

Use rulers, clocks and the like, when available and as appropriate.

Distance, Area and Volume

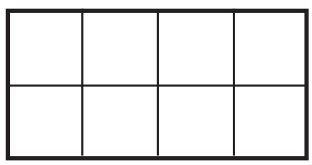
Distance

Distance is the shortest amount of space between two points and is measured in **linear** units (feet, metres , etc.).

Note that **distance** is either zero or a positive quantity; it is *never* negative.

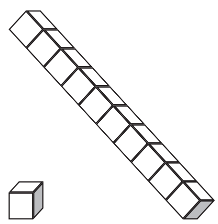
Area

In the early year groups, all students should experience counting the number of squares inside a rectangle.

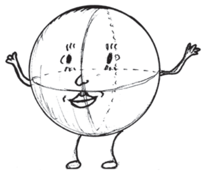


This image is invaluable when the subject of **area** is introduced in later year groups. **Area** is the amount of space in a **2-dimensional** figure, measured in **square** units.

Volume



Similarly, counting cubes should be part of every student's early experience, because it sets the stage for conceptualising **volume** and other space-related concepts. **Volume** is the amount of space in a **3-dimensional** figure, measured in **cubic** units.



• Counting Time •

By Year 2, most kids can count in 1s, 2s, 5s and 10s, up to at least 60.

We can make it easy for them to learn how to "tell time" by expanding these skills to include counting in 15s, 20s and 30s, as well as counting by 10s starting at *any* number (5, 15, 25…).

When you stop and think about it, all points on the clock can be reached quickly by counting various combinations of 30, 20, 15, 10, 5 and 1.

When kids are taught these counting skills *before* they deal with clocks, learning how to tell time is a very straightforward process. Without these skills, it's quite a chore.

Here's a way to teach kids these skills:

- Counting in 10s starting at any number goes like this:

  Start at 23: *twenty*-three (23), *thirty*-three (33), *forty*-three (43)…as though you are just counting in 10s, except that you say the *three*.

  So, counting in 10s starting at 37 becomes *thirty*-seven, *forty*-seven, *fifty*-seven…

- Counting in 15s can be done by counting in 10 and then counting in 5:

  15 + 15 = (15 + 10) + 5 = (25) + 5 = 30
  30 + 15 = (30 + 10) + 5 = (40) + 5 = 45
  45 + 15 = (45 + 10) + 5 = (55) + 5 = 60

- Counting in 20s is almost the same as counting in 2s:

  20 (*tw*enty), 40 (*for*ty), 60 (*six*ty), 80 (*eight*y)…

- Counting in 30s should be presented after the child can count in 3s.

  Counting in 3s is similar to counting by 2s:

  Counting in 2s can be explained as, "Say the number, don't say (skip) the next number, say the number..."

  Then counting by 3s becomes: "Say a number, don't say (skip) the next two numbers…"

  Now counting in 30s becomes: 30 (*th*irty), 60 (*six*ty), 90 (*nine*ty)…

These exercises can be led by both parents and teachers and can be started as early as Year 1. When young children learn to count quickly using the methods shown above, expressions like "half past," "three quarters of an hour," and "quarter till" are easily comprehended.

• Things to Point Out to Kids •

- Percentages for young children:

  - "100% fruit juice" means that it is *all* fruit juice and nothing else.
  - "10% fruit juice" means out of every 100 cups of the juice, only 10 cups are fruit juice.
  - "50% off" means half price.

- 10 hundredths make a tenth.
- 10 tenths make one "whole" (ones).
- 10 "wholes" (ones) make ten.
- 100 tens make a thousand.
- 10 hundreds make a thousand.
- 100 hundreds make ten thousand.
- 1,000 thousands make a million.

- Some things *make sense* to cut in half: a candy bar, a piece of wood, numbers...
  Other things don't: people, coins, cars...

- If twice as many people as you expected attend a picnic, then you will need twice as much food but if you are baking bread, twice the regular temperature will not bake twice as fast.

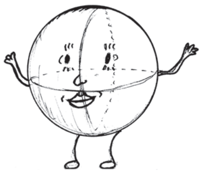
- When you *double* the number of pieces, the size of each piece is *half* as much as it was.
  (This is the *inverse relationship*: when one thing goes *up*, the other goes *down*.)

- On a regular ruler, whole inches are divided into parts:

  - halves ("2 parts the same")
  - fourths/quarters ("half of a half")
  - eighths ("half of a half of a half," two parts the same applied twice to $\frac{1}{2}$)
  - sixteenths ("a quarter of a quarter").

- "mono–" means 1: The *mono*rail at Disneyland runs on 1 rail.
- "bi–" means 2: A *bi*cycle has 2 wheels.
- "tri–" means 3: *Tri*ceratops has 3 horns.
- "qua–" means 4: A *qua*rtet has 4 players

- A quarter of an hour is 15 minutes (not 25).



• Problem Solving Strategy •

The idea that the whole is equal to the sum of its parts provides a framework for problem solving. The basic form of "attack" in problem solving is to ask yourself:

- What is the *whole* in this question? Is its value *known* or *unknown*?
- What are the *parts* in this question? Are their values *known* or *unknown*?
- What is the *relationship* between the whole and its parts? Which remains *constant* in the question? Which *changes*? How does it change?

By identifying what constitutes the whole and its parts, we can decide whether we need to perform a synthesis ("building up") or an analysis ("breaking down") to solve the problem at hand.

Synthesis:

If the whole is unknown, then the task is to build it up from its known parts:

- If the parts are equal, we *multiply*.
- If the parts are not equal, we *add*.

Key Concept: "The whole is equal to the sum (total) of its parts."

Analysis:

If the whole and one or more of its parts are known, then the task is to find the remaining part(s) by breaking down the whole, using the known part(s):

- If the known parts are not equal, we *subtract*."
- If the known parts are equal, we *divide*.

Key Concept: "Each individual part is equal to the whole minus the sum of all the other parts."

By determining which elements remain constant and which ones change, we can establish relationships between them and then decide on a method of attack to solve the problem at hand.

Virtually every mode of solution in primary and secondary curriculum (and beyond) can be modeled using the *whole-part* model.

Example: A box contains some marbles. 6 of the marbles are red, 5 are green and 14 are orange. How many marbles are in the box?

In this question, the *whole* (the total number of marbles) is unknown. Since the *parts* (the number of red, green and orange marbles) are known, we can use the Key Concept of Synthesis to find the whole:

$$\begin{aligned}\textbf{total \# of marbles} &= (\text{\# of red}) - (\text{\# of green}) - (\text{\# of orange})\\ &= 5 + 6 + 14\\ &= \textbf{25 marbles.}\end{aligned}$$

Example: A box contains 20 marbles. 5 of the marbles are red, 3 are green and the rest are orange. How many of the marbles are orange?

In this question, 20 represents the *whole*—all of the marbles. The *parts* are the respective quantities of red, green and orange marbles. Since the whole is known and one of the parts (the number of orange marbles) is unknown, we can use the Key Concept of Analysis to find the missing part:

$$\begin{aligned}\textbf{\# of orange marbles} &= (\text{total \#}) - (\text{\# of red}) - (\text{\# of green})\\ &= 20 - 5 - 3\\ &= \textbf{12 marbles.}\end{aligned}$$

Example: A train averages 60 kilometres per hour for a 5 hour trip. How far did the train travel?

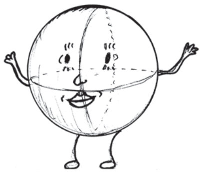
In this question, the distance travelled is the *whole* and is unknown. In each hour the train travelled an average of 60 kilometres so in 5 hours:

$$\begin{aligned}\textbf{distance travelled} &= (\text{average rate of speed}) \times (\text{time of trip})\\ &= \text{60 kilometres per hour} \times \text{5 hours}\\ &= \textbf{300 kilometres.}\end{aligned}$$

Example: A train travelled 200 kilometres at an average speed of 50 kilometres per hour. How long did the trip take?

In this question, the distance travelled is the *whole* and is known. In each hour the train travelled an average of 50 kilometres so:

$$\begin{aligned}\textbf{time of trip} &= (\text{distance travelled}) \div (\text{average rate of speed})\\ &= \text{200 kilometres} \div \text{50 kilometres per hour}\\ &= \textbf{4 hours.}\end{aligned}$$



Example: A sandbox contains 200 kilograms of mixed sand. 12% of the sand is brown and the rest is white. What is the percentage of brown sand after another 200 kilograms of white sand is added to the sandbox?

The *whole* (the 200 kilograms of mixed sand) has 2 *parts*: white sand and brown sand.

"Per"—"cent" means "for each"—"100," so if 12% of the 200 kilograms of sand is brown, then there are 24 kilograms of brown sand (12 kilograms of brown sand in the first 100 kilograms and 12 kilograms in the second 100 kilogram).

Now, if another 200 kilograms of white sand is added, we will have **a total of 400 kilograms of mixed sand, 24 kilograms of which is brown.**

Notice that the whole *changed* (from 200 kilograms to 400 kilograms), while the amount of brown sand remained *constant* (24 kilograms) when we added the second 200 kilograms of white sand.

24 out of **400** $= \frac{24}{400} = \frac{6}{100} = 6\%$

So, **6%** of the new mixture is brown sand.

Example: Another sandbox contains 100 kilograms of mixed sand, 15% of which is brown sand. The rest is white. How much white sand must be added to make the mixture only 5% brown?

In this question, there are 15 kilograms of brown sand (15% of 100 kilograms). As we add white sand, the *whole* (the total amount of sand) *changes* but the amount of brown sand remains *constant* (at 15 kilograms).

We want 15 out of the (new) total to equal 5 out of 100 (5%).

We can ask the equivalent questions: 15 is to what number as 5 is to 100?
5 out of 100 = 15 out of what number?
$\frac{5}{100} = \frac{15}{\text{what number?}}$

All of these methods yield the same answer: 300 kilograms. Since the box already has 100 kilograms of sand, **200 kilograms** (300 – 100) must be added.

Try these:

1) A box contains 50 marbles. Ten are red, five are blue and the rest are while. How many of the marbles are white? ______________________

2) A 25-kilogram bag of red sand is mixed with a 50-kilogram bag of white sand. What percentage of the final mixture is red? ______________________

1) Teach NUMBER FACTS as "the process of COUNTING IN GROUPS."

- ADDITION is "counting–up the total."
- SUBTRACTION is "counting 'how far apart' two numbers are."
- MULTIPLICATION is "a fast way to count equal groups."
- DIVISION is "counting the number of *this* group that are inside of *that* group."

Focus on 10, multiples of 10 and powers of 10.

Include counting by *common fractions* ($\frac{1}{2}$s, $\frac{1}{4}$s, $\frac{3}{4}$s, $\frac{1}{10}$s, $1\frac{1}{2}$s, $1\frac{3}{4}$s…) and by *decimal fractions* (0.1s, 0.5s, 0.25s, 0.01s, 0.75s, 1.5s, 1.75s…)

The goal is to be able to count *from* any number, *to* any number, *by* any number.

2) Teach the concepts of DENOMINATION and QUANTITY.

The goal is to realise that every *thing* in maths. (number, variable, term…) has a *name* and *amount* associated with it.

3) Teach students to figure–out HALF of various even and odd numbers *mentally*. Expand to include common and decimal fractions. Expand again to include FOURTHS, EIGHTHS, THIRDS, TENTHS, SIXTHS… of any number.

The goal is to be able to split *any* number into any fractional part.

4) Teach PLACE VALUE as "the art of 'seeing' 10s."

The goal is to be able to "see" how many:

- *10s* (100s, 1,000s…1), *0.1s* (0.01s, 0.001…1) and $\frac{1}{10}$*s* ($\frac{1}{100}$s, $\frac{1}{1000}$s…1)

there are in *any* number (whole number, decimal fraction and common fraction).

5) Teach students to COMPARE numbers by SUBTRACTION (the *difference*; "how far apart"; the distance between) and by DIVISION (the *ratio*; "relative size"; proportional relationship).

The goal is to be able to know when each type of comparison (*difference* or *ratio*) applies in a given context and to "see" the various relationships between the *size* of any two numbers.

6) Teach PROPORTIONAL THINKING in various contexts.

The goal is to be able to use PROPORTIONAL REASONING to solve:

- all forms of the "three-for-a-quarter"–type question,
- problems involving all aspects of *fractional parts* and
- the three (3) types of questions involving *percentages*

in any context.

7) Teach the meaning of the **Law of SAMEness**:

> "You can only add and subtract things of the same *denomination*, things that have the same *name*."

The goal is to understand why "you cannot add apples and oranges" in various contexts (common denominators, lining–up decimal points, combining similar terms…)

8) Teach the **Laws of Equality** (the **Laws of Transformation**):

- "Equals operated on by equals remain equal.",
- "Equals may anywhere be substituted for equals." and
- "Things equal to the same thing are equal to each other."

The goal is to be able use these laws in the process of transforming one number into another solving equations and problem solving.

9) Teach the important **properties of numbers**:

- the concept of the *Identity Property* and how it is used to "rename numbers" (simplifying fractions, simplifying expressions…)
- the concept of the *Distributive Property* and how it is used to "break–down" a question into manageable parts. and
- the concept of the *Commutative Property* as the "Order Property: it doesn't matter in what *order* we add and multiply two numbers but order does matter when we subtract and divide."

The goal is to be able to use "the properties" of numbers as a tool in problem solving and in mathematical discovery.

10) Teach that the **words** used in mathematics *make sense*. Teach the Latin and Greek roots of words commonly used in the classroom.

- *per cent* = per – cent = "for each" – "100"
- *denominator* = *de*–nom–inator = "*de* name of *de* fraction"
- *polygon* = *poly* (*many*)–gon (angle) = "a figure with *many angles*"

The goal is to develop a wide–ranging mathematics vocabulary.

11) Teach the **Laws of Wholes and Parts**:

- "The whole is equal to the *sum* of its parts."
- "Any one part is equal to the whole *minus* all of the other parts."

The goal is to be able to see "the one composed of the many," and to be able to use these laws as the basis for many diverse problem solving strategies.

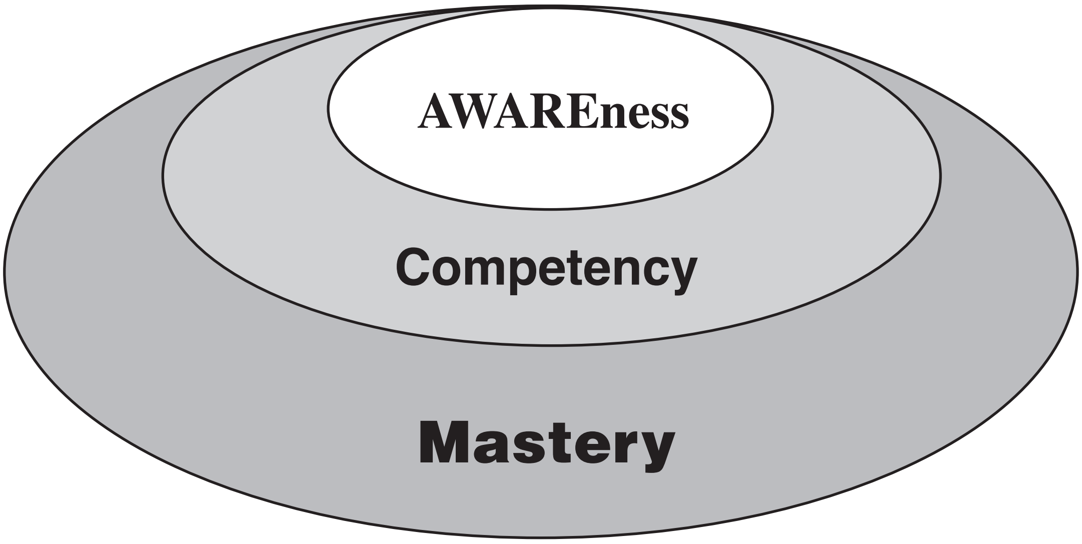
12) Teach that **ARITHMETIC** is "the process of transforming one number into another, using given rules."

The goal is to see arithmetic as the stepping–stone to understanding higher levels of mathematics (number theory, algebra, geometry, trigonometry, calculus…).

• The Mathnasium Glossary •

addition	counting "how many altogether." The process of forming a whole.
complement	"the rest of it." the remaining part with respect to the whole.
counting	the process of determining quantity (how many, how much).
denomination	the collective *name* of a group of similar things (apples, dogs, inches).
division	counting "how many of *these* are there inside of *that*." The process of separating a whole into equal parts.
fraction	the result of breaking a whole into *equal* parts; one or more of those equal parts. (Compare with **division**.)
group	one or more of the *same* thing (1 apple, 9 dogs, 5 inches, 8 things).
half	the first fraction. A whole divided into two equal parts; one of those parts. "Two parts the same."
interval	the distance from one number (or unit) to another. The space between two numbers.
Law of SAMEness	It is only possible to add and subtract things of the *same* kind, things with the same *name*, the same *denomination* (an apple plus a banana is *not* a "banapple").
mathematics	the study of wholes and parts and the relationship between them.
matrix	litreally, "mother." That which gives form origin or foundation to something enclosed or embedded in it. A place of origin and growth. Related words include environment, framework, womb, structure, model; enclosed, enveloping, surrounding.
measurement	the determining of quantity.
multiplication	counting "in equal groups." The process of forming a whole from equal groups.
part	a component of the whole. A fragment, fraction, section, portion, region; a piece broken off.
per cent	litreally, "for each 100," "parts per hundred," "how many for each hundred,"
proportion	litreally, "according to amount." The relation of one part to another or to the whole; relative size. Comparison, analogy, balance, symmetry.
ratio	a comparison of two numbers *by division*.
subtraction	counting "how far apart two numbers are" and "how much is left." The process of removing a part(s) from a whole.
whole	undivided. The one composed of the many. That which can be broken–down into parts. *All* of the quantity under consideration.
zero	the number that counts *none*. That which has no parts.

The MATHNASIUM Learning Principle:
***Awareness* precedes *Skills*.**



The Three Stages of Learning

There is a need for a "sense check" at each step in the learning process.

Otherwise:

Instructor says: "The *whole* is equal to the *sum* of its parts and is greater than *any one*."

Student hears: "The *hole* equals some of its *parts* and is greater than *anyone*."

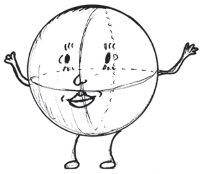
Instructor says: "Two cans are worth $1."

Student hears: "Toucans are worth $1."
(That's awfully cheap for such a pretty bird.)

The MATHNASIUM Teaching Principle:

K.I.S.S.

Kept **I**t **S**hort and **S**imple



• How We Teach •

With an observable sense of humor when talking—
outward, energetic, *animated*, when teaching.

Use these expressions as often as appropriate:

1) How far apart are the two numbers? (for **SUBTRACTION**)

2) How many of these are inside of that? (for **DIVISION**)

3) How many groups of ____ can you make?...Will there be any leftovers?

4) How many times as big/small? (for comparison by **DIVISION**)

5) How far apart are the two numbers? (for comparison by **SUBTRACTION**)

6) Do they have the same name or different names? Can the names be made the same? (for use with **THE LAW OF SAMEness**)

7) How many—of what? (for use with **QUANTITY** and **DENOMINATION**)

8) What is the whole...What are the parts? What is the relationship between them? (for analysing word problems.)

9) What *changes* and what stays the *same*? (for analysing word problems)

10) The *denominator* of a fraction is the *name* of the fraction and tells us how many equal parts are in the whole.

11) The *numerator* of a fraction tells us *how many* parts we are considering.

12) A *proper fraction* is a fraction whose value is *less than* one whole (1).

13) An *improper fraction* is a fraction whose value is *greater than* one whole (1). Mixed numbers are another name for improper fractions.

14) *Unity* is a fraction whose value is *equal to* one whole (1).

• A Warning About Symbols •

When my son was seven years old, he could answer questions like:

- How much is half of $3\frac{1}{2}$?,
- Half of what number is $2\frac{3}{4}$?
- How much is half of a half of a half?

One day he looked at an address in our neighbourhood and asked, “Dad, what does the ‘one-slash-two’ mean?”

He had learned the *concept* that half means “two parts the same”**verbally**, not as a symbol ($\frac{1}{2}$). At age 11, he mastered written (symbolic) fractions.

I know a Year 4 teacher who wrote this problem on the board: “$\frac{1}{2}$ of 20 = _____.”

Not one hand was raised. Silence reigned. Perplexed by the lack of response, she said, “Doesn’t anyone know what half of 20 is?” In a near deafening chorus, the entire class said, “10!”

These two stories illustrate a very important teaching principle:

Awareness (knowledge of concepts) *precedes* skills (manipulating symbols).

This is especially true in the early year groups of mathematics education.

Make sure that your students understand the concepts embodied in the symbols before you attempt to use those symbols to evaluate the students’ performance.

“There are three major factors at work that contribute to the frustrations of teachers and students involved in teaching and learning mathematics.

- One contributing factor is that we teach as though symbols have obvious and inherent meaning.
- The second is that we too often teach without considering the individual student’s level of cognitive maturity, not recognising that what seems so obvious to us adults may not be obvious to the child.
- And the third is that in our search for the ever clearer explanation, we often overlook the importance of the students’ need to construct their own understanding.” *(Mathematics Model Curriculum Guide, K–12*, published by the California Department of Education.) [Format changed.]

The material in this section minimises the use of symbols. It allows you to focus on verbal reasoning and understanding and on creating mental images.

• Thoughts on **Content** and **Delivery** •

Delivery: Verbal Teaching

The techniques and tips presented in this book are most valuable when used to create and augment oral, interactive presentations.

> "Socrates was on the right track: Students may retain more information when teachers *ask them questions* rather than simply lecturing…When children are actively involved, they remember more than when they're just sitting in front of the teacher…The effort involved *engages* students in the material [emphasis added]." (From *Teacher* Magazine, May, 1990.)

One of the main reasons many children do not learn number facts (and mathematics in general) is that practice questions are most often presented in written form. Verbal questioning presents a more easily digested image of the concept at hand. It is therefore usually less confusing to all students, be they beginning, remedial or advanced. It is also the first step in the the process of saying, seeing, writing and finally, understanding.

> Verbal questioning, manipulatives (coins, sticks, blocks, etc.) and written exercises could be considered three different "languages of learning." To be able to translate from one language to another, the child must first know one of them well. Since the child's natural language of learning is oral, I suggest that verbal questioning is the best place to start.

Schools would be wise to train teachers, classroom aides, volunteers and parents in verbal approaches to learning.

Content: Reverse Thinking

$7 + 3 = \square$ is a *straight* question. $7 + \square = 10$ is a *reverse* question.

Answering the reverse question $7 + \square = 10$ requires a higher-level thought process than does completing the straight question $7 + 3 = \square$, yet it involves the same basic set of facts. Similarly, the following problems:

$7 - \square = 5 \qquad \square - 4 = 6 \qquad \square \times 3 = 12 \qquad \square \div 3 = 5 \qquad 10 \div \square = 5$

require students to *think* about and explore number relationships, rather than to merely parrot memorised facts.

Students learn number facts best when both *straight* and *reverse* questions are presented *verbally* and in *written* form, as an integral part of the curriculum.

• Number Systems •

Natural Numbers ($\mathbb{N}$)
(sometimes called *Counting Numbers*)

The ***Natural Numbers*** are the positive numbers that do not contain common or decimal fractions.

$$\mathbb{N} = \{1, 2, 3\ldots\}$$

Whole Numbers ($\mathbb{W}$)

The ***Whole Numbers*** are all of the ***Natural Numbers*** and ***Zero*** (0).

$$\mathbb{W} = \{0, 1, 2, 3\ldots\}$$

Integers ($\mathbb{Z}$)

The ***Integers*** are the POSITIVE and NEGATIVE ***Whole Numbers***, including *Zero*.

$$\mathbb{Z} = \{\ldots{}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3\ldots\}$$

Rational Numbers ($\mathbb{Q}$)

The ***Rational Numbers*** have the form $\frac{a}{b}$, where a and b are ***integers*** and $b \neq 0$. That is, they are all the numbers that can be written as ***a ratio*** (***a fraction***).

All the ***Rational Numbers*** can be written as ***common fractions***, as ***decimal fractions*** and as ***percentages***.

The decimal form of a ***Rational Number*** is either a ***terminating decimal*** or a ***repeating decimal***.

EXAMPLES: $\frac{1}{4} = 0.25 = 25\%$ $\quad \frac{1}{3} = 0.333... = 0.\overline{3} = 33\frac{1}{3}\%$

Irrational Numbers ($\mathbb{I}$)

The ***Irrational Numbers*** are numbers that *cannot* be written as ***ratio***. Their decimal representations *do not repeat* and *do not terminate*.

EXAMPLES: $\pi = 3.14159\ldots \quad \sqrt{2} = 1.414213\ldots \quad x = 0.010010001\ldots$

Real Numbers ($\mathbb{R}$)

The ***Real Numbers*** are all of the ***Rational*** and ***Irrational Numbers*** taken together.

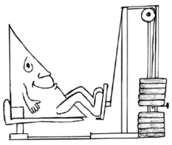
EXAMPLES: ***All of the above***.

Complex Numbers ($\mathbb{C}$)

The ***Complex Numbers*** are all the ***Real Numbers*** and numbers that involve the square root of negative one ($\sqrt{{}^{-}1}$). ***Complex Numbers*** have the general form $a + bi$, where a and b are ***real*** numbers and $i = \sqrt{{}^{-}1}$. a is the ***real*** part and bi is the called the ***imaginary*** part.

(For ***Real Numbers***, $b = 0$. For PURE ***Imaginary*** numbers, $a = 0$.)

EXAMPLES: ***All of the above*** AND ${}^{-}3 + 2i,\ 5 - 6i,\ 4i\ (a = 0),\ \frac{1}{2}\ (b = 0)$.



• Reflective and Critical Questions •

Complete the following.

1) Complete all the "***Try these***" questions on the following pages:

Page 22: Three Ways to Write a Number
Page 23: Rate
Page 24: Relationships: Direct & Inverse
Page 25: Variations: Direct & Inverse
Page 33: Problem Solving Strategy

Use this space if you need more room to explain your answers to the Problem Solving Strategy exercises (p. 33).

2) Solve the Algebraic Wind & Water problem. Give a full explanation using Mathnasium language.

Zelda can swim at 50 metres per minute upstream and twice as fast downstream. How fast can she swim in still water? What is the rate of the current?

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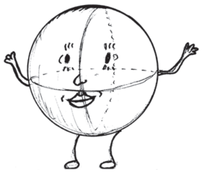


Lead Instruction
&
Progress Check

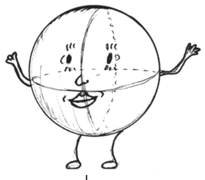
• Lead Instruction •

Today you will partner with another Instructor but instead of observing them you will lead the instruction.

Objectives:

1. Ability to fill out a student's Workout Plan correctly.
2. Communicate efficiently with the other Instructors.
3. Proactively engage with students when appropriate and disengage when appropriate.
4. Implement the Student Incentive Program.
5. Check student work.
6. Use the Mathnasium Teaching Constructs when appropriate.

Based on the objectives, what do you feel the most comfortable with? Least comfortable?



• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

Instructor Development/First Look

I have a good understanding of:					
1.	the importance of the language we use when teaching students.	**0**	**1**	**2**	**3**
2.	how to explain the meaning of per cent to a student.	**0**	**1**	**2**	**3**
3.	how to explain fractional parts to a student.	**0**	**1**	**2**	**3**
4.	the difference between a part-to-part comparison and a part-to-whole comparison.	**0**	**1**	**2**	**3**

Instructing

While Instructing I:		YES	NO
1.	filled out a student's Workout Plan.		
2.	proactively engaged with students.		
3.	disengaged and left students with a meaningful task.		
4.	communicated with other Instructors.		
5.	gave students rewards punches and/or helped them trade in their cards for something in the Rewards Cabinet.		
6.	made sure students were following their Mathnasium Hour.		
7.	used the Mathnasium Teaching Constructs when appropriate.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

Larry's Do's and Don'ts (section 10, p. 3)
First Look (Parts 3 and 4) (section 10, p. 21)
Instructing Experience (section 11)

NOTES:

____

____

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A First Look Course, Questions Based on the *First Look at Curriculum* & Practical Exam Prep

Watch the <u>Right Instruction - Teach the Mathnasium Way: A First Look at the Mathnasium Curriculum and Instruction</u> course. Then, read and complete the following pages:

Questions Based on the *First Look at Curriculum*

The First Look at the Mathnasium Curriculum & Instruction course gives a first look at the Mathnasium curriculum, with focus on The 10 Mathnasium Rules of Engagement and their correct application within the curriculum.

Objectives:

In this course, you will learn about:

1. The important elements of the Mathnasium curriculum.
2. The 10 Mathnasium Rules of Engagement.
3. Mathnasium Teaching Constructs.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the "Training Videos" section, click on "Right Instruction - Teach the Mathnasium Way: A First Look at Mathnasium Curriculum and Instruction".

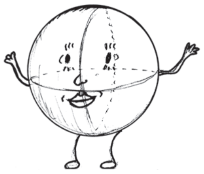
Notes:

Questions Based on the *First Look at Curriculum*

Instructions for the Instructor:

Complete the following pages after completing the *First Look at Curriculum and Instruction* online course in Mathnasium University. Use the space provided to explain or demonstrate the method you used to solve the exercise.

Bring your responses to the facilitator for review and discussion.



• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) $99 + 99 + 99 =$ ____________

____

2) $100 - 76 =$ ____________

____

3) $7 \times 3 \times 33 =$ ____________

____

4) $300 \div 25 =$ ____________

____

5) $6 \times 1\frac{1}{4} =$ ____________

____

6) $10 \div 2\frac{1}{2} =$ ____________

____

7) $\frac{3}{4}$ of $24 =$ ____________

____

• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) 7% of 350 = ____________

____

2) half of 99 = ____________

____

3) $\frac{3}{4}$ of what number is 10? = ____________

____

4) Arrange in order from *smallest* to *largest*:

$0,\ 1,\ \frac{5}{8},\ \frac{2}{7},\ \frac{1}{2},\ \frac{9}{10}$ ____________

____

5) $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \frac{3}{4}$ = ____________

____

6) 4 is to 6 as 10 is to = ____________

____

• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) $99 + 98 + 97 =$ ______

______

2) $3{,}690 - 996 =$ ______

______

3) $4 \times 9 \times 2 \times 5 \times 2 =$ ______

______

4) $1{,}500 \div 12 =$ ______

______

5) half of 49 = ______

______

6) half of 5,280 = ______

______

7) 7% of 250 = ______

______

8) $\frac{1}{2} \times 120\frac{8}{97} =$ ______

______

• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) $15 \div 1\frac{1}{2}$ = ____________________

____

2) $\frac{3}{4}$ of what number is 18? = ____________________

____

3) Arrange in order from *smallest* to *largest*:

$1,\ 0,\ \frac{5}{7},\ \frac{1}{10},\ \frac{3}{8},\ \frac{10}{11},\ \frac{1}{2},\ \frac{19}{20}$ ____________________

____

4) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{3}{7}$ = ____________________

____

5) 12 is to _______ as 8 is to 18

____

6) 1 + 2 + 3 + ... + 98 + 99 + 100 = ____________________

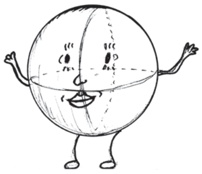
____

7) $\frac{1}{8}$ + 0.5 + _____ = 1.75

____

8) $210 \div 0.6$ = ____________________

____



• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) Halfway through the second quarter, what (fractional) part of the game is left?

2) Half $\left(\frac{1}{2}\right)$ of the marbles in a bag are red. A third $\left(\frac{1}{3}\right)$ are white. The other six (6) are blue. How many marbles are in the bag?

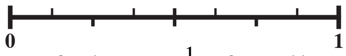
3) Two–thirds $\left(\frac{2}{3}\right)$ of a kilogram of candy costs \$6.40. How much does a whole kilogram of candy cost?

4) Mario received 15% of the votes for class president. He got 60 votes. How many people voted in the election?

• Mental Maths •

Find a "Number Sense" way to solve each exercise. Explain your reasoning.

1) A furlong is an eighth $\left(\frac{1}{8}\right)$ of a mile. What part of a mile is $7\frac{1}{2}$ furlongs?



1 furlong = $\frac{1}{8}$ of a mile

____

____

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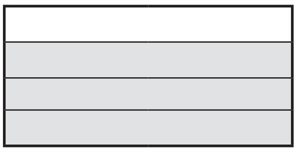
2) A fish tank is three-fourths full. Two-thirds of the water leaks out. After the water leaks out, 10 litres are left in the tank. How many litres of water does the tank hold when full?



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3) Rachel bought stock on Monday. By Wednesday the value of the stock had decreased by 20%. By Friday the value of the stock had increased by 8% from its value on Wednesday. If the value of the stock on Friday is \$216. What was the value of the stock on Monday?

____

____

____

• Algebraic and Number Sense Solutions •

Give one or more "Algebraic" and "Number Sense" solutions for each exercise.

1) Two-thirds of a kilogram of candy costs 40 cents. How much does $1\frac{1}{2}$ kilograms of candy cost?

ALGEBRAIC SOLUTION ______________________

____

NUMBER SENSE SOLUTION ______________________

____

2) A 6-foot man casts an 8-foot shadow. How tall is a tree that casts a 24-foot shadow?

ALGEBRAIC SOLUTION ______________________

____

NUMBER SENSE SOLUTION ______________________

____

3) A 6-foot man casts an 8-foot shadow. How tall is a tree that casts a 20-foot shadow?

ALGEBRAIC SOLUTION ______________________

____

NUMBER SENSE SOLUTION ______________________

____

• Algebraic and Number Sense Solutions •

Give an "Algebraic" and a "Number Sense" solution for each exercise.

1) 6% of 150 equals what number?

Algebraic Solution ______________________

____

Number Sense Solution ______________________

____

2) 12% of what number is 30?

Algebraic Solution ______________________

____

Number Sense Solution ______________________

____

3) 7 is what percentage of 20?

Algebraic Solution ______________________

____

Number Sense Solution ______________________

____

• Algebraic and Number Sense Solutions •

Give an "Algebraic" and a "Number Sense" solution for each exercise.

1) 2 is what fractional part of 16?

ALGEBRAIC SOLUTION ____________________

____

NUMBER SENSE SOLUTION ____________________

____

2) $\frac{3}{8}$ of what number is 9?

ALGEBRAIC SOLUTION ____________________

____

NUMBER SENSE SOLUTION ____________________

____

3) $\frac{3}{4}\%$ of 20 equals what number?

ALGEBRAIC SOLUTION ____________________

____

NUMBER SENSE SOLUTION ____________________

____

4) $\frac{2}{3}\%$ of what number is 10?

ALGEBRAIC SOLUTION ____________________

____

NUMBER SENSE SOLUTION ____________________

____

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Instruct Independently
&
Progress Check

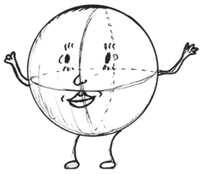
• Instruct Independently •

Today you have a chance to teach on the floor independently. Use the Practical Exam Checklist as a guide during your instruction today.

Objectives:

1. Ability to fill out a Student's Workout Plan correctly.
2. Communicate efficiently with the other Instructors.
3. Proactively engage students when appropriate and disengage when appropriate.
4. Implement the student incentive program.e.
5. Check student work.
6. Use the Mathnasium Teaching Constructs when appropriate.

Based on the objectives and the Practical Exam Checklist, what do you feel the most comfortable with? Least comfortable?



• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

A First Look at Curriculum/Practical Exam Checklist

I have a good understanding of:					
1.	how to use the 10 Rules of Engagement when on the instructional floor.	**0**	**1**	**2**	**3**
2.	the difference between Direct Teaching and Socratic Questioning.	**0**	**1**	**2**	**3**
3.	how the Mathnasium Constructs are used in instruction.	**0**	**1**	**2**	**3**

Instructing

While Instructing I:		YES	NO
1.	filled out a student's Workout Plan.		
2.	proactively engaged with students.		
3.	disengaged and left students with a meaningful tasks.		
4.	communicated with other Instructors.		
5.	gave students rewards punches and/or helped them trade in their cards for something in the Rewards Cabinet.		
6.	made sure students were following their Mathnasium Hour.		
7.	used the Mathnasium Teaching Constructs when appropriate.		

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

A First Look Video (section 12, p. 1)
Questions Based on the *First Look at Curriculum* (section 12, p. 3)
Practical Exam Checklist
Instructing Experience

NOTES:

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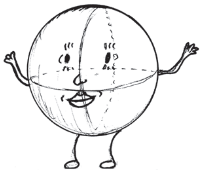

Practical Exam

Practical Exam Prep
Practical Exam (see facilitator)
Self-Reflection

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Practical Exam – Part 1 Prep

• Practical Exam Checklist •

On the last day of your Instructor Level 1 (New Hire) training you will demonstrate what you have learned during the practical test. Your facilitator will evaluate you on your ability to demonstrate the skills outlined in the **10 Mathnasium Rules of Engagement**. They will observe you for 60 minutes while you teach multiple students. Below is a list of 10 Rules and how you can prepare.
Note: Rules #5, 7 and 8 will be covered in the next phase of Instructor Training.

10 Mathnasium Rules of Engagement:

Rule #1

Use the Mathnasium teaching constructs. Use the Teaching Icons as guides.

To Prepare: Review the Mathnasium Teaching Constructs.

Notes:

____

____

____

____

Rule #2

Do not repeat an explanation if it is not working. Try another approach. Do not force students to keep working on material they have already mastered.

To Prepare: Review the Modes of Teaching document.

Notes:

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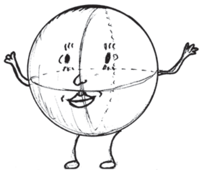


• Practical Exam Checklist •

Rule #3

Fall back on prerequisite knowledge when the student is having trouble. Extend knowledge when the student understands the concept.

To Prepare: Review The Meaning of Per Cent: Per Cent document (First Look Part 3) and Units of Measure document (First Look Part 4).

Notes:

Rule #4

Praise, encourage and constructively criticise when appropriate.

To Prepare: Review the Proactive Teaching document.

Notes:

Rule #6

Use Mathnasium vocabulary because it makes sense. Avoid confusing nomenclature.

To Prepare: Review the Mathnasium Teaching Principle K.I.S.S in the Proactive Teaching document, the Mathnasium Teaching Constructs and the Core Vocabulary (First Look Part 4).

Rule #9

Require students to use mental maths. to enhance numerical fluency and limit reliance on pencil and paper.

To Prepare: Review the Addition, Subtraction and Division Tips documents (First Look Part 1).

Notes:

____

____

____

Rule #10

Master Team Teaching.

To Prepare: Review the Team Teaching Checklist, Team Teaching Course and your shadowing experiences.

Notes:

____

____

____

You will NOT be evaluated on the following during this Practical. Rules #5, 7 and 8 will be covered in more detail during the next phase of your Instructor training.

5. Use Socratic Questioning when it's appropriate. Use Direct Teaching when it's appropriate.

7. Use drawings, diagrams, manipulatives and Desk Tools when appropriate to clarify and reinforce concepts visually.

8. Enable students to achieve metacognition, an awareness of one's own thinking process.

Practical Exam – Part 1

Meet with your facilitator, they will administer the Practical Exam.

Self Reflection

Reflect on your Practical Exam experience.

• Self-Reflection •

Evaluate how well you feel you demonstrated the following:

1– Needs Improvement **2**– Met Expectations **3**–Exceeded Expectations

Practical Exam

Mathnasium's Rules of Engagement (#1, 2, 3, 4, 6, 9, 10)				
#1.	Use the Mathnasium teaching constructs. Use the Teaching Icons as guides.	**1**	**2**	**3**
#2.	Do not repeat an explanation if it is not working. Try another approach. Do not force students to keep working on material they have already mastered.	**1**	**2**	**3**
#3.	Fall back on prerequisite knowledge when the student is having trouble. Extend knowledge when the student understands the concept.	**1**	**2**	**3**
#4.	Praise, encourage and constructively criticise when appropriate.	**1**	**2**	**3**
#6.	Use Mathnasium vocabulary because it makes sense. Avoid confusing nomenclature.	**1**	**2**	**3**
#9.	Require students to use mental maths. to enhance numerical fluency and limit reliance on pencil and paper.	**1**	**2**	**3**
#10.	Master Team Teaching.	**1**	**2**	**3**

Once you complete the Self-reflection meet with your facilitator to discuss your Practical and an Improvement Plan (if necessary).

Notes:

Numerical Fluency Course
&
NF Sampler

• Numerical Fluency •

Watch the Right Instruction - Teach the Mathnasium Way: Numerical Fluency 2.0 Instructional Techniques training video. Refer to the Numerical Fluency Program Overview and PK Sampler Training Packet (p. 5 – 77). Work through the sampler to familiarise yourself with the NF 2.0 program. Additionally, complete the Reflective and Critical Questions on pages 3 and 4 upon completion of the training video and sampler.

The goal of the video is to help you learn how to instruct and evaluate students in addition and subtraction fact fluency.

OBJECTIVES:

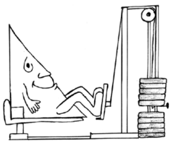
In this video, you will learn about:

1. number facts and numerical fluency.
2. the methodology behind teaching numerical fluency.
3. the Numerical Fluency curriculum.
4. how to evaluate mastery of addition and subtraction facts.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the "Training Videos" section, click on "Right Instruction - Teach the Mathnasium Way: Numerical Fluency 2.0 Instructional Techniques".

NOTES:



• Reflective and Critical Questions •

Answer the following questions AFTER completing the NF training video and PK sampler.

1) Explain two different techniques you would use to teach 7 + 6 to a student.

2) What will you do if you see a student finger counting?

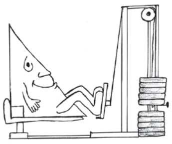
3) How can you continue to ensure students are using Numerical Fluency techniques even after they complete the Numerical Fluency Program?

• Reflective and Critical Questions •

4) What are some of the pitfalls of memorisation?

5) How can Numerical Fluency techniques be scaled for exercises like 47 + 8 and 54 – 7?

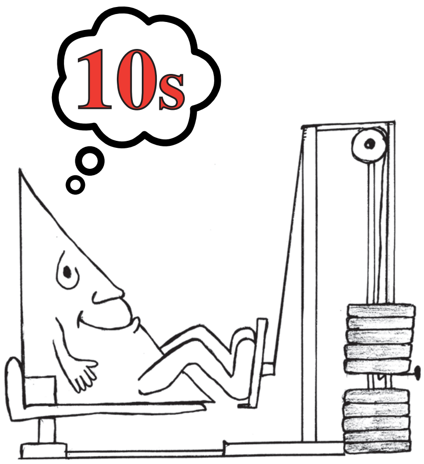
6) What is your biggest take away from the Numerical Fluency course?



Numerical Fluency

Addition & Subtraction Facts

Version 2.0



Program Overview & PK Sampler

Training Packet

Section 1:

The Mathnasium Perspective

Why is addition and subtraction fact fluency so important?

Without addition and subtraction fact fluency, students typically experience ongoing struggles in maths. If students can't add and subtract fluently with basic facts, then they can't multiply and divide. If they can't add, subtract, multiply and divide, then they can't do much of anything else successfully in maths.

Why do many students struggle to master addition and subtraction facts?

The root of this problem lies in the way number facts are typically taught in the early grades. Great emphasis is placed on acquiring knowledge of the facts through ***memorisation*** and demonstrating "mastery" through the use of anxiety-provoking timed tests. Not only is this a painful process for everyone involved (kids, teachers and parents), but in many cases it also does **NOT** result in the desired outcome. When students cannot recall a fact, the *only* recourse most of them have is to *rack their brain* to recall a "memorised" fact or ***count on their fingers by ones*** (or similar techniques like *counting by ones mentally*, *foot tapping*, *head bobbing*, *drawing dots*, etc.), because it is the only strategy available to them.

What are the problems with rote memorisation?

When students attempt to learn number facts by rote, they are drilling numbers that do not actually mean anything to them. They lack understanding of how the numbers relate to each other. Thus, when students ***forget*** a "memorised" fact, they typically make a guess and have no sense of whether their answer is right or wrong. Also, there is an opportunity cost to learning number facts through memorisation. How can we expect a student to effortlessly find the correct answer to 28 + 5 if they learned 8 + 5 by purely drilling the numbers into their head?

What are the problems with finger counting?

When kids count on their fingers or count by ones in some way to add or subtract, two things can typically go wrong: it takes them *too long* to provide the correct answer, or they get the *wrong* answer.

Here's what's going on:

Similar to the problems with rote memorisation, students don't really have a mental grasp on the numbers when they count on their fingers. As a result, they write down or say out loud whatever answer they get from *counting up or down by ones* and have no sense of whether their answer is right or wrong or whether it is reasonable or unreasonable. Additionally, they are most often providing answers to number facts so slowly that it will become extremely difficult for them to perform more complex tasks that rely on mastery of these number facts.

If memorisation and finger-counting are ineffective approaches, then how does Mathnasium help students to attain addition and subtraction fact fluency?

Instead of relying on finger counting or rote memorisation, Mathnasium helps students ***develop*** fact fluency by providing instruction and guided practice with ***efficient and reliable*** ways of thinking so that facts can be recalled on a moment's notice. The facts are, ultimately, ***committed to memory***, but the process of acquiring that knowledge is much more valuable and effective than the process of memorising unrelated bits of information.

How does Mathnasium customise a solution for each student?

The first step in providing students with the instruction they need is to ensure they encounter the *right materials* at the *right time*. Mathnasium accomplishes this by administering our diagnostic assessment which uncovers whether students have attained addition and subtraction fact fluency. For those students who have yet to attain fact fluency, the assessment also uncovers gaps in their knowledge that would prevent them from being ready to learn the techniques we teach. From there, a customised learning plan is designed in a way that *ensures students master all the necessary prerequisite skills* before we teach them our *efficient and reliable techniques* for mastering addition and subtraction facts.

SECTION 2:

The Mathnasium Methodology

Addition and subtraction fact fluency issues most commonly present themselves when students are solving addition facts with sums over 10 (e.g., 8 + 5 = __) and subtraction facts with minuends over 10 (e.g., 17 – 9 = __). This is where the lack of efficient techniques often leads students to rely on finger counting and other inefficient techniques.

Mathnasium materials are designed to teach the following three methods for solving **Addition Facts with Sums over 10**.

DOUBLES
+ *or* – 1

6 + 6 is 12, so **6 + 7** is 13.

9 + 9 is 18, so **9 + 8** is 17.

Up to & Over
10

8 + 5

$\underbrace{\boxed{8 \; 2}}_{10} + 3 = \underset{8\,+\,5}{13}$

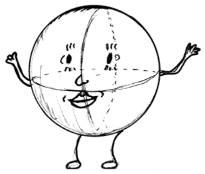
ADD 9
+ 10, – 1

7 + 10 is 18, so **7 + 9** is 17.

Mathnasium materials are designed to teach students the ***Up to and Over 10*** method for solving **Subtraction Facts (and Missing Addends) with Minuends over 10**.

$$\begin{array}{r} 17 \; (7) \\ 10 \\ -\;\; 9 \; (1) \\ \hline 8 \end{array}$$

$$7 + \underset{10}{\underline{5}} = 12 \qquad \boxed{3} \;\; 10 \;\; \boxed{2}$$



Building Prerequisite Knowledge

In order for students to learn our efficient techniques to master **Addition Facts with Sums over 10** and **Subtraction Facts with Minuends over 10**, they need to have mastery of the following prerequisite skills:

- **Doubles Facts**
- **Number Bonds to 10**
- **Partitioning Single-Digit Numbers**
- **10 Plus a Single-Digit Number**
- **Addition Facts: Sums up to 10**

Let's take a look at how these prerequisites apply.

Addition Facts: Sums Over 10

DOUBLES + *or* – 1

When asked 7 + 8, the student reasons that since 7 + 7 is 14, then 7 + 8 is one more (15).

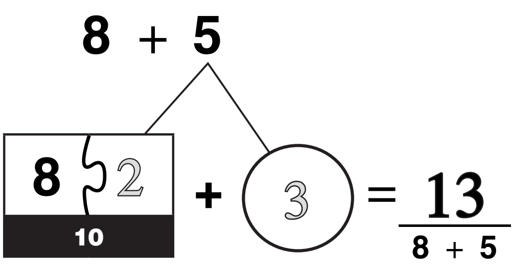
7 + **7** is __14__, so **7** + **8** is __15__.

In order to use this technique, students need to have mastery of:

- **Doubles Facts up to 12 + 12**

***Up to & Over* 10**

When asked 8 + 5, the student partitions the 5 into the part that brings 8 up to 10 (2), and then adds the leftover part (3) to 10 to make 13.



In order to use this technique, students need to have mastery of:

- **Number Bonds to 10** (8 + **2** = 10)
- **Partitioning Single-Digit Numbers** (**5** decomposes to **2** + **3**)
- **10 Plus a Single-Digit Number** (**10** + **3** = **13**)

ADD 9
+10, –1

When asked 6 + 9, the student reasons that since 6 + 10 is 16, then 6 + 9 is one less (15).

6 + **10** is <u>16</u>, so **6** + **9** is <u>**15**</u>.

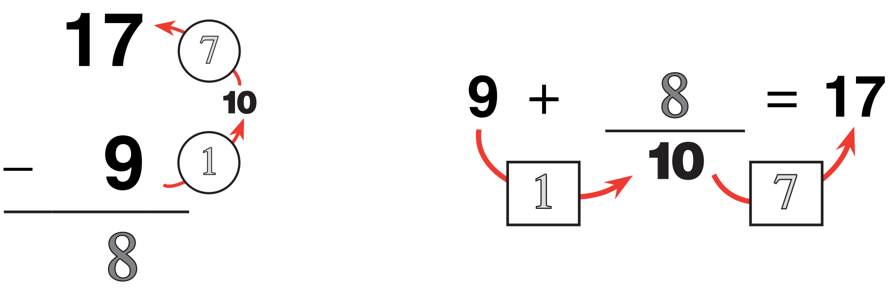
In order to use this technique, students need to have mastery of:

- **10 Plus a Single-Digit Number** (**6** + **10** = **16**)

Subtraction Facts/Missing Addends: Minuends Over 10

Up to & Over
10

When asked 17 – 9 *or* 9 + ___ = 17, the student reasons that 9 and 17 are 8 apart by taking two "jumps": from 9 ***up to*** 10 (**1**), and then from 10 ***up to*** 17 (**7**), and finds the sum of these two "jumps" (**1** + **7** = **8**).



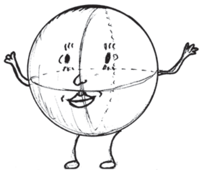
In order to use this technique, students need to have mastery of:

- **Number Bonds to 10** (9 + **1** = 10)
- **10 Plus a Single-Digit Number** (10 + <u>**7**</u> = 17)
- **Addition Facts: Sums Up to 10** (**1** + **7** = **8**)

Additional Prerequisite Skills

There are two additional prerequisite skills that support the program. They function as prereqs mainly for the purposes of sequential mathematical development and do not necessarily tie directly into the Sums Over 10 and Minuends Over 10 techniques. These two additional prerequisite skills include:

- **Adding On to the Larger Number**
- **Subtraction Facts: Minuends Up to 10**



Section 3:

Evaluating Assessment Responses
Levels of Proficiency

• *Levels of Proficiency* Guidelines Chart •

Skill Area	No Knowledge	Awareness	Competency	Mastery
1) **Adding On to the Larger Number**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
2) **10 Plus a Single-Digit Number**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
3) **Partitioning Single-Digit Numbers**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
4) **Addition Facts:** **Sums *up to* 10**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
5) **Number Bonds to 10**	All incorrect Exhibits no understanding of how to find the answer	3–9 Number Bonds that are not mastered	1–2 Number Bonds that are not mastered	All timely correct answers (No finger counting or similar technique)
6) **Doubles Facts**	All incorrect Exhibits no understanding of how to find the answer	3–9 Doubles Facts that are not mastered	1–2 Doubles Facts that are not mastered	All timely correct answers (No finger counting or similar technique)
7) **Subtraction Facts:** **Minuends *up to* 10**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
8) **Addition Facts:** **Sums *over* 10**	All incorrect Exhibits no understanding of how to find the answer	2–4 Indicators (Excluding 4 incorrect answers)	Only 1 Indicator	All timely correct answers (No finger counting or similar technique)
9) **Subtraction Facts/ Missing Addends:** **Minuends *over* 10**	All incorrect Exhibits no understanding of how to find the answer	3–8 Subtraction Facts or Missing Addends that are not mastered	1–2 Subtraction Facts or Missing Addends that are not mastered	All timely correct answers (No finger counting or similar technique)

Indicator: An **Indicator** is an *incorrect* response or a “Slow” correct response.

Mastered: Individual ***Number Bonds to 10***, ***Doubles Facts*** or ***Subtraction Facts/ Missing Addends*** are considered mastered if a student can effortlessly give a correct response within approximately 3 seconds.

SECTION 4:

PK Tiers

Tier 1 PKs

Adding On to the Larger Number
10 Plus a Single-Digit Number
Partitioning Single-Digit Numbers

Tier 1 skills must be mastered before moving on to Tier 2 skills.

Tier 2 PKs

Addition Facts: Sums Up to 10
Number Bonds to 10
Doubles Facts
Subtraction Facts: Minuends Up to 10

Tier 2 skills must be mastered before moving on to Tier 3 skills.

Tier 3 PKs

Addition Facts: Sums Over 10
Subtraction Facts/Missing Addends: Minuends over 10

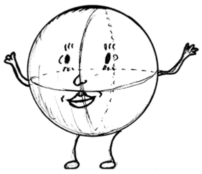
SECTION 5:

Samples of PK Pages

Tier 1
(1st PK)

Adding On to the Larger Number

8 + 1	3 + 2
1 + 7	2 + 5



• Adding On 1 or 2 More •

1) How much is **2**... *and* **1** MORE? ______

2) How much is **3**... *and* **2** MORE? ______

3) How much is **6**... *and* **1** MORE? ______

4) How much is **8**... *and* **2** MORE? ______

5) How much is **4**... *and* **1** MORE? ______

6) How much is **5**... *and* **2** MORE? ______

---

7) **7** + **2** = ______   8) **9** + **1** = ______

9) **5** + **1** = ______   10) **6** + **2** = ______

11) **4** + **2** = ______   12) **8** + **1** = ______

• Adding On to the **Larger** Number •

Direct Teaching

When adding two numbers, it's always best to find the ***larger*** number first, and then ***add on*** to it.

EXAMPLE: Circle the ***larger*** number and ***add on*** to it.

2 + **5** = **?**

5 and **2** MORE is 7, so **2** + **5** = **7**.

Try these: **Circle the larger number and add on to that number.**

1) **1** + **7** = ______

2) **3** + **2** = ______

3) **9** + **1** = ______

4) **2** + **6** = ______

5) **1** + **4** = ______

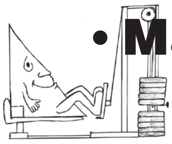
6) **1** + **2** = ______

7) **8** + **2** = ______

8) **5** + **1** = ______

9) **2** + **7** = ______

10) **1** + **3** = ______



• Mastery Check: Adding On to the Larger Number •

Part I:

1) $3 + 1 =$ ______

2) $2 + 4 =$ ______

3) $6 + 2 =$ ______

4) $1 + 5 =$ ______

5) $9 + 1 =$ ______

6) $2 + 3 =$ ______

7) $8 + 2 =$ ______

8) $1 + 6 =$ ______

Part II: This section is for Instructors to administer verbally.

9) $8 + 1$

S	Y / N
FC	

10) $3 + 2$

S	Y / N
FC	

11) $1 + 7$

S	Y / N
FC	

12) $2 + 5$

S	Y / N
FC	

TIER 1
(2nd PK)

10 Plus a Single-Digit Number

$10 + 3$

$6 + 10$

$10 + \underline{\ ?\ } = 19$

$10 + \underline{\ ?\ } = 17$

• 10 Plus a Single-Digit Number •

Direct Teaching

In the number **14**, the "**1**" ***means*** "**10**".

So, **10** *and* __4__ MORE = **14**.

---

1) **10** *and* _____ MORE = **15**

2) **10** *and* _____ MORE = **16**

3) _____ *and* **3** MORE = **13**

4) _____ *and* **7** MORE = **17**

5) **10** *and* _____ MORE = **19**

6) **10** *and* _____ MORE = **11**

7) _____ *and* **4** MORE = **14**

8) _____ *and* **8** MORE = **18**

9) **10** *and* _____ MORE = **12**

• 10 Plus a Single-Digit Number •

1) **10** *and* _____ MORE = **12**

2) _____ *and* **4** MORE = **14**

3) **10** *and* _____ MORE = **11**

4) _____ *and* **6** MORE = **16**

5) **10** *and* _____ MORE = **15**

6) _____ *and* **9** MORE = **19**

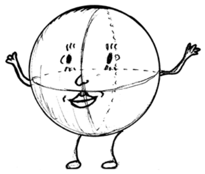
---

7) How much is **10** *and* **8** MORE? _____

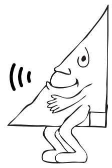
8) How much is **10** *and* **3** MORE? _____

9) How much is **10** *and* **7** MORE? _____

10) How much is **10** *and* **4** MORE? _____



• 10 Plus a Single-Digit Number •



Let's Talk About It!

1) 10 + ______ = 15 2) 10 + 5 = ______

3) ______ + 2 = 12 4) 2 + 10 = ______

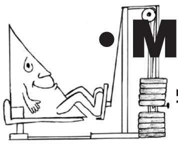
5) 10 + ______ = 19 6) 10 + 9 = ______

7) ______ + 6 = 16 8) 6 + 10 = ______

9) 10 + ______ = 18 10) 10 + 8 = ______

11) ______ + 4 = 14 12) 4 + 10 = ______

13) 10 + ______ = 17 14) 10 + 7 = ______



• Mastery Check: 10 Plus a Single-Digit Number •

☆

Part I:

1) $10 + 7 =$ _____

2) $2 + 10 =$ _____

3) $10 + 5 =$ _____

4) $10 +$ _____ $= 18$

5) $10 +$ _____ $= 11$

6) $10 +$ _____ $= 14$

Part II: This section is for Instructors to administer verbally.

7) $10 + 3$

S	Y / N
FC	

8) $6 + 10$

S	Y / N
FC	

9) $10 +$ __?__ $= 19$

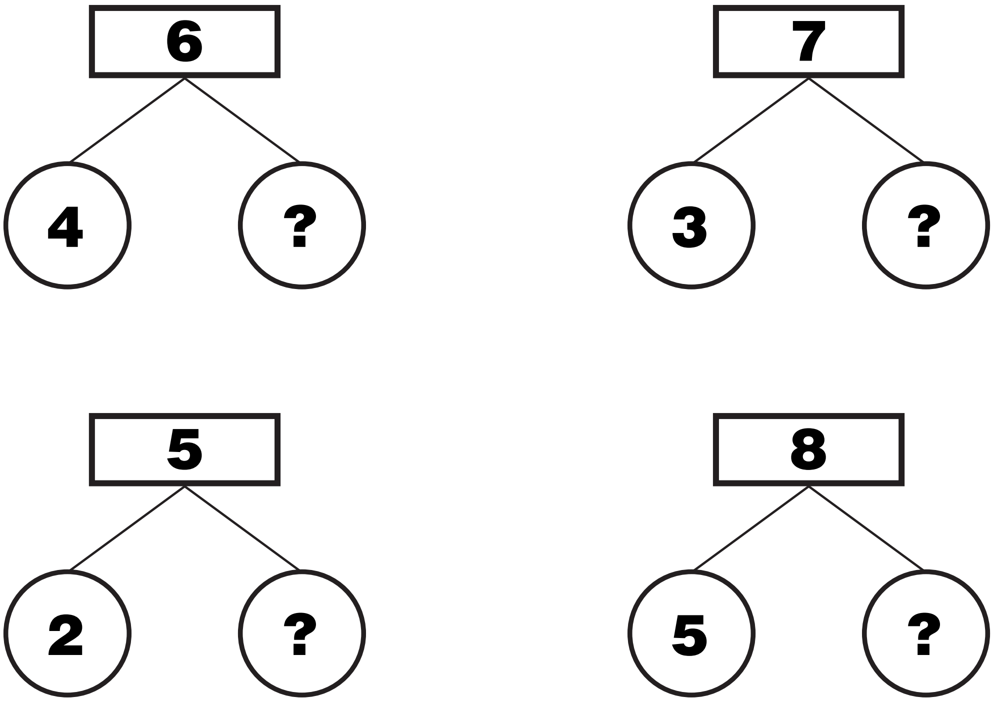
S	Y / N
FC	

10) $10 +$ __?__ $= 17$

S	Y / N
FC	

TIER 1
(3rd PK)

Partitioning Single-Digit Numbers



• Close Together or Far Apart •

Circle the pair of numbers that *feels* **CLOSE TOGETHER.**

1) 7 and 9 **or** 7 and 2

2) 6 and 1 **or** 6 and 5

3) 3 and 5 **or** 1 and 5

4) 9 and 2 **or** 9 and 6

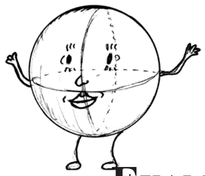
Circle the pair of numbers that *feels* **FAR APART.**

5) 8 and 2 **or** 8 and 5

6) 1 and 4 **or** 3 and 4

7) 5 and 7 **or** 2 and 7

8) 6 and 4 **or** 6 and 2



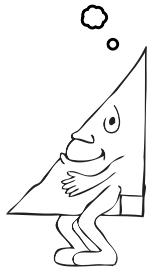
• *Going* Up To •

EXAMPLE:

7

5 ?

Let's find the
MISSING PART
by counting *up*.



Since **5** and **7** *feel* **CLOSE TOGETHER**...

Let's **THINK**...

How far is it from **5** ***up to*** **7**? __2__

Try these: **In each exercise, find the MISSING PART.**

1) **5**

4 ?

How far is it from

4 ***up to*** **5**? _____

2) **9**

7 ?

How far is it from

7 ***up to*** **9**? _____

3) **8**

6 ?

How far is it from

6 ***up to*** **8**? _____

4) **6**

5 ?

How far is it from

5 ***up to*** **6**? _____

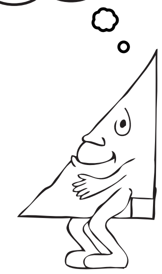
• Taking Away •

EXAMPLE:

8

2 ?

Let's find the **MISSING PART** by counting *back*.



Since **2** and **8** *feel* **FAR APART**...

Let's **THINK**...

Take away **2** from **8**. How much is **left**? 6

Try these: **In each exercise, find the MISSING PART.**

1) **9**

2 ?

Take away **2** from **9**.

How much is **left**? _____

2) **6**

1 ?

Take away **1** from **6**.

How much is **left**? _____

3) **7**

1 ?

Take away **1** from **7**.

How much is **left**? _____

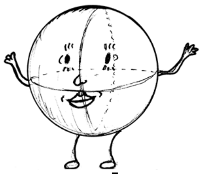
4) **5**

2 ?

Take away **2** from **5**.

How much is **left**? _____

Direct Teaching



• Up To or Take Away •

In each exercise, find the **MISSING PART.**

1)

9

7 ?

How far is it from

7 ***up to*** **9**? _____

2)

5

2 ?

Take away **2** from **5**.

How much is **left**? _____

3)

4

up to

3

4)

8

take away

2

5)

7

up to

5

6)

9

take away

1

7)

7

up to

4

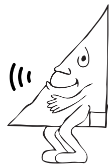
8)

6

take away

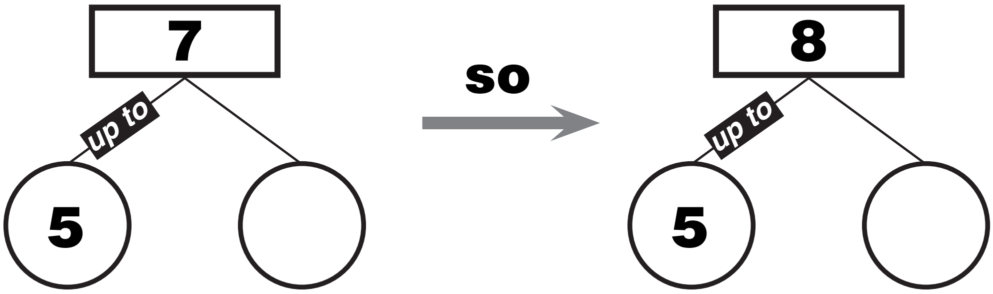
2

• Up To or Take Away in *Steps* •



Let's Talk About It!

1)



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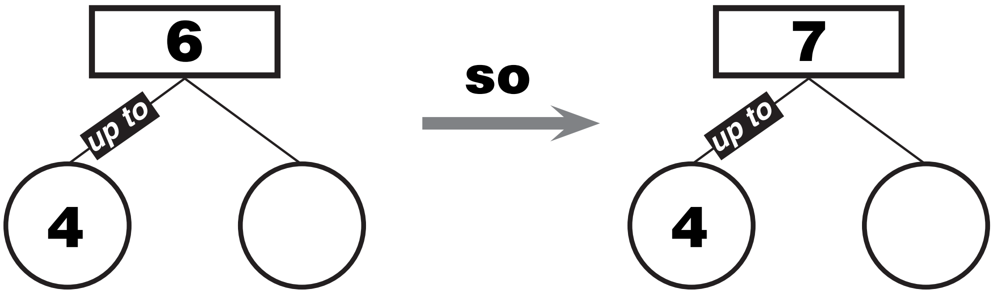

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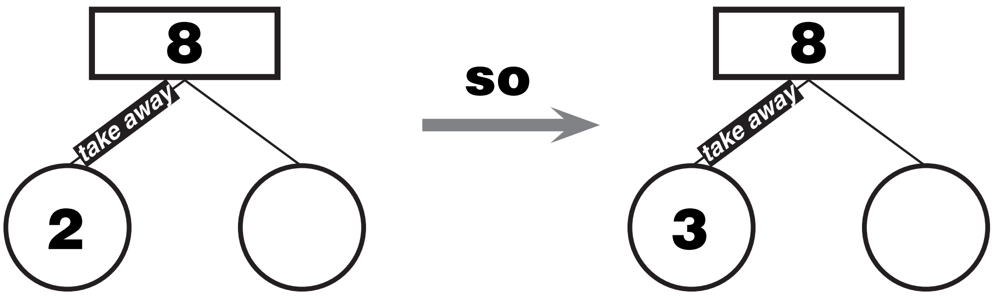
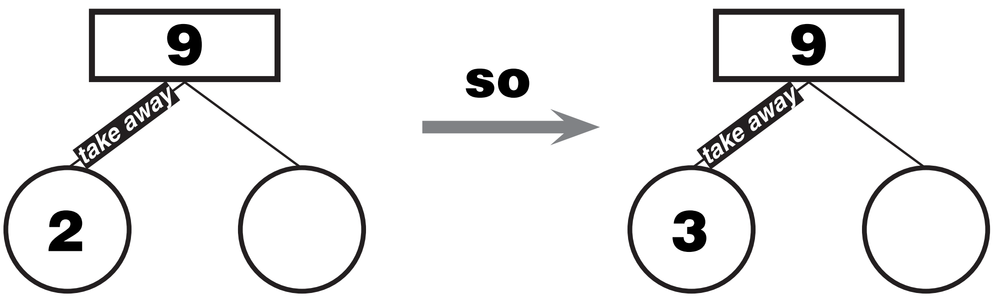
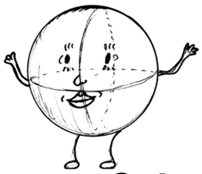


4)





• Up To or Take Away •

Solve. Then circle the phrase that explains how you found your answer.

1) 8
6 ◯
UP TO TAKE AWAY

2) 7
2 ◯
UP TO TAKE AWAY

3) 9
3 ◯
UP TO TAKE AWAY

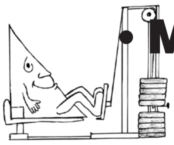
4) 4
3 ◯
UP TO TAKE AWAY

5) 8
5 ◯
UP TO TAKE AWAY

6) 5
1 ◯
UP TO TAKE AWAY

7) 6
5 ◯
UP TO TAKE AWAY

8) 7
4 ◯
UP TO TAKE AWAY



• Mastery Check: Partitioning Single-Digit Numbers •

1) 6
4

2) 7
3

3) 5
2

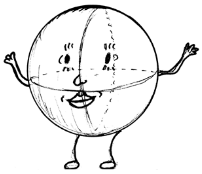
4) 8
5

5) 9
3

6) 4
3

7) 7
5

8) 8
2



Tier 2
(1st PK)

Addition Facts: Sums Up to 10

$6 + 2$ $\qquad$ $4 + 3$

$3 + 7$ $\qquad$ $5 + 4$

• Adding On 2 More •

1) 8 + 1 is ______, so 8 + 2 is ______.

2) 3 + 1 is ______, so 3 + 2 is ______.

3) 7 + 1 is ______, so 7 + 2 is ______.

4) 6 + 1 is ______, so 6 + 2 is ______.

---

5) 5 + 2 = ______

6) 3 + 2 = ______

7) 8 + 2 = ______

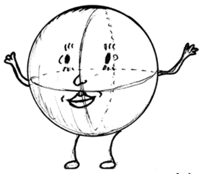
8) 6 + 2 = ______

9) 2 + 4 = ______

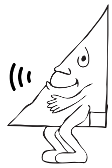
10) 2 + 2 = ______

11) 7 + 2 = ______

12) 2 + 5 = ______



• Adding On 3 or 4 More •



1) **7** *and* **2** MORE is ______, so **7** *and* **3** MORE is ______.

2) **6** *and* **2** MORE is ______, so **6** *and* **3** MORE is ______.

3) **4** *and* **2** MORE is ______, so **4** *and* **4** MORE is ______.

4) **5** *and* **2** MORE is ______, so **5** *and* **4** MORE is ______.

---

5) **7** + **2** is ______, so **7** + **3** is ______.

6) **2** + **2** is ______, so **2** + **3** is ______.

7) **4** + **2** is ______, so **4** + **4** is ______.

8) **5** + **2** is ______, so **5** + **3** is ______.

9) **6** + **2** is ______, so **6** + **4** is ______.

• Adding On 3 or 4 More •

1) 6 + 2 is ______, so 6 + $\underline{4}$ is ______.

2) 5 + 2 is ______, so 5 + 3 is ______.

3) 3 + 3 is ______, so 3 + $\underline{4}$ is ______.

4) 7 + 2 is ______, so 7 + 3 is ______.

---

5) 4 + 3 = ______

6) 4 + 4 = ______

7) 6 + 3 = ______

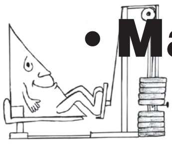
8) 2 + 3 = ______

9) 5 + 4 = ______

10) 3 + 7 = ______

11) 4 + 6 = ______

12) 3 + 5 = ______



• Mastery Check: Addition Facts – Sums Up to 10 • ☆

Part I:

1) $2 + 3 =$ ______

2) $3 + 6 =$ ______

3) $5 + 2 =$ ______

4) $1 + 7 =$ ______

5) $2 + 4 =$ ______

6) $8 + 2 =$ ______

7) $3 + 5 =$ ______

8) $4 + 6 =$ ______

Part II: This section is for Instructors to administer verbally.

9) $6 + 2$

S	Y / N
FC	

10) $4 + 3$

S	Y / N
FC	

11) $5 + 4$

S	Y / N
FC	

12) $3 + 7$

S	Y / N
FC	

TIER 2
(2nd PK)

Number Bonds to 10

6	?
10	

3	?
10	

1	?
10	

8	?
10	

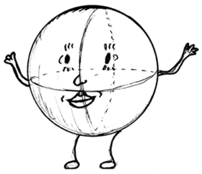
5	?
10	

7	?
10	

4	?
10	

9	?
10	

2	?
10	



• Up To or Take Away •

In each exercise, find the **MISSING PART.**

1) 10

7 ?

How far is it from

7 ***up to*** **10**? _____

2) 10

4 ?

Take away **4** from **10**.

How much is **left**? _____

3) 10

take away

2

4) 10

up to

9

5) 10

up to

6

6) 10

take away

3

7) 10

take away

1

8) 10

up to

8

• Number Bonds to **10** •

In each exercise, circle the two numbers that add up to **10**.

1) 7 6 3

2) 2 8 4

3) 6 5 4

4) 9 5 5

5) 1 9 3

6) 8 3 7

Complete the **10 PUZZLES**.

7) 8 | 10

8) 1 | 10

9) 3 | 10

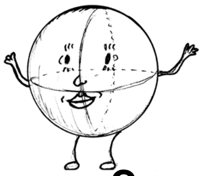
10) 6 | 10

11) 9 | 10

12) 7 | 10

13) 2 | 10

14) 4 | 10



• Number Bonds to 10 •

Complete the **10 PUZZLES**.

1) 7 | 10

2) 3 | 10

3) 8 | 10

4) 2 | 10

5) 6 | 10

6) 4 | 10

---

7) 10 + _____ = 10

9 + _____ = 10

8 + _____ = 10

7 + _____ = 10

6 + _____ = 10

5 + _____ = 10

8) 0 + _____ = 10

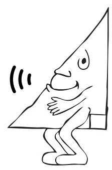
1 + _____ = 10

2 + _____ = 10

3 + _____ = 10

4 + _____ = 10

5 + _____ = 10



Let's Talk About It!

• Number Bonds to 10 •

☆

1) 7 + _____ = 10

2) 4 + _____ = 10

3) 8 + _____ = 10

4) 9 + _____ = 10

Complete the **10 PUZZLES**.

5) 1 / 10

6) 10 / 10

7) 6 / 10

8) 3 / 10

9) 2 / 10

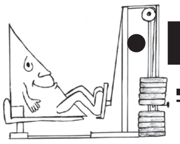
10) 4 / 10

11) 9 / 10

12) 8 / 10

13) 7 / 10

14) 5 / 10



• Mastery Check: Number Bonds to 10 •

☆

1) 6 | 10

2) 3 | 10

3) 1 | 10

4) 8 | 10

5) 5 | 10

6) 7 | 10

7) 0 | 10

8) 4 | 10

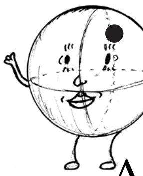
9) 9 | 10

10) 2 | 10

TIER 2
(3rd PK)

Doubles Facts

4 + 4	9 + 9	6 + 6
8 + 8	3 + 3	11 + 11
7 + 7	10 + 10	12 + 12

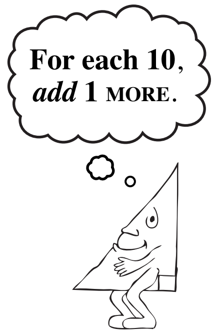


• Solving **Doubles** Facts with **5 + 5** or **10 + 10** •

Direct Teaching

A great way to solve some of the **DOUBLES FACTS** is to start with either **5 + 5** or **10 + 10**, and then ***add*** more or ***take away*** some.

EXAMPLE: To solve **11 + 11**, you can start with **10 + 10**.

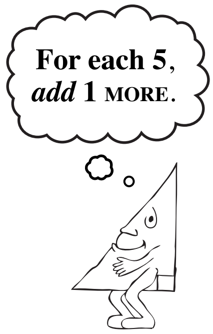


$$\begin{array}{c} 10 + 10 \\ {\scriptstyle +1}\downarrow \quad \downarrow{\scriptstyle +1} \\ 11 + 11 \end{array}$$

11 + 11 *is:* $\underset{10\ +\ 10}{\underline{\ 20\ }}$ ***and*** $\underset{\text{HOW MUCH}}{\underline{\ 2\ }}$ MORE = $\underset{11\ +\ 11}{\underline{\ 22\ }}$

Try these:

1) To solve **6 + 6**, you can start with **5 + 5**.

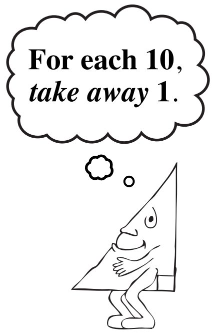


$$\begin{array}{c} 5 + 5 \\ {\scriptstyle +1}\downarrow \quad \downarrow{\scriptstyle +1} \\ 6 + 6 \end{array}$$

6 + 6 *is:* $\underset{5\ +\ 5}{\underline{\qquad\quad}}$ ***and*** $\underset{\text{HOW MUCH}}{\underline{\qquad\quad}}$ MORE = $\underset{6\ +\ 6}{\underline{\qquad\quad}}$

---

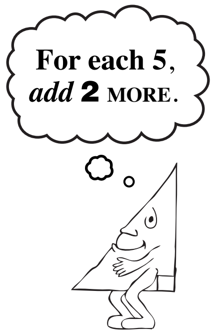
2) To solve **9 + 9**, you can start with **10 + 10**.



$$\begin{array}{c} 10 + 10 \\ {\scriptstyle -1}\downarrow \quad \downarrow{\scriptstyle -1} \\ 9 + 9 \end{array}$$

9 + 9 *is:* $\underset{10\ +\ 10}{\underline{\qquad\quad}}$ ***take away*** $\underset{\text{HOW MUCH}}{\underline{\qquad\quad}}$ = $\underset{9\ +\ 9}{\underline{\qquad\quad}}$

• Solving **Doubles** Facts •



1) To solve **7 + 7**, you can start with **5 + 5**.

$$\begin{matrix} & 5 + 5 & \\ +2 \downarrow & & \downarrow +2 \\ & 7 + 7 & \end{matrix}$$

7 + 7 *is:* _______ (**5 + 5**) *and* _______ (HOW MUCH) MORE = _______ (**7 + 7**)

---

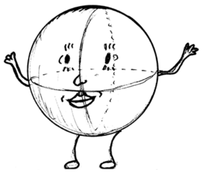
2) **9 + 9** *is:* _______ (**10 + 10**) *take away* _______ (HOW MUCH) = _______ (**9 + 9**)

3) **11 + 11** *is:* _______ (**10 + 10**) *and* _______ (HOW MUCH) MORE = _______ (**11 + 11**)

4) **7 + 7** *is:* _______ (**5 + 5**) *and* _______ (HOW MUCH) MORE = _______ (**7 + 7**)

5) **6 + 6** *is:* _______ (**5 + 5**) *and* _______ (HOW MUCH) MORE = _______ (**6 + 6**)

6) **12 + 12** *is:* _______ (**10 + 10**) *and* _______ (HOW MUCH) MORE = _______ (**12 + 12**)



• Comparing **Doubles** Facts •

1) **Which doubles fact is *bigger*? (Circle your answer.)**

5 + 5 **or** **6 + 6**

By ***how much***? ______

5 + 5 is ______, so **6 + 6** is ______.

---

2) **Which doubles fact is *bigger*? (Circle your answer.)**

10 + 10 **or** **9 + 9**

By ***how much***? ______

10 + 10 is ______, so **9 + 9** is ______.

---

3) **Which doubles fact is *bigger*? (Circle your answer.)**

10 + 10 **or** **12 + 12**

By ***how much***? ______

10 + 10 is ______, so **12 + 12** is ______.

• Comparing **Doubles** Facts •

1) **Which doubles fact is *bigger*? (Circle your answer.)**

10 + 10 **or** **9 + 9**

By ***how much***? ______

10 + 10 is ______, so **9 + 9** is ______.

---

2) **5 + 5** is ______, so **6 + 6** is ______.

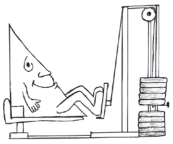
3) **10 + 10** is ______, so **8 + 8** is ______.

4) **5 + 5** is ______, so **7 + 7** is ______.

5) **10 + 10** is ______, so **12 + 12** is ______.

6) **5 + 5** is ______, so **4 + 4** is ______.

7) **10 + 10** is ______, so **11 + 11** is ______.



• Mastery Check: Doubles Facts •

Part I:

1) $3 + 3 =$ ______

2) $10 + 10 =$ ______

3) $7 + 7 =$ ______

4) $5 + 5 =$ ______

5) $9 + 9 =$ ______

6) $11 + 11 =$ ______

Part II: This section is for Instructors to administer verbally.

7) $4 + 4$

S	Y / N
FC	

8) $8 + 8$

S	Y / N
FC	

9) $6 + 6$

S	Y / N
FC	

10) $12 + 12$

S	Y / N
FC	

TIER 2
(4th PK)

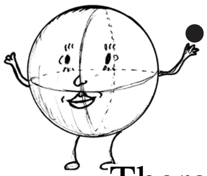
Subtraction Facts: Minuends Up to 10

9 – 7

8 – 2

10 – 3

7 – 4

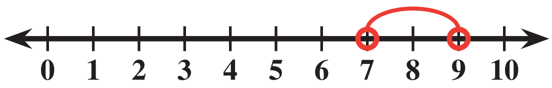
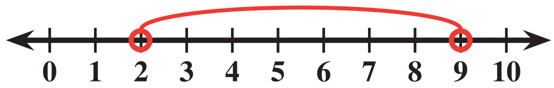


• *Two* Ways to Think About **Subtraction** •

☆

Direct Teaching

There are ***two*** ways we can think about **subtraction**.

UP TO	TAKE AWAY
EXAMPLE 1:	**EXAMPLE 2:**
$9 - 7 = \underline{\ ?\ }$	$9 - 2 = \underline{\ ?\ }$
Since **9** and **7** *feel* **CLOSE TOGETHER**	Since **9** and **2** *feel* **FAR APART**
	
Let's **THINK**...	Let's **THINK**...
How far is it from **7** ***up to*** **9**? __2__	***Take away*** **2** from **9**. How much is **left**? __7__

Try these:

1) $5 - 3 = \underline{\ ?\ }$

How far is it from **3** ***up to*** **5**? ______ 5 – 3

2) $7 - 2 = \underline{\ ?\ }$

Take away **2** from **7**. How much is **left**? ______ 7 – 2

3) $10 - 9 = \underline{\ ?\ }$

How far is it from **9** ***up to*** **10**? ______ 10 – 9

4) $8 - 3 = \underline{\ ?\ }$

Take away **3** from **8**. How much is **left**? ______ 8 – 3

• Subtract by *Going* **Up To** •

1) **10 – 8 = ?**

How far is it from
8 *up to* 10? ______ 10 – 8

2) **8 – 5 = ?**

How far is it from
5 *up to* 8? ______ 8 – 5

3) **5 – 4 = ?**

How far is it from
4 *up to* 5? ______ 10 – 9

4) **9 – 7 = ?**

How far is it from
7 *up to* 9? ______ 9 – 7

Solve by *going* UP TO the bigger number.

5) 7 – 4 = ______

6) 8 – 6 = ______

7) 9 – 8 = ______

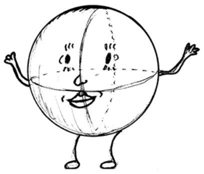
8) 10 – 7 = ______

9) 7 – 5 = ______

10) 4 – 2 = ______

11) 8 – 4 = ______

12) 6 – 5 = ______



• Subtract by **Taking Away** •

1) $9 - 1 =$ **?**

Take away **1** from **9**.

How much is **left**? ______
$9 - 1$

2) $7 - 3 =$ **?**

Take away **3** from **7**.

How much is **left**? ______
$7 - 3$

3) $8 - 3 =$ **?**

Take away **3** from **8**.

How much is **left**? ______
$8 - 3$

4) $10 - 2 =$ **?**

Take away **2** from **10**.

How much is **left**? ______
$10 - 2$

Solve by TAKING AWAY.

5) $7 - 2 =$ ______

6) $9 - 4 =$ ______

7) $10 - 1 =$ ______

8) $5 - 2 =$ ______

9) $9 - 3 =$ ______

10) $8 - 2 =$ ______

11) $6 - 1 =$ ______

12) $10 - 3 =$ ______

• Up To or Take Away •

Solve by *going* **UP TO** the bigger number.

1) $10 - 8 =$ _____

2) $7 - 6 =$ _____

3) $9 - 7 =$ _____

4) $8 - 5 =$ _____

5) $10 - 6 =$ _____

6) $6 - 4 =$ _____

7) $5 - 3 =$ _____

8) $9 - 5 =$ _____

Solve by **TAKING AWAY**.

9) $7 - 2 =$ _____

10) $8 - 1 =$ _____

11) $10 - 3 =$ _____

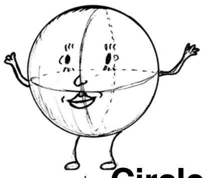
12) $5 - 2 =$ _____

13) $8 - 3 =$ _____

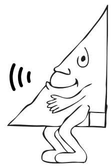
14) $10 - 4 =$ _____

15) $6 - 2 =$ _____

16) $9 - 1 =$ _____



• Choosing the Best Method •



Let's Talk About It!

Circle the fact that would be best solved by *going* **UP TO** the bigger number since the numbers *feel* **CLOSE TOGETHER.**

1)	$9 - 2$	$8 - 3$	$9 - 7$
2)	$7 - 4$	$10 - 1$	$6 - 2$
3)	$8 - 2$	$9 - 6$	$6 - 1$
4)	$5 - 1$	$8 - 6$	$10 - 2$
5)	$7 - 1$	$7 - 2$	$8 - 4$
6)	$8 - 2$	$9 - 5$	$9 - 3$

Solve by *going* **UP TO** the bigger number.

7) $9 - 7 =$ _____

8) $10 - 6 =$ _____

9) $5 - 4 =$ _____

10) $7 - 4 =$ _____

11) $8 - 6 =$ _____

12) $9 - 5 =$ _____

• Up To or Take Away •

Solve. Then circle the phrase that explains how you found your answer.

1) $8 - 6 =$ _____

UP TO TAKE AWAY

2) $9 - 2 =$ _____

UP TO TAKE AWAY

3) $7 - 1 =$ _____

UP TO TAKE AWAY

4) $6 - 5 =$ _____

UP TO TAKE AWAY

5) $10 - 7 =$ _____

UP TO TAKE AWAY

6) $5 - 2 =$ _____

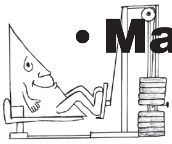
UP TO TAKE AWAY

7) $9 - 5 =$ _____

UP TO TAKE AWAY

8) $7 - 3 =$ _____

UP TO TAKE AWAY



• **Mastery Check:** Subtraction Facts – Minuends Up to 10 • ☆

Part I:

1) $6 - 4 =$ _____

2) $9 - 5 =$ _____

3) $8 - 3 =$ _____

4) $10 - 8 =$ _____

5) $7 - 2 =$ _____

6) $5 - 3 =$ _____

7) $10 - 6 =$ _____

8) $6 - 2 =$ _____

Part II: This section is for Instructors to administer verbally.

9) $9 - 7$

S	Y / N
FC	

10) $8 - 2$

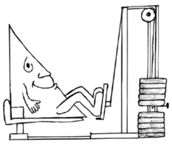
S	Y / N
FC	

11) $10 - 3$

S	Y / N
FC	

12) $7 - 4$

S	Y / N
FC	



TIER 3

(1st PK)

Addition Facts: Sums Over 10

$7 + 8$

$6 + 9$

$8 + 5$

$4 + 7$

• Doubles *Plus* 1 •

Direct Teaching

When the numbers in an addition fact are **1** ***apart***, you can use DOUBLES FACTS as a starting point and then ***add*** **1**.

EXAMPLE: **6** + **7** = **?**

Since **6** and **7** are **1** ***apart***, let's start with **6** + **6** and then ***add*** **1**.

$$\begin{array}{c} 6 + 6 \\ \big\downarrow {+1} \\ 6 + 7 \end{array}$$

6 + **6** is 12, so **6** + **7** is 13.

Try these:

1) **4** + **4** is _____, so **4** + **5** is _____.

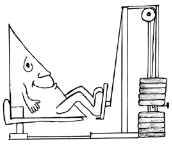
2) **7** + **7** is _____, so **7** + **8** is _____.

3) **8** + **8** is _____, so **8** + **9** is _____.

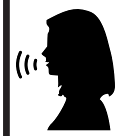
4) **5** + **5** is _____, so **5** + **6** is _____.

5) **3** + **3** is _____, so **3** + **4** is _____.

6) **11** + **11** is _____, so **11** + **12** is _____.



• Doubles *Minus* 1 •



Direct Teaching

Another way to solve an addition fact with numbers that are **1** ***apart*** is to start with a DOUBLES FACT and then ***take away*** **1**.

EXAMPLE: **9** + **8** = __?__

Since **9** and **8** are **1** ***apart***, let's start with **9** + **9** and then ***take away*** **1**.

$$9 + 9 \xrightarrow{-1} 9 + 8$$

9 + **9** is __18__, so **9** + **8** is __17__.

Try these:

1) **5** + **5** is _____, so **5** + **4** is _____.

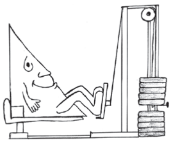
2) **7** + **7** is _____, so **7** + **6** is _____.

3) **4** + **4** is _____, so **4** + **3** is _____.

4) **8** + **8** is _____, so **8** + **7** is _____.

5) **12** + **12** is _____, so **12** + **11** is _____.

6) **6** + **6** is _____, so **6** + **5** is _____.



• Doubles *Plus* or *Minus* 1 •

Think about the **double** and then *add* **1**.

1) $4 + 5 =$ _____

2) $6 + 7 =$ _____

3) $10 + 11 =$ _____

4) $8 + 9 =$ _____

5) $7 + 8 =$ _____

6) $3 + 4 =$ _____

7) $12 + 13 =$ _____

8) $5 + 6 =$ _____

Think about the **double** and then *take away* **1**.

9) $8 + 7 =$ _____

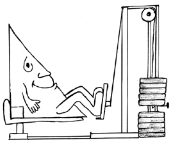
10) $5 + 4 =$ _____

11) $9 + 8 =$ _____

12) $6 + 5 =$ _____

13) $12 + 11 =$ _____

14) $7 + 6 =$ _____



• Comparing to **10** •

Circle your answers.

1) Is the answer to **6** + **5** *more than* **10**? YES / NO

2) Is the answer to **3** + **6** *more than* **10**? YES / NO

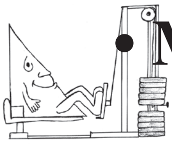
3) Is the answer to **9** + **1** *more than* **10**? YES / NO

4) Is the answer to **7** + **5** *more than* **10**? YES / NO

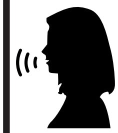
5) Is the answer to **8** + **6** *more than* **10**? YES / NO

In each exercise, circle all the facts with answers that are MORE THAN 10.

6)	**7 + 3**	**7 + 4**	**7 + 2**
7)	**3 + 9**	**6 + 2**	**8 + 4**
8)	**4 + 3**	**9 + 5**	**3 + 8**
9)	**5 + 6**	**7 + 6**	**4 + 5**

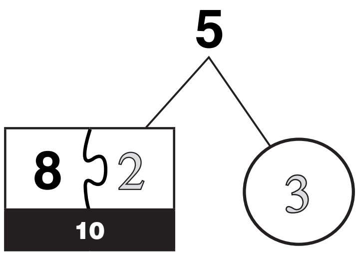


• Making 10 and Finding the *Leftover* Part •



Direct Teaching

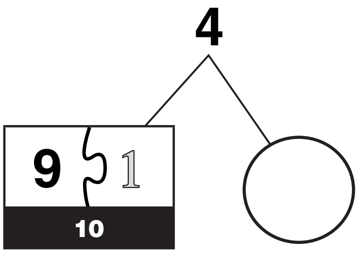
Example: Break **5** into the part that brings **8** *up to* **10** (2). Then find the ***leftover*** part of **5** (3).



Getting ***really good*** at breaking up numbers in this way helps us to solve addition facts with answers that are *more than* 10!

Try these: **In each exercise, make 10 and then find the *leftover* part.**

1)



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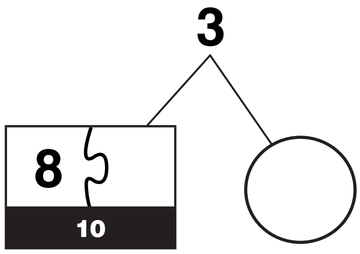

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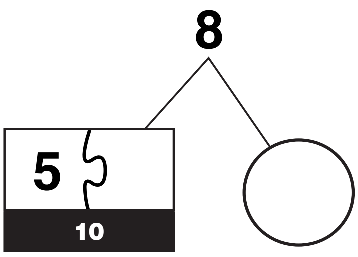
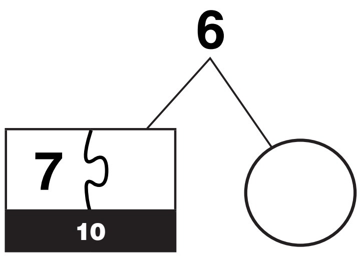
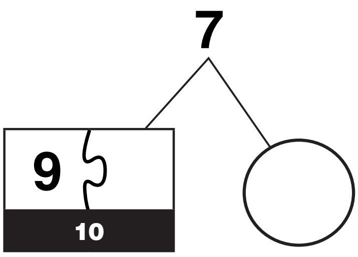


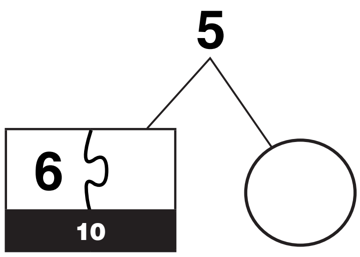
4)

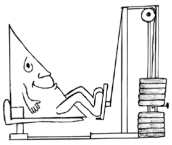


5)

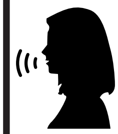


6)





• Adding *Up to* and *Over* 10 •



A great way to solve ADDITION FACTS with answers bigger than **10** is to add ***up to*** and ***over*** **10**.

EXAMPLE:

8 + 5

8 | 2 (10) + 3 = 13
(8 + 5)

Try these: **Solve using the "*Up to* and *Over* 10" method.**

1) **9 + 6**

9 | (10) + ◯ = ____
(9 + 6)

2) **7 + 5**

7 | (10) + ◯ = ____
(7 + 5)

3) **8 + 4**

8 | (10) + ◯ = ____
(8 + 4)

4) **5 + 9**

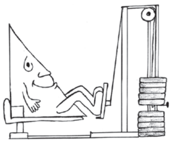
5 | (10) + ◯ = ____
(5 + 9)

5) **6 + 8**

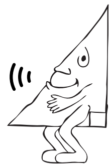
6 | (10) + ◯ = ____
(6 + 8)

6) **4 + 7**

4 | (10) + ◯ = ____
(4 + 7)



• Adding *Up to* and *Over* 10 •



Let's Talk About It!

Solve using the "*Up to* and *Over* 10" method.

1) 7 + 4 = _____
3 1

2) 9 + 7 = _____

3) 8 + 5 = _____

4) 5 + 6 = _____

5) 8 + 7 = _____

6) 9 + 3 = _____

7) 6 + 8 = _____

8) 7 + 6 = _____

9) 5 + 7 = _____

10) 6 + 9 = _____

11) 9 + 5 = _____

12) 8 + 4 = _____

• Another Way to Add **9** •

Direct Teaching

Another way to add **9** is to add **10** to the number and then ***take away*** **1**.

EXAMPLE: **6** + **9** = ___?___

Since **9** is **10** – **1,** let's start with **6** + **10** and then ***take away*** **1**.

$$\begin{array}{c} 6 + 10 \\ \big\downarrow {-1} \\ 6 + 9 \end{array}$$

6 + **10** is ___16___, so **6** + **9** is ___15___.

Try these:

1) **4** + **10** is _____, so **4** + **9** is _____.

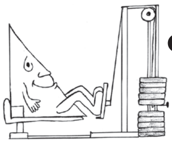
2) **5** + **10** is _____, so **5** + **9** is _____.

3) **8** + **10** is _____, so **8** + **9** is _____.

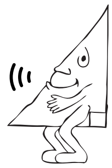
4) **7** + **10** is _____, so **7** + **9** is _____.

5) **3** + **10** is _____, so **3** + **9** is _____.

6) **6** + **10** is _____, so **6** + **9** is _____.



• Which **Methods** Work Best for *You*? •



Solve. Then circle the method you used to find your answer.

1) $8 + 5 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

2) $6 + 9 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

3) $7 + 8 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

4) $9 + 8 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

5) $8 + 4 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

6) $5 + 6 =$ _____

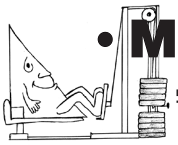
DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

7) $6 + 8 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1

8) $9 + 7 =$ _____

DOUBLES + *or* – 1 | *Up to & Over* 10 | ADD 9 + 10, – 1



• Mastery Check: Addition Facts – Sums Over 10 • ☆

Part I:

1) $7 + 5 =$ _____

2) $9 + 8 =$ _____

3) $5 + 9 =$ _____

4) $8 + 6 =$ _____

5) $7 + 4 =$ _____

6) $6 + 7 =$ _____

7) $4 + 8 =$ _____

8) $7 + 9 =$ _____

9) $9 + 3 =$ _____

10) $6 + 5 =$ _____

Part II: This section is for Instructors to administer verbally.

11) $7 + 8$

S	Y / N
FC	

12) $6 + 9$

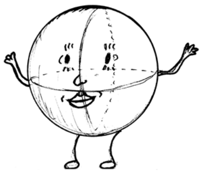
S	Y / N
FC	

13) $8 + 5$

S	Y / N
FC	

14) $4 + 7$

S	Y / N
FC	



TIER 3
(2nd PK)

Subtraction Facts/Missing Addends: Minuends Over 10

13 – 8

17 – 9

11 – 5

15 – 6

7 + __?__ = 16

9 + __?__ = 13

8 + __?__ = 14

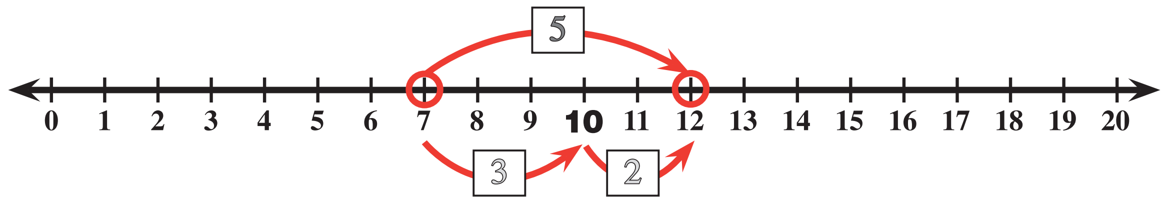
5 + __?__ = 12

• Missing Addends: *Up to* and *Over* 10 •

Direct Teaching

EXAMPLE: $7 + \underline{\ ?\ } = 12$

THINK: "*How far* is it from **7** ***up to*** **12**?"



First, let's *go* from **7** ***up to*** **10** (3). Then *go* from **10** ***up to*** **12** (2).

$$7 + \underline{\ 5\ } = 12$$

$\boxed{3}$ 10 $\boxed{2}$

To get our answer, we ***add*** the two parts together (3 + 2 = 5).

Try these: **In each exercise, solve by going *Up to* and *Over* 10.**

1) $8 + \underline{\qquad} = 13$

$\boxed{}$ 10 $\boxed{}$

2) $9 + \underline{\qquad} = 16$

$\boxed{}$ 10 $\boxed{}$

3) $7 + \underline{\qquad} = 11$

$\boxed{}$ 10 $\boxed{}$

4) $6 + \underline{\qquad} = 15$

$\boxed{}$ 10 $\boxed{}$

5) $5 + \underline{\qquad} = 12$

$\boxed{}$ 10 $\boxed{}$

6) $9 + \underline{\qquad} = 14$

$\boxed{}$ 10 $\boxed{}$

• Subtraction Facts: *Up to* and *Over* 10 •

Direct Teaching

EXAMPLE: 17 – 9 = __?__

THINK: "*How far* is it from **9** *up to* **17**?"

First, let's *go* from **9** ***up to*** **10** (1). Then *go* from **10** ***up to*** **17** (7).

$$\begin{array}{r} 17 \; (7) \\ 10 \\ -\;\; 9 \; (1) \\ \hline 8 \end{array}$$

To get our answer, we ***add*** the two parts together (1 + 7 = 8).

Try these: **In each exercise, solve by going *Up to* and *Over* 10.**

1) $$\begin{array}{r} 13 \\ 10 \\ -\;\; 8 \\ \hline \end{array}$$

2) $$\begin{array}{r} 11 \\ 10 \\ -\;\; 5 \\ \hline \end{array}$$

3) $$\begin{array}{r} 15 \\ 10 \\ -\;\; 6 \\ \hline \end{array}$$

4) $$\begin{array}{r} 12 \\ 10 \\ -\;\; 7 \\ \hline \end{array}$$

• Missing Addends: *Up to* and *Over* 10 •

☆

In each exercise, solve by going *Up to* and *Over* **10**.

1) **7** + ______ = **12**

$\square$ **10** $\square$

2) **8** + ______ = **15**

$\square$ **10** $\square$

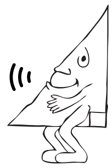
3) **9** + ______ = **17**

$\square$ **10** $\square$

4) **6** + ______ = **13**

$\square$ **10** $\square$

In each exercise, solve by going *Up to* and *Over* **10**.



Let's Talk About It!

5) 8 + _____ = 14

6) 4 + _____ = 12

7) 6 + _____ = 15

8) 7 + _____ = 11

9) 4 + _____ = 13

10) 8 + _____ = 16

11) 5 + _____ = 11

12) 9 + _____ = 12

• Subtraction Facts: *Up to* and *Over* 10 •

☆

In each exercise, solve by going *Up to* and *Over* 10.

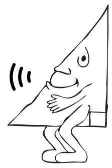
1)
$$\begin{array}{r} 11 \\ -\ \ 5 \\ \hline \end{array}$$
10

2)
$$\begin{array}{r} 12 \\ -\ \ 8 \\ \hline \end{array}$$
10

3)
$$\begin{array}{r} 15 \\ -\ \ 7 \\ \hline \end{array}$$
10

4)
$$\begin{array}{r} 16 \\ -\ \ 9 \\ \hline \end{array}$$
10

In each exercise, solve by going *Up to* and *Over* 10.



5)
$$\begin{array}{r} 14 \\ -\ \ 6 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 17 \\ -\ \ 8 \\ \hline \end{array}$$

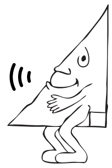
7)
$$\begin{array}{r} 12 \\ -\ \ 9 \\ \hline \end{array}$$

8)
$$\begin{array}{r} 11 \\ -\ \ 4 \\ \hline \end{array}$$

9)
$$\begin{array}{r} 16 \\ -\ \ 8 \\ \hline \end{array}$$

10)
$$\begin{array}{r} 13 \\ -\ \ 7 \\ \hline \end{array}$$

• Missing Addends and Subtraction Facts •



Let's Talk About It!

1) 9 + ______ = 12   2) 4 + ______ = 11

3) 7 + ______ = 13   4) 9 + ______ = 17

5) 8 + ______ = 11   6) 7 + ______ = 16

7) 6 + ______ = 15   8) 9 + ______ = 14

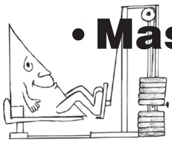
---

9) 12 – 7 = ______   10) 18 – 9 = ______

11) 15 – 9 = ______   12) 13 – 5 = ______

13) 12 – 6 = ______   14) 11 – 7 = ______

15) 14 – 5 = ______   16) 16 – 9 = ______



• Mastery Check: Sub Facts/Missing Addends – Minuends Over 10 • ☆

Part I:

1) $13 - 8 =$ _____

2) $15 - 6 =$ _____

3) $12 - 4 =$ _____

4) $14 - 5 =$ _____

5) $9 +$ _____ $= 13$

6) $7 +$ _____ $= 11$

7) $8 +$ _____ $= 16$

8) $5 +$ _____ $= 12$

Part II: This section is for Instructors to administer verbally.

9) $17 - 9$

S	Y / N
FC	

10) $11 - 5$

S	Y / N
FC	

11) $7 + \underline{\ ?\ } = 16$

S	Y / N
FC	

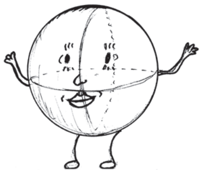
12) $8 + \underline{\ ?\ } = 14$

S	Y / N
FC	

Reflection
&
Progress Check

• Reflection: Numerical Fluency •

Share an experience you had working with a Numerical Fluency student. Are there any students who would benefit from Numerical Fluency instruction that are not currently working on the material? How will you continue to use Numerical Fluency techniques in your instruction?



• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

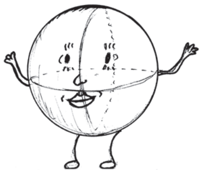
Numerical Fluency

I have a good understanding of:					
1.	Doubles ±1 technique.	**0**	**1**	**2**	**3**
2.	Up to and Over 10 technique.	**0**	**1**	**2**	**3**
3.	Add 9 (+10, –1) technique.	**0**	**1**	**2**	**3**
4.	the PK tier structure.	**0**	**1**	**2**	**3**
5.	the “Up To” and “Take Away” language for subtraction.	**0**	**1**	**2**	**3**
6.	the importance of the “Let’s Talk About It” icon.	**0**	**1**	**2**	**3**
7.	how to administer the verbal part of the Mastery Checks.	**0**	**1**	**2**	**3**

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

- ☐ Numerical Fluency Training Video (section 15, p. 1)
- ☐ Numerical Fluency Overview & PK Sampler (section 15, p. 7)
- ☐ Numerical Fluency Reflective & Critical Questions (section 15, p. 3,5)
- ☐ Numerical Fluency Reflection (section 16, p. 2)

NOTES:



Multiplication Fact Fluency Course & Instructor Development

• Multiplication Fact Fluency •

Watch the <u>Right Instruction - Teach the Mathnasium Way: Multiplication Fact Fluency</u> training video. Refer to the Multiplication Fact Fluency Instruction Manual and Curriculum Sampler (p. 7 – 65) as you complete the course. Work through the manual to familiarise yourself with the MFF program. Additionally, complete the Reflective and Critical Questions on pages 3 and 5 upon completion of the course.

This course will prepare you to instruct students that are working on Multiplication Fact Fluency curriculum.

OBJECTIVES:

In this course, you will:

1. learn about Number Facts and Numerical Fluency.
2. receive an overview of the Multiplication Fact Fluency methodology and curriculum.
3. observe real student engage in practice multiplication activities.
4. learn how to evaluate Multiplication Fact Mastery.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the "Training Videos" section, click on "Right Instruction - Teach the Mathnasium Way: Multiplication Fact Fluency Instruction".

NOTES:

___

___

___

___

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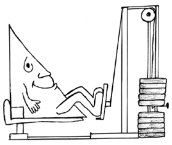

• Reflective and Critical Questions •

Answer the following questions AFTER completing the MFF Course and Manual.

1) How are the MFF Prerequisite Skills leveraged in the MFF techniques?

2) How can the MFF techniques be scaled for larger multiplication exercises like 40×53?

3) How can MFF techniques be applied to division exercises like $1{,}000 \div 8$ and $207 \div 23$?

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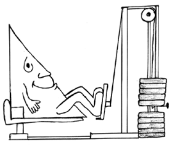
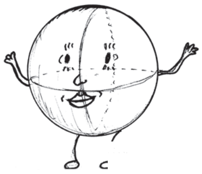


• Reflective and Critical Questions •

4) Explain the method you would use to teach 6×12 to a student.

5) Illustrate the visual or use of manipulatives that would accompany your 6×12 explanation.

6) What multiplication activity/game would you play with a student who is working on their 12s facts?



The
Multiplication Fact Fluency Program

Instruction Manual and Curriculum Sampler

Our story begins with:

When asked, "How much is 8×7?" two adults responded in perfect unison: "56." When asked how they knew that, Adult #1 said, "I memorised it when I was a kid." Adult #2 responded, "7 times 7 is 49, plus 7 is 56."

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Two brothers, a Year 5 student and a Year 7 student, came into a Mathnasium. The Year 5 student needed help with maths. according to Dad, whereas the Year 7 student didn't. While the Year 5 student was doing his Assessment, our savvy Centre Director went over to the Year 7 student and said, "So, I hear you're pretty good at maths."

The boy responded, "Yeah."

The Centre Director asked, "How much is 12×12?"

"144," the boy shot back.

"Good job… So, how much is 12×13?"

Knowing 12×12, the boy could not come up with 12×13. As fate would have it, sitting next to the boy was a Year 4 student who had learned multiplication at Mathnasium. He said, "If twelve 12s (note that he did not say "12 times 12") are 144, then thirteen 12s is just one more 12 on top, so the answer is 156." We got two sign ups that morning because Dad realised that his oldest son was trapped in the Times Table chart and that, while the boy had instant recall of the facts in the box, he had no idea how to leverage that knowledge to promote mathematical learning.

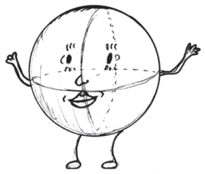
I tell this story to make clear the difference between sterile, static knowledge—knowledge usable only in its own right—and pregnant, dynamic knowledge—knowledge that has progeny, i.e., knowledge that forms the basis for future learning.

Type I Learning — Memorisation

In the story above, the Year 7 student learned the Times Tables by rote memorisation. Considering the amount of time and effort it takes to learn the Times Tables, there is definitely an appeal to learning this way. The problem, as we have seen, is that it does not automatically translate into a foundation for future learning.

Type 2 Learning — Construction

Adult #2 and the Year 4 student above did what nearly all do with respect to their Times Tables, namely, they constructed the facts they NEED from facts they KNOW.



Quick, what is 11 × 12?

> Did you know "132" from memory or did you construct it ("12 × 10 = 120 and 120 + 12 = 132," or perhaps "12 × 12 = 144 and 144 – 12 = 132")?

Unless you recently needed to learn that 11 × 12 = 132, chances are that you constructed the answer.

Next question: 9 × 12 = ____? Do you have "108" memorised or did you construct it (by actually multiplying 9 × 12 or from 12 × 10 = 120 and 120 – 12 = 108 or ...)?

The 25 to 144 Zone

The range of numbers used in Addition and Subtraction Fluency is 0 to 24, numbers familiar to students in the early year groups. This range is extended in Multiplication Fluency to include numbers from 0 to 144. This observation presents some unexpected and important challenges.

> Learning Times Tables often begins at the end of Year 2. Many kids do not have a working relationship with 48, 60, 81, 96, 132 or 144, to name just a few.
>
> Kids interact on a daily basis with numbers in the 0 to 24 range: 5 people in the family, 15 kids on a sports team, 20 minutes until dinner, a $2.00 a week allowance.
>
> Interaction with numbers in the 25 to 144 range is not an everyday experience for most kids. (Try to imagine 76 trombones... How long will it take to drink those 99 bottles of beer on the wall?...) As a result, when students are told to memorise "11 × 12 = 132," they often have no real idea what 132 represents. For this child, 132 is more or less a nonsense word like ishCaDabble—just numbers without any context.

It's a bit of 1, 2, 3, infinity (many).

As a result, the process of learning (internalising, mastering, learning by heart) the Times Tables takes on a new dimension compared with mastering addition and subtraction facts and we must adjust our expectations of how kids are going to learn them.

Prerequisite Skills

Students must demonstrate mastery of the Numerical Fluency: Addition and Subtraction Facts program.before beginning the Multiplication Fact Fluency. This program.leverages the addition and subtraction techniques found in the Numerical Fluency Program to build fluency with multiplication.

Additionally, the students are evaluated on these prerequisite skills:

- Half of Multiples of 10
- Doubles up to 50
- 10 More Than and 10 Less Than a Number
- Adding over Multiples of 10
- Subtracting through Multiples of 10

Multiplication Fact Order

Students are introduced to multiplication facts in the following order: 0s, 1s, 2s, 5s, 10s, then 3s, 4s, 11s, 12s, 9s, 6s, 7s and 8s.

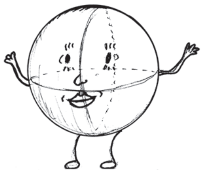
Multiplication Games

Each Multiplication Prescriptive contains a page designed to have the child go off and practice their multiplication facts is a more “fun” way than just answering questions on a page.

This page is placed right before the Mastery Check and requires 4 punches completed on at least two dates to pass.

It will also be necessary for a students to practice between PKs and periodically after completion to maintain efficiency.

See **Multiplication Activities** on page 49.



Strategies Used in Prescriptives

The materials provided here are broken up into numerical PKs but utilise a standards set of strategies across them. These include:

> **Multiplication as repeated addition** – This is a fundamental concept that many students who "have" most of their multiplication facts do not always grasp. It is presented at least in review in each PK, though has decreasing emphasis as a student progresses.
>
> **Commutative property of Multiplication** – Standards across the country are requiring recognition and usage of the idea of what we are calling "Turnaround facts"; a requirement for students to not only understand the commutative nature of multiplication but to immediately recognise it and use it for problem solving.
>
> **Launching points** – the strategy of launching points is presented in different ways throughout the materials, without the actual term. What we want students to come away with is the idea that using what you know to find what you don't is always a good strategy. We will in each case present them with an efficient way to do this but do not want to limit the concept to one specific path.
>
> **Doubles** – the Mathnasium focus on doubling of numbers is continued to assist with 2s, 4s and 8s.

For more specific information on the strategies implemented in each PK please refer to the MU Right Instruction Course on Multiplication.

Prerequisite Skills

• Addition: *Up to* and *Over Multiples of* 10 •

EXAMPLE:

Break up the 5 into two parts so that one of the parts plus 18 equals 20. Then add the leftover part to 20 to get the answer.

18 + 5

10

18 2 + 3 = 23

---

1) **39 + 4**

40

39 + ◯ = ____

2) **68 + 6**

70

+ ◯ = ____

3) **47 + 8**

50

+ ◯ = ____

4) **59 + 3**

60

+ ◯ = ____

5) **85 + 7**

90

+ ◯ = ____

6) **28 + 9**

30

+ ◯ = ____

• ADDING 10 *first* •

When adding a number that is *close* to 10, we can **ADD 10** ***first***.

EXAMPLE 1:

Let's add 9 to a number by **ADDING 10** ***first*** and then *subtracting* **1** (since 9 = 10 – **1**).

43 + 9 = ?

43 + 10 = $\boxed{53}$, $\boxed{53}$ – 1 = **52**

ADD 10 *first*	*Then* SUBTRACT 1	43 + 9

EXAMPLE 2:

Let's add **12** to a number by **ADDING 10** ***first*** and then *adding* **2** (since **12** = **10** + **2**).

29 + 12 = ?

29 + 10 = $\boxed{39}$, $\boxed{39}$ + 2 = **41**

ADD 10 *first*	*Then* ADD 2	29 + 12

Try these:

1) **15 + 9 = ?**

15 + 10 = $\boxed{}$, $\boxed{}$ – 1 = ____

ADD 10 *first*	*Then* SUBTRACT 1	15 + 9

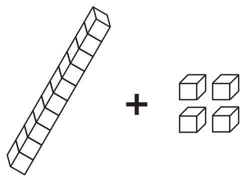
2) **37 + 12 = ?**

37 + 10 = $\boxed{}$, $\boxed{}$ + 2 = ____

ADD 10 *first*	*Then* ADD 2	37 + 12

• 10 *More Than* a Number •

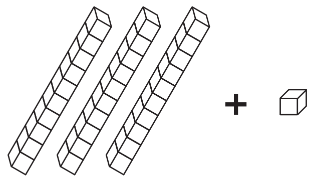
1) **14**



Add 10 by drawing another TEN in the picture.

10 *more than* **14** is ______

2) **31**



Add 10 by drawing another TEN in the picture.

10 *more than* **31** is ______

3) What number is **10** *more than* **23**? ______

4) What number is **10** *more than* **19**? ______

5) What number is **10** *more than* **55**? ______

6) What number is **10** *more than* **36**? ______

7) What number is **10** *more than* **88**? ______

8) What number is **10** *more than* **47**? ______

9) 43 + 10 = ______

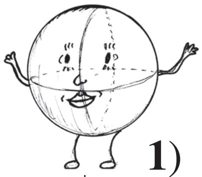
10) 78 + 10 = ______

11) 82 + 10 = ______

12) 29 + 10 = ______

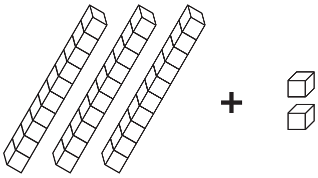
13) 50 + 10 = ______

14) 64 + 10 = ______



• 10 *Less Than* a Number •

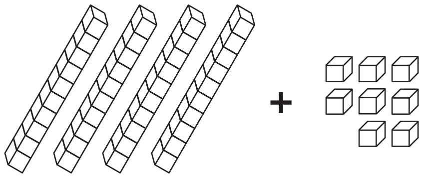
1) **32**



Take away 10 by crossing out a TEN in the picture.

10 *less than* **32** is ______

2) **48**



Take away 10 by crossing out a TEN in the picture.

10 *less than* **48** is ______

3) What number is **10** *less than* **69**? ______

4) What number is **10** *less than* **25**? ______

5) What number is **10** *less than* **30**? ______

6) What number is **10** *less than* **83**? ______

7) What number is **10** *less than* **77**? ______

8) What number is **10** *less than* **12**? ______

9) 49 – 10 = ______

10) 84 – 10 = ______

11) 61 – 10 = ______

12) 11 – 10 = ______

13) 76 – 10 = ______

14) 60 – 10 = ______

• Half of Large Numbers •

1) Half of **190** = ?

_____ + _____ = _____

half of **100** | half of **90** | half of **190**

2) Half of **250** = ?

_____ + _____ = _____

half of **200** | half of **50** | half of **250**

3) Half of **180** = half of 100 + half of _____

_____ + _____ = _____

half of **180**

4) Half of **150** = half of _____ + half of _____

_____ + _____ = _____

half of **150**

5) Half of **230** = half of _____ + half of _____

_____ + _____ = _____

half of **230**

6) Half of **260** = half of _____ + half of _____

_____ + _____ = _____

half of **260**

• Double of Large Numbers *Practice* •

1) Double of **36** = ?

36

☐ + ◯

Value of the **10s** | Value of the **1s**

______ + ______ = ______

double of **30** | double of **6** | double of **36**

2) Double of **13** = ?

13

☐ + ◯

Value of the **10s** | Value of the **1s**

______ + ______ = ______

double of **10** | double of **3** | double of **13**

---

3) Double of **29** = double of 20 + double of ______

______ + ______ = ______
double of **29**

4) Double of **27** = double of ______ + double of ______

______ + ______ = ______
double of **27**

5) Double of **46** = double of ______ + double of ______

______ + ______ = ______
double of **46**

6) Double of **48** = double of ______ + double of ______

______ + ______ = ______
double of **48**

• Subtraction through Multiples of 10 •

When subtracting mentally, it may be helpful to subtract until the ones place is a 0. Then subtract the leftover part to get the answer.

Example: 34 – 6
Break up 6 into two parts, 4 and 2. That way, 34 minus 4 is 30, which has a 0 is the ones place. Then subtract 2 from 30 to get the final answer of 28.

$$34 - 6$$

$$34 - 4 - 2$$

$$30 - 2 = 28$$

Try these:

1)

$$32 - 5$$

$$32 - \bigcirc - \boxed{}$$

$$30 - \boxed{} = \underline{\qquad}$$

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2)

$$21 - 7$$

$$21 - \bigcirc - \boxed{}$$

$$20 - \boxed{} = \underline{\qquad}$$

Common Techniques in all Multiplication Fact Prescriptives

• Multiplication by 10 •

The "×" sign is called the *multiplication sign*.

10×3 reads as

"10 times 3" or

"10, *three* times"

$10 + 10 + 10 = 30$

10, *three* times is $10 \times 3 = 30$.

The answer to a multiplication problem is called the *product*.

Try these:

Rewrite as repeated addition.

1) $10 \times 5 =$ ____

2) $10 \times 7 =$ ____

Rewrite as multiplication problems.

3) $10 + 10 + 10 + 10 =$ _______________

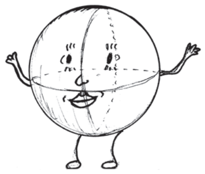
4) $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 =$ _______________

Rewrite as repeated addition, then find the sum.

5) $10 \times 6 =$ ________________________________ = __________

Rewrite as a multiplication fact, then find the product.

6) $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 =$ _____ × _____ = _____



• 10 *Less Than* a Number •

Rewrite as repeated addition, then find the sum.

1) $10 \times 2 =$ ______________________________ = __________

2) $10 \times 6 =$ ______________________________ = __________

3) 10, five times = ________________________ = __________

Rewrite as multiplication, then find the product.

4) $10 + 10 + 10 + 10 =$ _____ $\times$ _____ = __________

5) $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 =$ ____ $\times$ _____ = _________

6) 10, three times = _____ $\times$ _____ = __________

Count in 10s, seven times.

7) 0, _____, _____, _____, _____, _____, _____, _____

8) So 10, seven times is _____.

• Turnaround Facts •

3 groups of 5
5, three times
5 × 3

Total number: **15**

5 groups of 3
3, five times
3 × 5

Total number: **15**

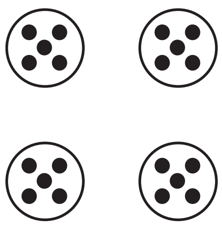
Notice that 3 groups of 5 has the same number of dots as 5 groups of 3.

So 5 × 3 has the same product as 3 × 5.

When multiplying, *the product is the same if the numbers we are multiplying switch places*.

Try these: **Draw groups that model the multiplication. Then find the total number of dots.**

1) 5 × 4



Total number: ________

2) 4 × 5

Total number: ________

3) 5 × 6

Total number: ________

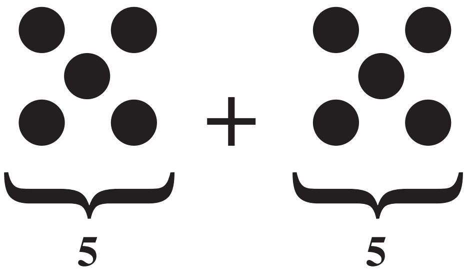
4) 6 × 5

Total number: ________

Techniques Used for Each Multiplication Fact

• Multiplication by 5 Using Halving •

What is 5 doubled?



5 doubled = 10
5 doubled is the same as 5, *two* times, 5 × 2.
So 5 × 2 = 10.

Multiplying by **2** is the same as **doubling**.

Example: 4 × 2

4 × 2 is the same as 4 doubled = 4 + 4 = 8.
So 4 × 2 = 8.

***Try these*:**

1) 10 × 2 = 10 doubled = _____ **2)** 7 × 2 = 7 doubled = _____

3) 3 × 2 = __________ = _____ **4)** 2 × 2 = __________ = _____

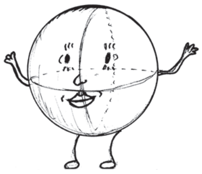
5) 6 × 2 = 6 + 6 = _____ **6)** 1 × 2 = 1 + 1 = _____

7) 9 × 2 = ___ + ___ = _____ **8)** 8 × 2 = ___ + ___ = _____

9) What is another way of thinking about multiplying by 2?

____

10) 11 doubled = ______ × ______ = ______



• Multiplication by 10 •

What is 1×6?

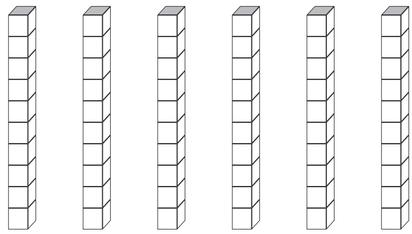
1, *six* times is 6.

$1 \times 6 = 6$

What is 10×6?

10, *six* times is 60.

$10 \times 6 = 60$

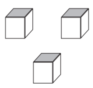


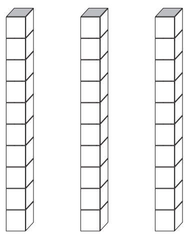
Notice that 1×6 has a **6** in the *ones* place and 10×6 has a **6** in the *tens* place.

When we multiply a number by 10, it's the same as moving all the digits one place to the left, then putting a zero in the empty ones place.

Try these:

1) $1 \times 3 =$ **3**, so $10 \times 3 =$ ___________.





2) $1 \times 7 =$ **7**, so $10 \times 7 =$ ___________.

3) $10 \times 4 =$ ___________

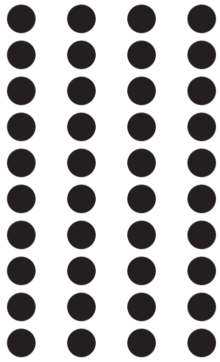
4) $10 \times 9 =$ ___________

5) What is the difference between multiplying by 1 and multiplying by 10?

____

• Multiplication by 5 Using Halving •

What is 10 × 4?



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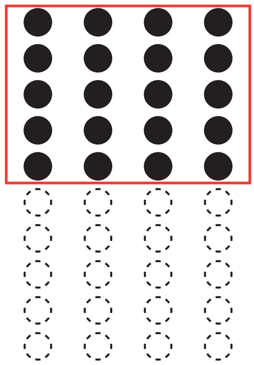
What is 5 × 4?



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Notice that the product of 5 × 4 is ***half*** the product of 10 × 4.

Because 5 is half of 10, when we multiply by 5, the product is half the product when we multiply with 10.

Try these:

1) 10 × 6 = **60**, so 5 × 6 = __________.

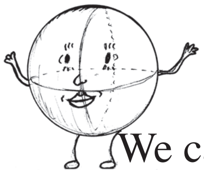
2) 10 × 9 = **90**, so 5 × 9 = __________.

3) 5 × 8 = __________

4) 5 × 3 = __________

5) Why is half of 10 times a number the same as 5 times the same number?

____



• Doubling and Adding On •

We can also use our 2s facts to help us find multiplication facts with 3.

What is 6×2?

6, two times is the same as 6 doubled.

$$6 \times 2 = 12$$

What is 6, 3 times?

Since we know 6, two times,

we can find 6, three times by adding 6 more.

$$6 \times 3 = 12 + 6 = 18$$

So, $6 \times 3 = 18$.

Since $6 \times 3 = \mathbf{18}$, $3 \times 6 = \mathbf{18}$.

Try these:

1) $4 \times 3 = \underbrace{\underline{\quad 8 \quad}}_{\text{4 doubled}} + \underbrace{\underline{\quad 4 \quad}}_{\text{1 more 4}} = \underline{\qquad\qquad}$

2) $7 \times 3 = \underbrace{\underline{\qquad\qquad}}_{\text{7 doubled}} + \underbrace{\underline{\qquad\qquad}}_{\text{1 more 7}} = \underline{\qquad\qquad}$

3) $5 \times 2 = \mathbf{10}$, so $5 \times 3 =$ ___________.

4) $8 \times 2 = \mathbf{16}$, so $8 \times 3 =$ ___________.

• Multiplication by Doubling •

What is 3×2?

3, *two* times is 6.

$3 \times 2 = 6$

What is 3×4?

3, *four* times is 12.

$3 \times 4 = 12$

When we multiply a number by 4, it is the same as doubling the number and then doubling the answer.

Try these:

1) $5 \times 4 = \underbrace{\ 10\ }_{\text{5 doubled}} + \underbrace{\ 10\ }_{\text{5 doubled}} =$ __________

2) $7 \times 4 = \underbrace{\qquad}_{\text{7 doubled}} + \underbrace{\qquad}_{\text{7 doubled}} =$ __________

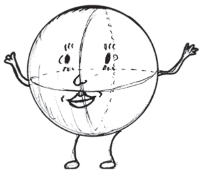
3) $9 \times 2 =$ **18**, so $9 \times 4 =$ __________.

4) $11 \times 2 =$ **22**, so $11 \times 4 =$ __________.

5) What is another way of thinking about multiplying by 4?

____

____



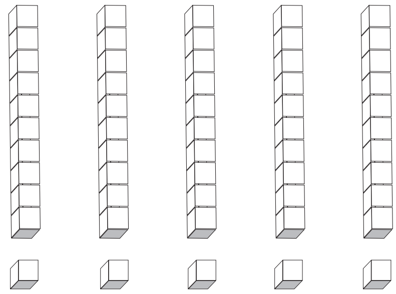
• Multiplication by 10 and Then 1 •

Because $10 + 1 = 11$, multiplying by 11 is the same as multiplying by 10, multiplying by 1 and then adding those two answers together.

EXAMPLE 1: To find 11×5, find 10×5 and then add 1×5.

$10 \times 5 = \mathbf{50}$

$1 \times 5 = \mathbf{5}$



$50 + 5 = \mathbf{55}$

So, 11 × 5 = 55.

Notice that there is a 5 in both the tens place and the ones place.

EXAMPLE 2: 11×2

$\boxed{\mathbf{11}}$

10 × 2 + 1 × 2

$20 + 2 = 22$

So, $11 \times 2 = \mathbf{22.}$

Try these:

1) $11 \times 9 =$ 90 (10×9) + 9 (1×9) = ______

2) $11 \times 4 =$ ______ (10×4) + ______ (1×4) = ______

3) $10 \times 7 = \mathbf{70}$ and $1 \times 7 = \mathbf{7}$, so $11 \times 7 =$ ______.

4) $11 \times 8 =$ ______

5) $11 \times 6 =$ ______

• Patterns with 11 •

There is a pattern for multiplying by 11. The answer is the number we are multiplying by, twice.

11 × 1	11 × 2	11 × 3	11 × 4	11 × 5
11	**22**	**33**	**44**	**55**

This pattern works when we multiply 11 by numbers from 0 to 9.

What happens for numbers bigger than 9?

Let's find 11 × 10.

We know 11, *nine* times is 99.
To find 11, *ten* times, add 10 and then add 1
99 + 10 + 1 = 109 + 1 = 110

We can also multiply by 11 by first multiplying by 10, multiplying by 1 and then adding those two answers together.

To find 11 × 10, multiply by 10 and then multiply by 1.

$\boxed{11}$

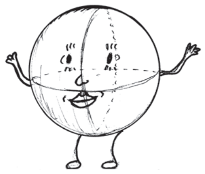
10 × 10 + 1 × 10

So, 11 × 10 = **110.**

Try these:

1) 11 × 4 = ___________ **2)** 11 × 9 = ___________

3) 11 × 2 = ___________ **4)** 11 × 7 = ___________

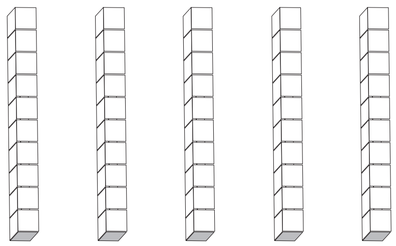


• Multiplication by 10 and Then 2 •

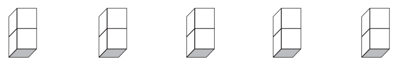
Because $10 + 2 = 12$, multiplying by 12 is the same as multiplying by 10, multiplying by 2 and then adding those two answers together.

EXAMPLE 1: To find 12×5, find 10×5 and then add 2×5.

$10 \times 5 = \mathbf{50}$



$2 \times 5 = \mathbf{10}$



$50 + 10 = \mathbf{60}$

So, 12 × 5 = 60

EXAMPLE 2: 12×7

$\boxed{\mathbf{12}}$

$\mathbf{10 \times 7 + 2 \times 7}$

70 + 14 = 84

So, 12 × 7 = 84.

Try these:

1) $12 \times 9 =$ 90 (10 × 9) + 18 (2 × 9) = __________

2) $12 \times 4 =$ __________ (10 × 4) + __________ (2 × 4) = __________

3) $10 \times 10 = \mathbf{100}$ and $2 \times 10 = \mathbf{20}$, so $12 \times 10 =$ __________.

4) $12 \times 8 =$ __________

5) $12 \times 6 =$ __________

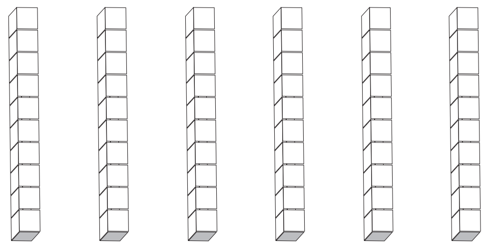
• Multiplication by 9 — Decomposing •

☆

Because 10 – 1 = 9, multiplying by 9 is the same as multiplying by 10, multiplying by 1 and then finding the difference of the two numbers.

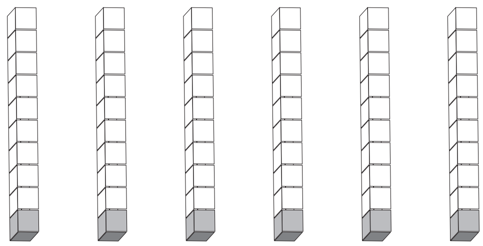
EXAMPLE: To find 9 × 6, find 10 × 6 and then subtract 1 × 6.

10 × 6 = **60**



1 × 6 = **6**

9 = **10** – **1**
10 × 6 – **1** × 6
60 – 6
So, 9 × 6 = **54**



Try these:

1) 9 × 5 = ___50___ – ___5___ = __________
(10 × 5) (1 × 5)

2) 9 × 7 = __________ – __________ = __________
(10 × 7) (1 × 7)

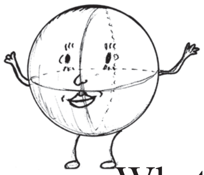
3) 10 × 8 = **80** and 1 × 8 = **8**, so 9 × 8 = __________.

4) 10 × 11 = **110** and 1 × 11 = **11**, so 9 × 11 = __________.

5) What is another way to multiply with 9?

____

____



• Pattern in Multiplication by 9 •

What pattern do you notice about the answers?

____

____

____

9 × 1	**9**
9 × 2	**18**
9 × 3	**27**
9 × 4	**36**
9 × 5	**45**
9 × 6	**54**
9 × 7	**63**
9 × 8	**72**
9 × 9	**81**
9 × 10	**90**

As we multiply 9 by the next larger number, the *tens* place increases by 1 and the *ones* place decreases by 1. This is because adding 9 is the same as adding 10 and subtracting 1. To find 9 × 11, start with 9 × 10 and then add 10 and subtract 1.

$$9 \times 10 = 90$$
$$90 + 10 = 100$$
$$100 - 1 = 99$$

So, 9 × 11 = **99**.

9 × 11	**99**
9 × 12	**108**
9 × 13	**117**
9 × 14	**126**

Let's continue the pattern.

Try these:

1) 9 × 3 = **27**, so 9 × 4 = __________.

2) 9 × 14 = **126**, so 9 × 15 = __________.

3) How can you use 9 × 15 to solve 9 × 16?

____

____

• Multiplication with Decomposing •

We have learned that we can use facts we know to help us find facts that we do not.

Because 6 = 5 + 1, we can use our 5s facts and 1s facts to find 6s facts.

What is 6 × 8?

$$\boxed{6}$$

5 × 8 + 1 × 8

40 + **8**

So, 6 × 8 = **48**

Multiplication with **5** and **1** is often easier than with 6, so it helps to break 6 into **5** + **1**.

What is 6 × 12?

$$\boxed{6}$$

5 × 12 + 1 × 12

60 + **12**

So, 6 × 12 = **72.**

Try these:

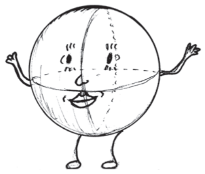
1) 6 × 7 = $\underset{5 \times 7}{\underline{\quad 35 \quad}}$ + $\underset{1 \times 7}{\underline{\quad 7 \quad}}$ = ________

2) 6 × 9 = $\underset{5 \times 9}{\underline{\qquad\quad}}$ + $\underset{1 \times 9}{\underline{\qquad\quad}}$ = ________

3) 5 × 3 = **15** and 1 × 3 = **3**, so 6 × 3 = ________.

4) 6 × 11 = ________

5) 6 × 6 = ________



• Multiplication with Decomposing •

We have learned that we can use facts we know to help us find facts that we do not.

Because 7 = 5 + 2, we can use our 5s facts and 2s facts to find 7s facts.

What is 7 × 6?

$$\boxed{7}$$

5 × 6 + 2 × 6

30 + **12**

So, 7 × 6 = **42**

Because multiplication with **2** and **5** is often easier, it helps to use **5 + 2** to make 7.

What is 7 × 12?

$$\boxed{7}$$

5 × 12 + 2 × 12

60 + **24**

So, 7 × 12 = **84**

Try these:

1) 7 × 9 = $\underbrace{\underline{\ \ 45\ \ }}_{5 \times 9}$ + $\underbrace{\underline{\ \ 18\ \ }}_{2 \times 9}$ = __________

2) 7 × 8 = $\underbrace{\underline{\qquad\qquad}}_{5 \times 8}$ + $\underbrace{\underline{\qquad\qquad}}_{2 \times 8}$ = __________

3) 5 × 3 = **15** and 2 × 3 = **6**, so 7 × 3 = __________.

4) 7 × 11 = __________ **5)** 7 × 5 = __________

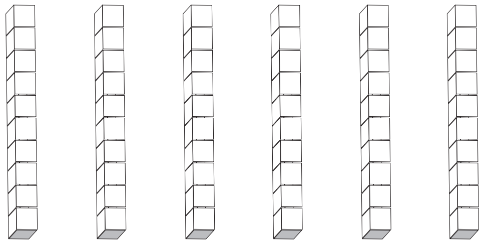
• Multiplication with Decomposing •

We have learned that we can use facts we know to help us find facts that we do not.

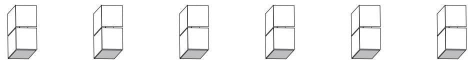
Because $8 = 10 - 2$, we can use our 10s facts and 2s facts.

EXAMPLE: To find 8×6, find 10×6 and then subtract 2×6.

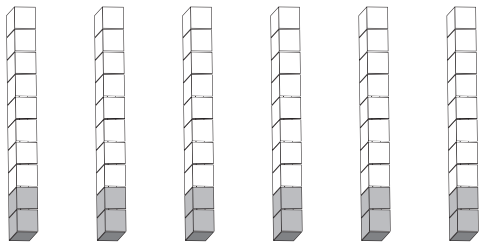
$10 \times 6 = \mathbf{60}$



$2 \times 6 = \mathbf{12}$



$8 = \mathbf{10} - \mathbf{2}$
$\mathbf{10} \times 6 - \mathbf{2} \times 6$
$60 - 12$
So, $8 \times 6 = \mathbf{48}$.



Try these:

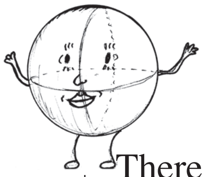
1) $8 \times 9 = \underset{10 \times 9}{\underline{\quad 90 \quad}} - \underset{2 \times 9}{\underline{\quad 18 \quad}} = \underline{\qquad\qquad}$

2) $8 \times 4 = \underset{10 \times 4}{\underline{\qquad\qquad}} - \underset{2 \times 4}{\underline{\qquad\qquad}} = \underline{\qquad\qquad}$

3) $10 \times 11 = \mathbf{110}$ and $2 \times 11 = \mathbf{22}$, so $8 \times 11 = \underline{\qquad\qquad}$.

4) $8 \times 8 = \underline{\qquad\qquad}$

5) $8 \times 5 = \underline{\qquad\qquad}$



• Multiplication by 8 — Doubling •

There is another strategy we can use when we are multiplying by 8!

When multiplying by 2, we double.

When we multiply by 4, we double, then double the answer (double - double).

When we multiply by 8, we double, then double the answer, then double the answer again (double - double - double).

Example: 4×8

4 doubled is **8**.

8 doubled is **16**.

16 doubled is **32**.

So, $4 \times 8 =$ **32**.

Example: 25×8

25 doubled is **50**.

50 doubled is **100**.

100 doubled is **200**.

So, $25 \times 8 =$ **200**.

Try these:

1) 7×8

7 doubled is **14**.

14 doubled is $\boxed{}$.

$\boxed{}$ doubled is $\triangle$.

So, $7 \times 8 =$ ___________.

2) 9×8

9 doubled is $\bigcirc$.

$\bigcirc$ doubled is $\boxed{}$.

$\boxed{}$ doubled is $\triangle$.

So, $9 \times 8 =$ ___________.

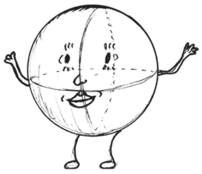
3) What is another way to multiply by 8?

____

Practice Grids

• 3s Facts •

Multiplier	2 × ___	1 × ___	3 × ___	3s Fact
2	4	2	6	3 × 2 = 6
5				
8				
4				
6				
	2			
		10		
9				
	14			
		0		
12				
	6			
		11		



• 4s Facts •

Multiplier	2 Times	4 Times	4s Fact
9	18	36	4 × 9 = 36
5			
8			
4			
6			
	2		
		40	
2			
	14		
		0	
12			
	6		
		44	

Application Exercises

• Multiplication by 12 •

Find the multiplication fact that helps answer the question, then solve.

1) A baker is making 12 loaves of bread. He needs 3 cups of flour for each loaf. How many cups of flour does the baker need?

Multiplication Fact: ________ × ________ = ________

2) There are a dozen cookies in a box. Each cookie has 12 chocolate chips. How many chocolate chips are there in the box?

Multiplication Fact: ________ × ________ = ________

3) The baker makes 8 cakes and sells each for $12. How much money does he make from the cakes?

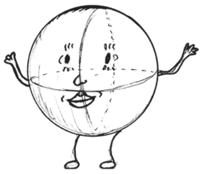
Multiplication Fact: ________ × ________ = ________

4) It takes 12 minutes to bake a batch of muffins. How long will it take to bake 5 batches of muffins?

Multiplication Fact: ________ × ________ = ________

5) Nine people each buy 12 cupcakes from the bakery. How many cupcakes does the baker sell?

Multiplication Fact: ________ × ________ = ________



• Multiplication by 9 •

Find the multiplication fact that helps answer the question, then solve.

1) A cat has 9 lives. How many lives do 8 cats have?

Multiplication Fact: ________ × ________ = ________

2) Whiskers has 9 toy mice and each toy mouse has 2 ears. How many ears do Whisker's mice have altogether?

Multiplication Fact: ________ × ________ = ________

3) How many legs do 9 cats have?

Multiplication Fact: ________ × ________ = ________

4) Each bag of cat food has enough food in it to last 11 days. How many days will 9 bags of cat food last?

Multiplication Fact: ________ × ________ = ________

5) Connor has 7 cats and each cat has 9 kittens. How many kittens does Connor have?

Multiplication Fact: ________ × ________ = ________

Practice Activities

In addition to standard multiplication activities, the following activities have been devised by the Education Department to provide short (2–5 minutes), strategic multiplication practice and can be implemented at various stages in their acquisition of facts. A short summary of each activity, as well as video of student/instructor interactions can be found in the "Multiplication Fact Fluency Instruction" module found in the MU training portal.

Multiplication War: This game is played between either student-and- instructor or student-and- student.

Each player is dealt $\frac{1}{2}$ of a Mathnasium deck of cards and simultaneously take two cards off the top of their pile to be placed face-up on the table. If an instructor is playing, the student should be prompted to recite both facts and the player showing the largest product gets to keep all the cards. This game can be played with a time limit or until the cards run out (if time allows).

To focus on a single family of multiplication facts, pull two cards from the deck associated with the factor to be practiced (e.g. if a student is practicing 6s facts, pull two 6s from the deck). Lay one face-up in front of each player. Players will then pull one card from the top of their draw pile and lay it next to the factor being practiced and find the product. For cumulative practice, players could have cards arranged in a pile associated with the "facts" they have acquired. For example, if a student has recently completed their 3s facts, an instructor could create a split pile of only 0s, 1s, 2s, 10s, 5s and 3s with the other pile being cards of any type, drawing a card from each pile to form the pair. This would force one of each pair being played to correlate with a fact they have knowledge of.

Potential variations could include having students "sum up" their cards to determine a winner or turning three cards face-up for enrichment students.

Multiplication Head's Up: This activity requires either a student-and-student or student-and- instructor and a deck of Mathnasium playing cards. A third person must also be involved to assist with the game. Remove all 0 cards from the deck.

The players will (at the same time) place a card on their forehead, facing out, without looking at the card. The third person will announce the product of the two numbers with the first person to correctly identify the card against their own forehead being the winner. Play as many rounds as the time allows for, stars can be given at the instructor's discretion.

To focus on a single family of multiplication facts, place a card associated with the facts to be practiced on the table. Acting as a single player, the student could then place a card on their forehead with the instructor announcing the product of the student's card and the chosen card on the table. The student would then have to identify their card. For cumulative practice, the chosen cards on the table could vary in accordance with those "facts" they have acquired. Students would then be limited to the facts they have been exposed to in the Multiplication Fact Fluency program.

Multiplication with dice: Students can explore multiplication facts using six- or twelve-sided dice depending on where they are with facts acquisition. This activity could be as simple as rolling two dice and having the student state the product or if a student is practicing a specific fact they could roll one die and state the product (e.g., if a student is practicing 12s they could roll one die, if a 5 is facing up they could state the product of 60). Alternatively, they could roll two dice and find the sum first before extending to the product. Laying a Mathnasium playing card on the table is valuable to identify specific families of facts to be practiced. Students should earn stars at the instructor's discretion for completing this activity.

Potential variations could include laying one Mathnasium playing card face-up on the table, then rolling one die (the card times what is showing on the die) or a pair of dice (the card times the sum of what is showing on the dice); roll a die and have the student double (2s), double/double (4s) or double/double/double (8s); or, roll one die, the instructor states a product that would use that die and the student is prompted to identify the missing value.

Go Fish-Tac-Toe: This activity requires either a student-and-student or student-and-instructor, a Mathnasium deck of cards and a prepared 3 × 3 Tic-Tac-Toe board of products based upon the student's current level, created by the instructor. Once players determine who will be Xs and Os, each player should be dealt 7 cards and a draw pile created. If either player is currently holding a pair of cards whose product is found on the Tic-Tac-Toe board, they can play the pair and claim the product. Players will then alternate turns (with X going first) requesting a "complement" to a card they are holding from the other player to make a product found on the board. For example, if one of the values on the grid is 35 and the player is holding a 7, they could ask the other player for a 5 to complete that product. If the other player is not holding a 5, they may then draw one from the draw pile ("go fish") and the turn switches to the other player. After drawing, the player may then play a pair of cards to claim a product before proceeding to the other player's turn. Alternatively, a player can choose to draw a card without requesting one from other other player, which immediately ends his or her turn without the opportunity to play a pair of cards. If a player has fewer than four cards at the end of his or her turn, the player draws cards until their hand has the minimum hand size of four cards.

If the student has difficulty finding pairs of factors that claim products (e.g., keeps passing the turn), Instructors should assist students and remind them of the techniques demonstrated in the Multiplication Fact Fluency prescriptives. Instructors can also encourage students to use three cards, finding the sum or difference of two of the cards before multiplying by the third to find a product.

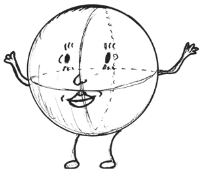
Players continue to alternate turns until one player has claimed three products in a row, winning the game; if that becomes impossible, then the first player to claim five products on the board wins.

To generate the board it is important to understand where a student is along the path of acquiring their facts. For example, if a student is at the end of their 0s, 1s, 2s, 10s and 5s progression, then any fact associated with those are fair to be placed on the board. Additionally, when choosing products, the instructor compiling the board must be careful to not exceed the number of available cards. A student may not reasonably be able to practice solely their 5s facts using this game as creating a board of 0, 5, 10, 15, 20, 25, 30, 35, 45 could require more 5s than a standard deck would have. Some sample boards are available at the end of this document.

Multiploganos: This game can be played individually, student-and-student or student-and-instructor.

The instructor will arrange a grid of dominos on the table with the goal of the student identifying as many facts as they can in any direction (horizontally, vertically and diagonally). Students should be prompted to write down the full fact (e.g., $7 \times 5 = 35$) as they find them and can also identify "turnaround" facts, provided they are found in a different location on the board. If they are playing alone, the number of facts they find can be converted to stars (for rewards); if they are playing against another student they can be counted as points (highest total wins). Potential variations could include allowing the students to double the dominos to include facts above 6s or other rectangular configurations.

To focus on a single family of multiplication facts, an instructor could lay a single Mathnasium playing card on the table with the student being prompted to instead find the sum in any direction (horizontally, vertically, diagonally), then find the product.



• Go Fish-Tac-Toe •

Grid A

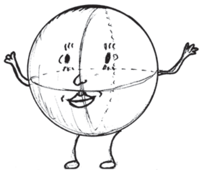
Suitable for Masters of 0s, 1s, 2s, 10s and 5s Facts

20	16	40
8	0	12
60	24	10

Grid B

Suitable for Masters of 0s, 1s, 2s, 10s and 5s Facts

6	24	4
30	0	16
8	40	20



• Go Fish-Tac-Toe •

Grid C

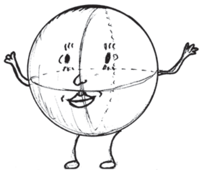
Suitable for Masters of 0s, 1s, 2s, 10s, 5s, 3s, 4s, 11s, 12s and 9s Facts

35	50	32
22	0	27
24	132	18

Grid D

Suitable for Masters of 0s, 1s, 2s, 10s, 5s, 3s, 4s, 11s, 12s and 9s Facts

121	81	64
25	0	49
1	100	144



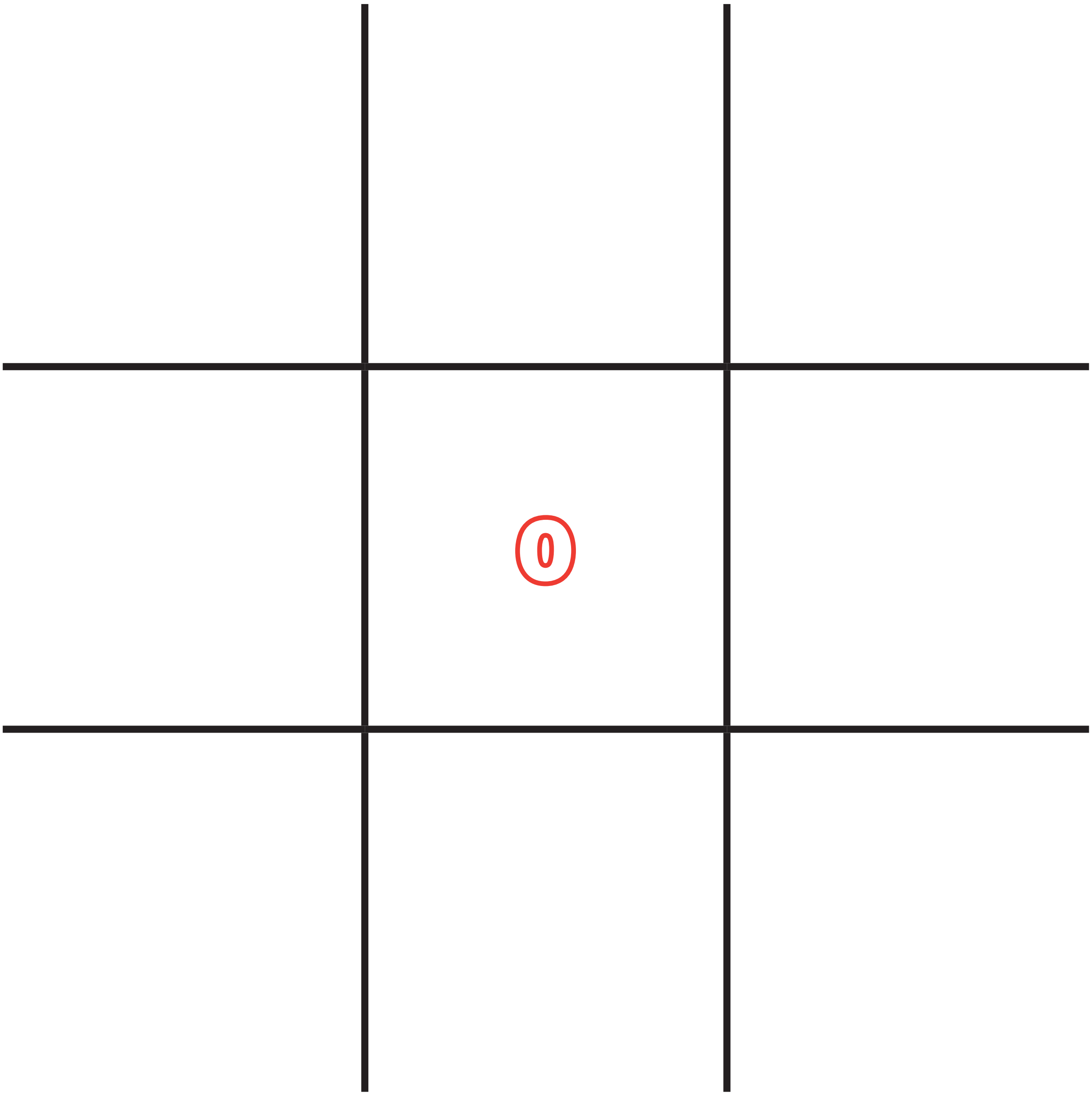
• Go Fish-Tac-Toe •

Grid E

Suitable for Masters of Multiplication Facts

108	42	6
72	0	144
50	110	56

• Go Fish-Tac-Toe •



Mastery Checks

• Mastery Check: Multiplication by 6 •

☆

Part I:

1) Rewrite as repeated addition and solve: 6×8

____

2) Rewrite as multiplication and solve: $6 + 6 + 6 + 6$

____

3) 6×12 has the same product as _______ $\times$ _______.

$6 \times 12 =$ _______

Multiply.

4) $6 \times 0 =$ __________

5) $6 \times 5 =$ __________

6) $6 \times 2 =$ __________

7) $6 \times 7 =$ __________

Part II:

This section is for Instructors to administer verbally.

8) 6×10

S	Y / N

9) 6×3

S	Y / N

10) 6×6

S	Y / N

11) 6×11

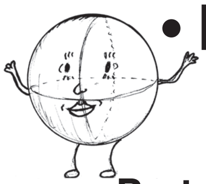
S	Y / N

12) 6×1

S	Y / N

13) 6×9

S	Y / N



• Mastery Check: Multiplication by 8 •

Part I:

1) Rewrite as repeated addition and solve: 8×7

____

2) Rewrite as multiplication and solve: $8 + 8 + 8 + 8 + 8 + 8$

____

3) 8×11 has the same product as ______ $\times$ ______.

$8 \times 11 =$ ______

Multiply.

4) $8 \times 9 =$ __________

5) $8 \times 2 =$ __________

6) $8 \times 5 =$ __________

7) $8 \times 0 =$ __________

Part II:

This section is for Instructors to administer verbally.

8) 8×4

S Y N

9) 8×10

S Y N

10) 8×1

S Y N

11) 8×12

S Y N

12) 8×8

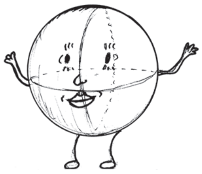
S Y N

13) 8×3

S Y N

MFF Activity Games

Practice playing the MFF games



time to PLAY with MULTIPLICATION

Seek opportunities to play as many of the Multiplication Activity Games as possible. This can be with other Instructors, students or even with your family and friends. You'll pick you favourite to play with your facilitator during your next Progress Check!

Students are asked to play 4 multiplication practice activities over the course of at least 2 visits. Instructors make note of each game on these stars.

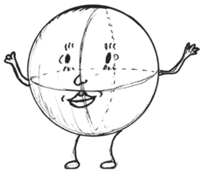
See page 47 of the MFF Manual for a list of games and instructions.

Date: ______ Date: ______ Date: ______ Date: ______

Reflection
&
Progress Check

• Reflection: Multiplication Fact Fluency •

Share an experience you had working with a student on Multiplication Fact Fluency curriculum. How can you use Multiplication Fact Fluency techniques during your instructional time?



• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

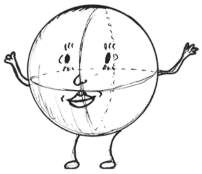
Multiplication Fact Fluency

I have a good understanding of:					
1.	the issue just memorising multiplication facts.	**0**	**1**	**2**	**3**
2.	the common techniques used for each fact	**0**	**1**	**2**	**3**
3.	the MFF games.	**0**	**1**	**2**	**3**
4.	why students don't learn the facts in order from 0 to 12.	**0**	**1**	**2**	**3**
5.	what the "S" means on the Mastery Check.	**0**	**1**	**2**	**3**
6.	how to administer the verbal part of the Mastery Checks.	**0**	**1**	**2**	**3**

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

- ☐ **Multiplication Fact Fluency Course (section 17, p. 1)**
- ☐ **The Multiplication Fact Fluency Manual (section 17, p. 7)**
- ☐ **MFF Reflective and Critical Questions (section 17, p. 3, 5)**
- ☐ **Multiplication Fact Fluency Reflection (section 18, p. 2)**
- ☐ **Play a multiplication game with your facilitator. (section 17, p. 49)**

Notes:



DeskTools and Manipulatives Course & Instructor Development

• DeskTools and Manipulatives •

Watch the <u>Right Instruction - Teach the Mathnasium Way: DeskTools and Manipulatives</u> training video. Then, complete the Reflective and Critical Questions on pages 3. Additionally, complete the DeskTools and Manipulatives activity on page 39.

In this course, you will learn how to use DeskTools and Manipulatives to help reinforce student understanding.

Objectives:

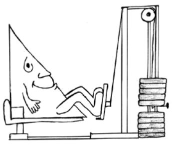
In this course, you will see video examples of:

1. Larry using various DeskTools with students.
2. Larry using manipulatives with students.

To access the training video:

1. Log into your Radius account.
2. On your homepage, click on the red training button to open your Training Path.
3. In your Training Path under the “Training Videos” section, click on “Right Instruction - Teach the Mathnasium Way: DeskTools and Manipulatives”.

Notes:



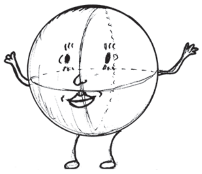
• Reflective and Critical Questions •

Answer the following questions AFTER completing the DeskTools and Manipulatives course.

1) What are visual manipulatives and how to they appear in Mathnasium curriculum?

2) What precaution do Instructors need to take when using manipulatives?

3) How can you use the clock manipulative to teach a student how to tell time?

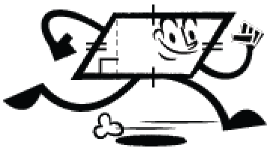


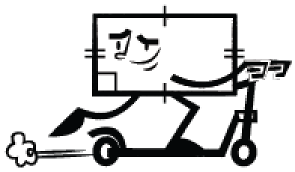
Proactive Teaching Tools

MATHNASIUM®

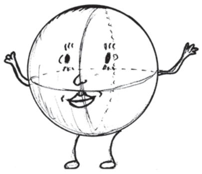
Desk Tools

• Visual Manipulatives •



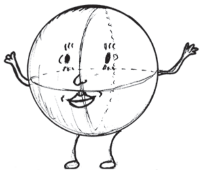


• Contents •

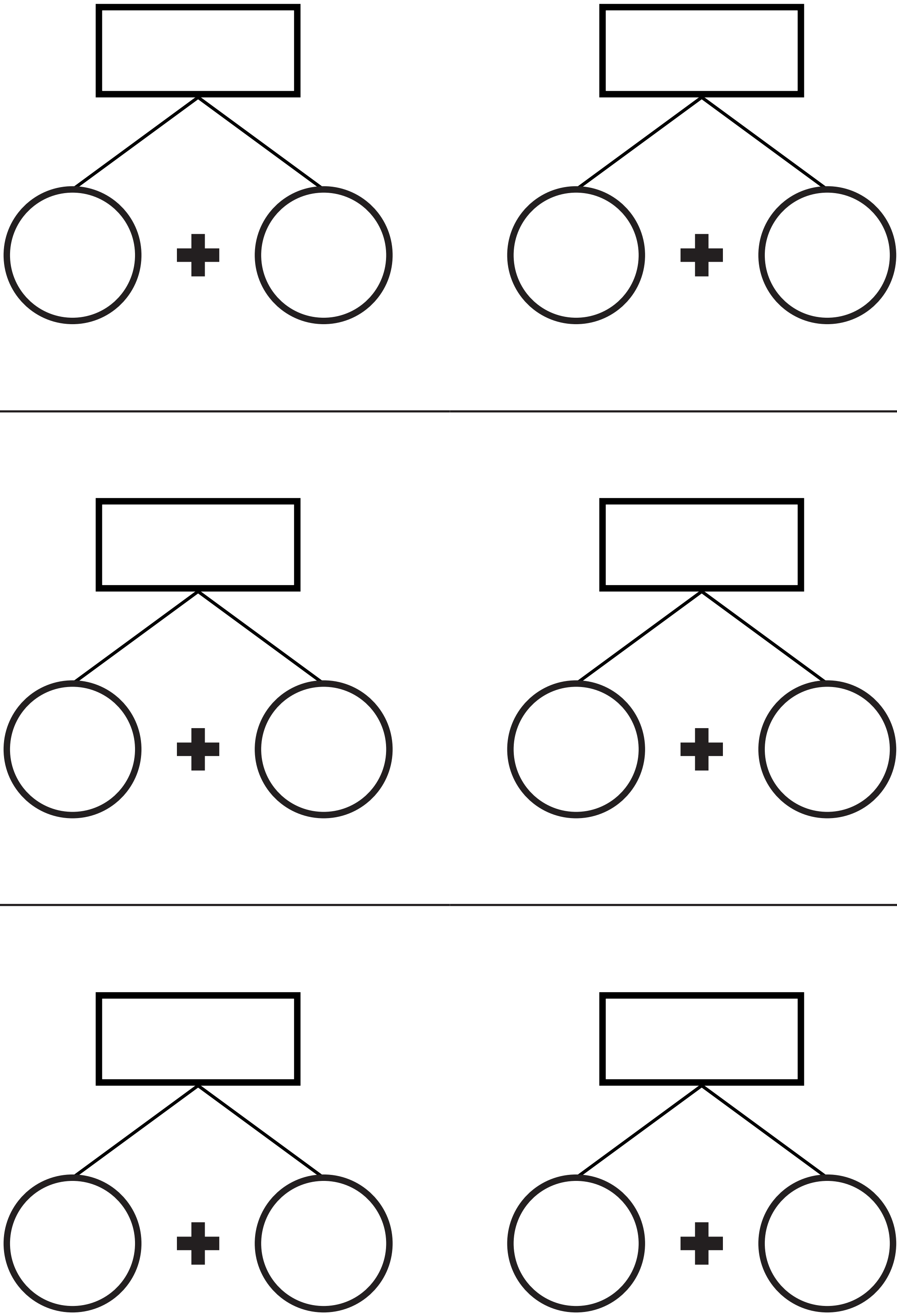


• Contents •

• Five and Ten Frames •

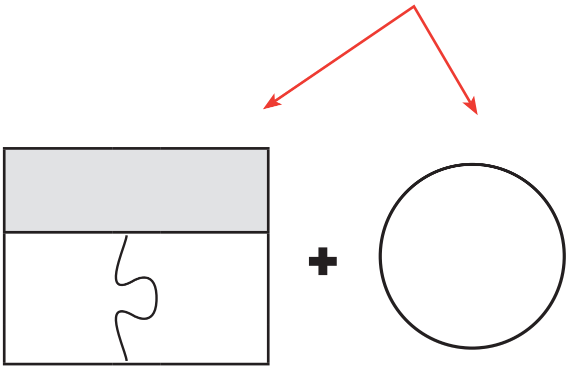


• Decomposing •



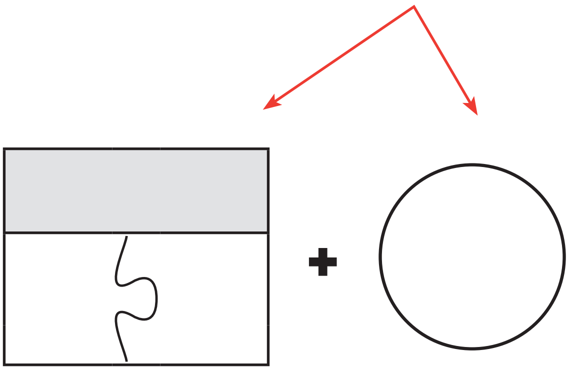
• Counting *Up* and *Over* Multiples of 10 •

________ + ________



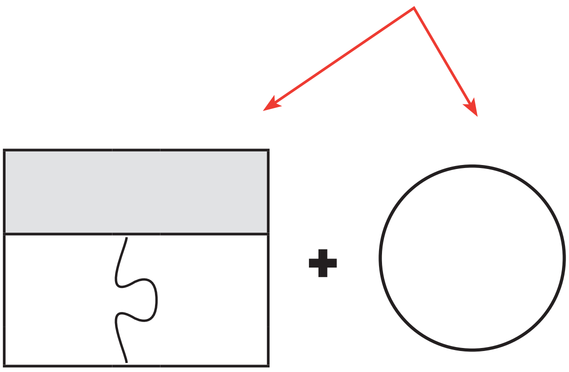
= ______

________ + ________

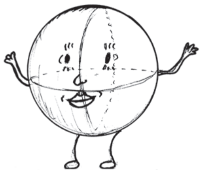


= ______

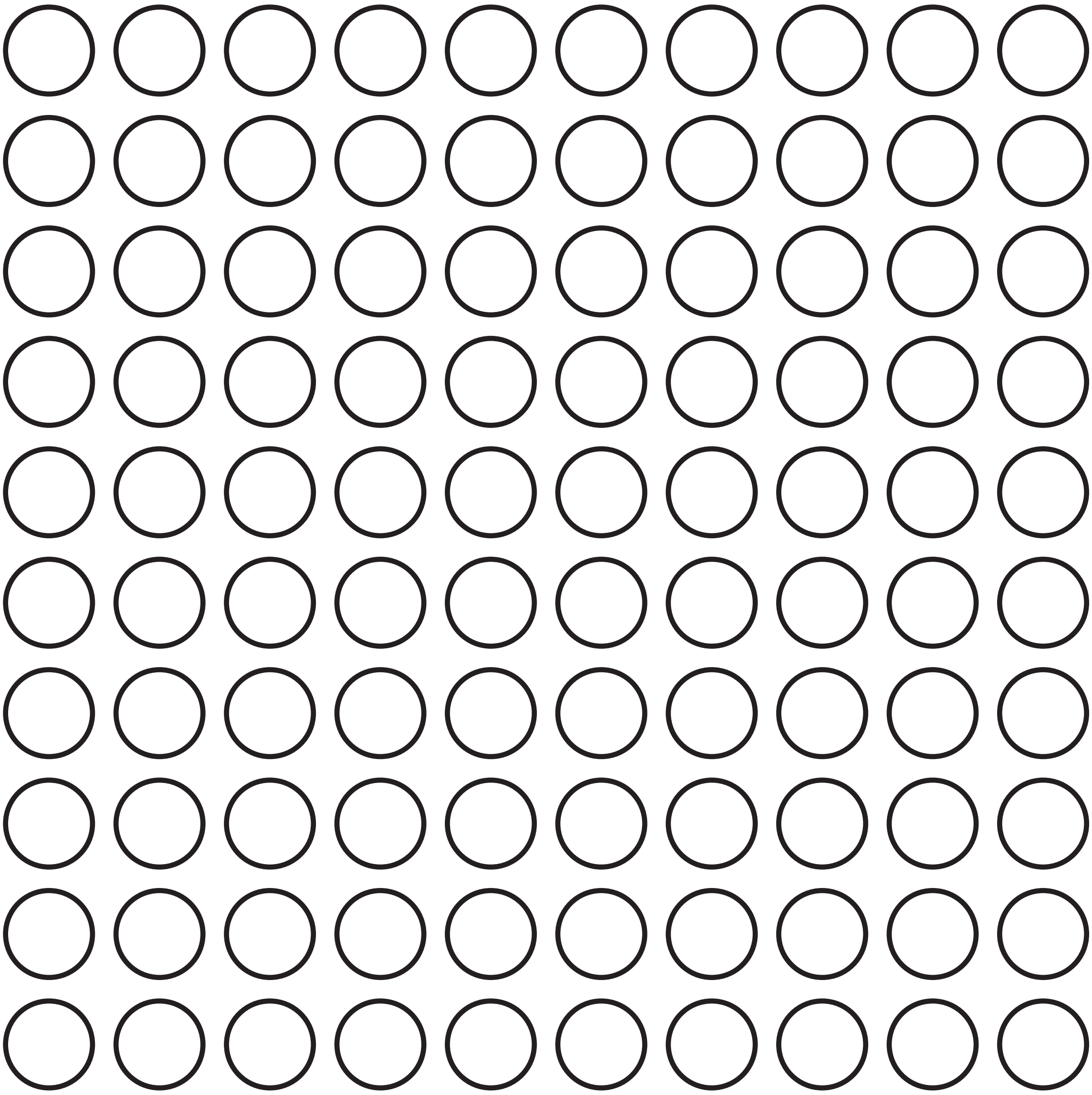
________ + ________



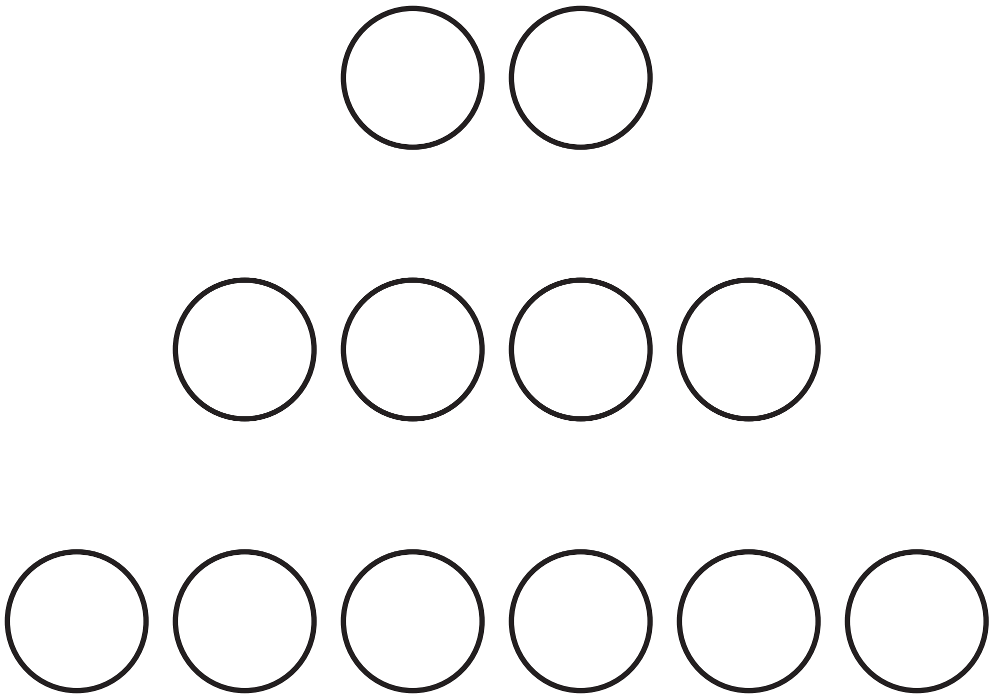
= ______

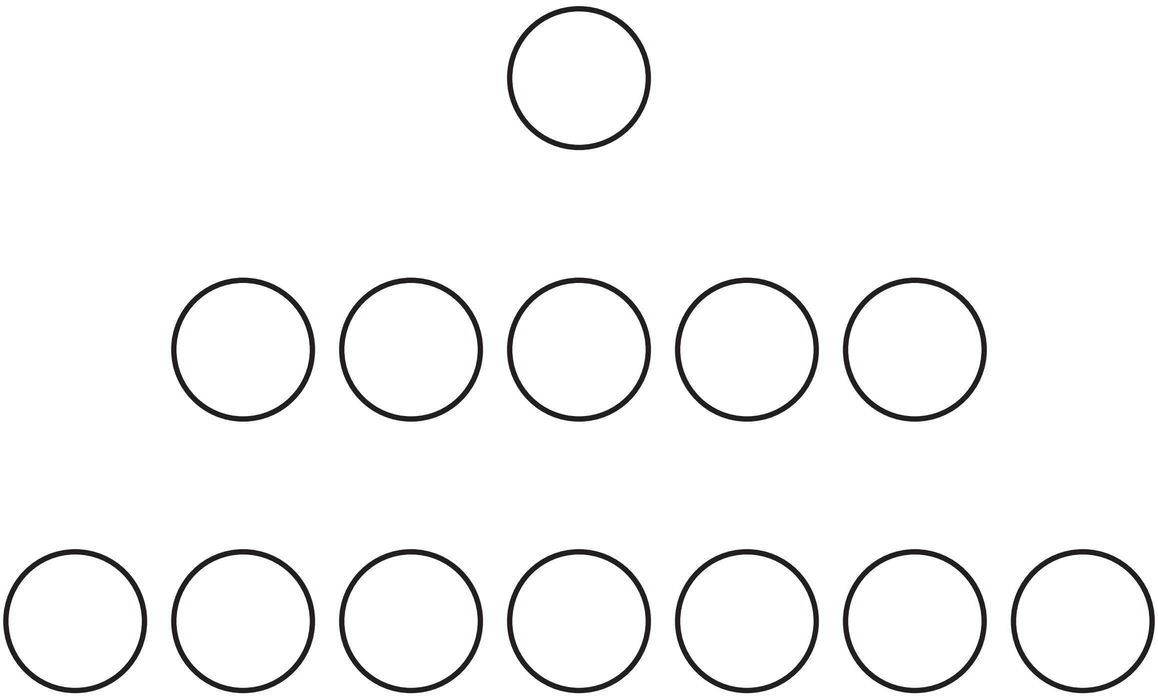


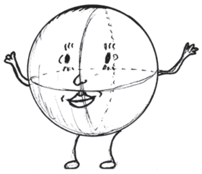
• 100 Circles •



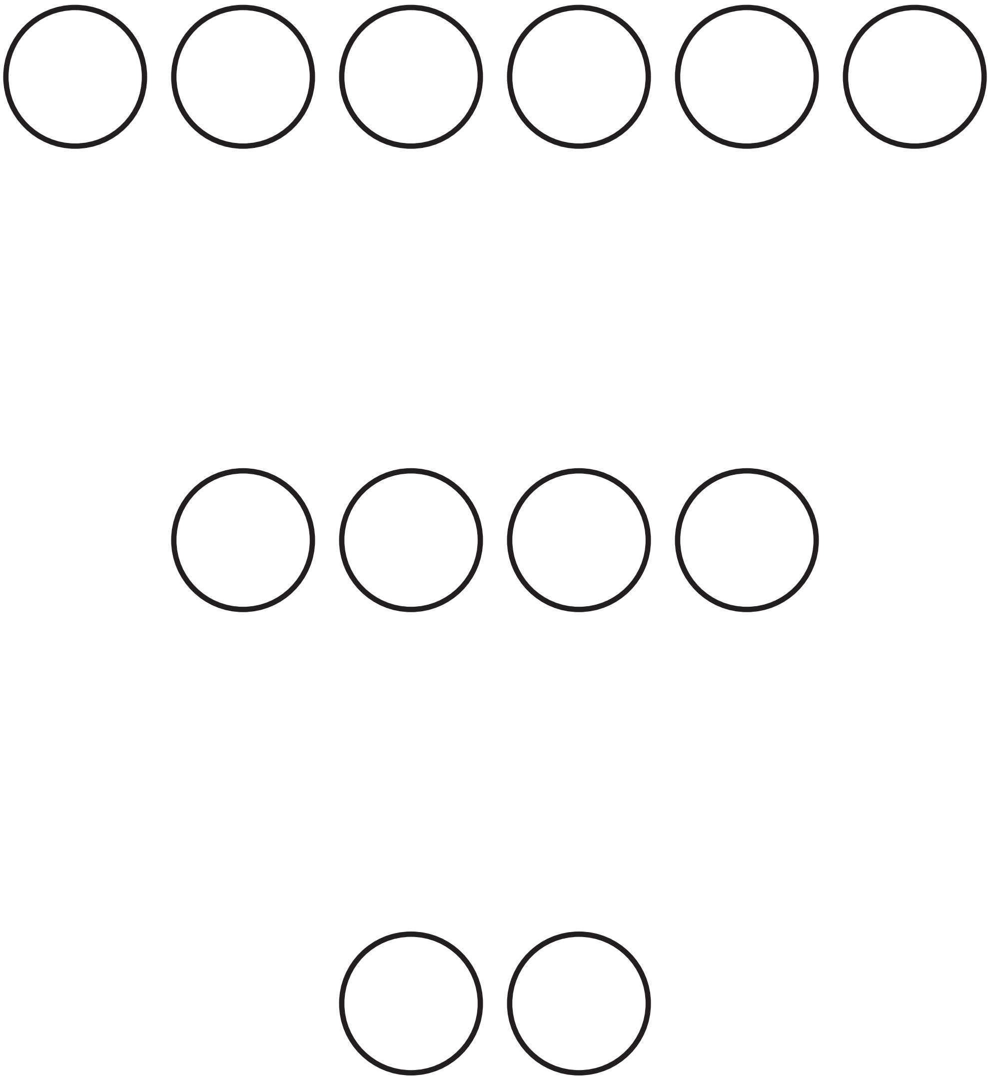
• Half of Even and Odd Numbers •



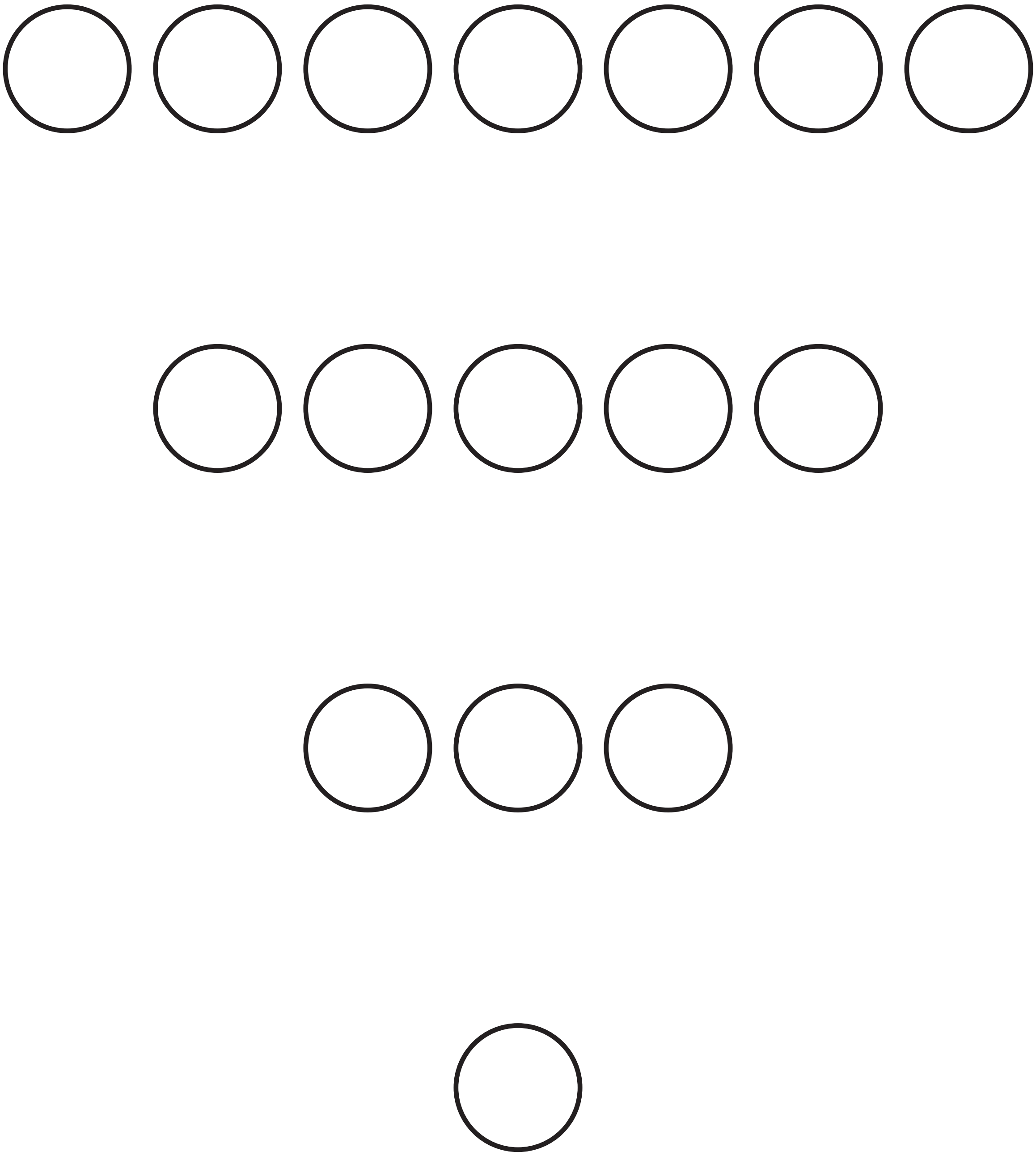


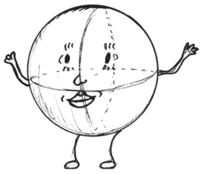


• Half of Even Numbers •



• Half of Odd Numbers •





• Hundred Chart •

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100									

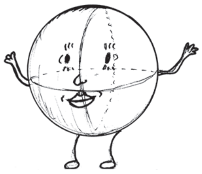
• Counting in... •

0, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

____, ____, ____, ____, 0, ____, ____, ____, ____, ____, ____, ____

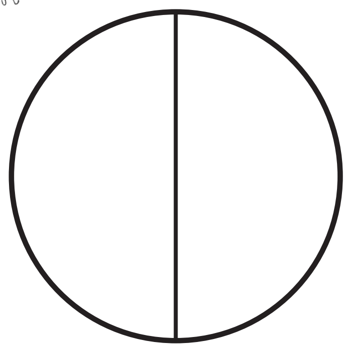
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

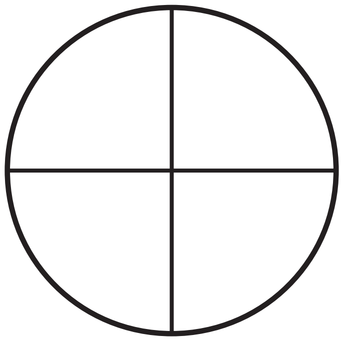
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____

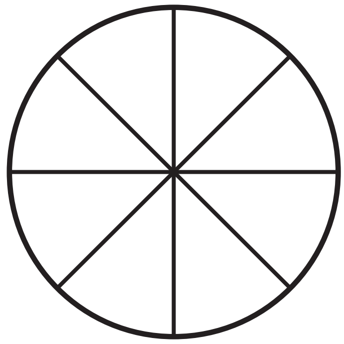


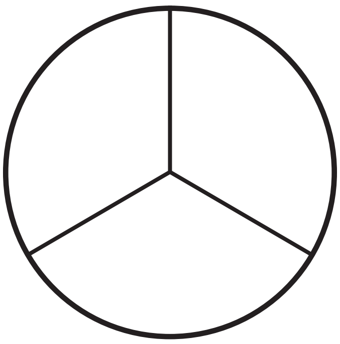
• Grid •

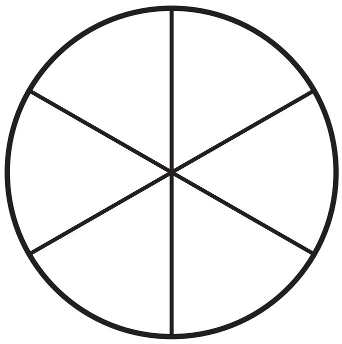
• Fraction Circles 1 •

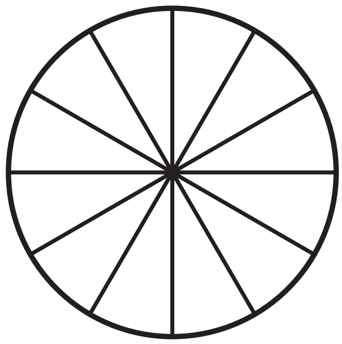


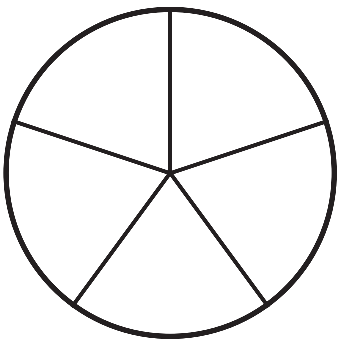


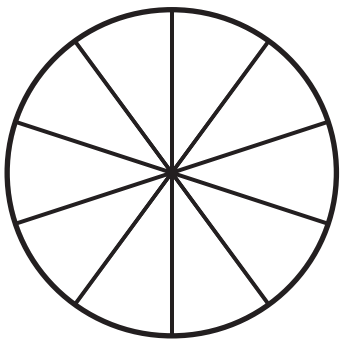




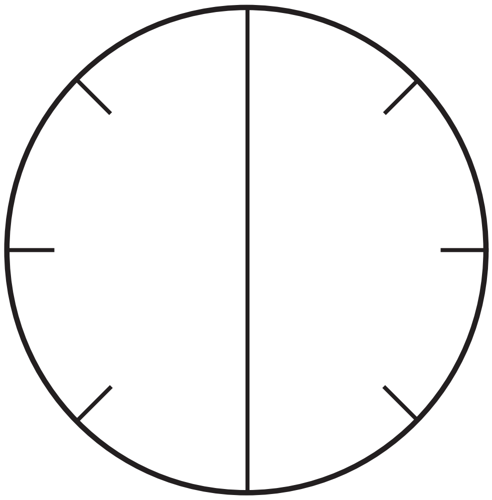


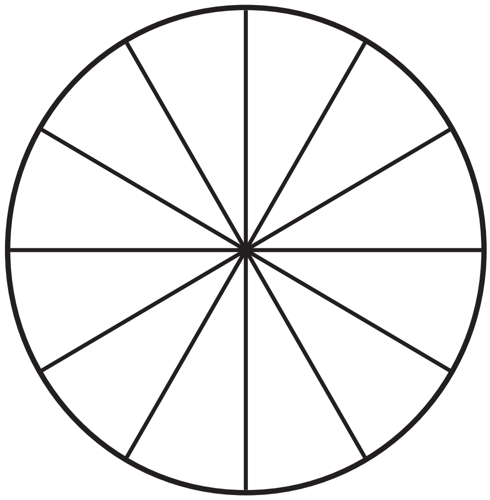


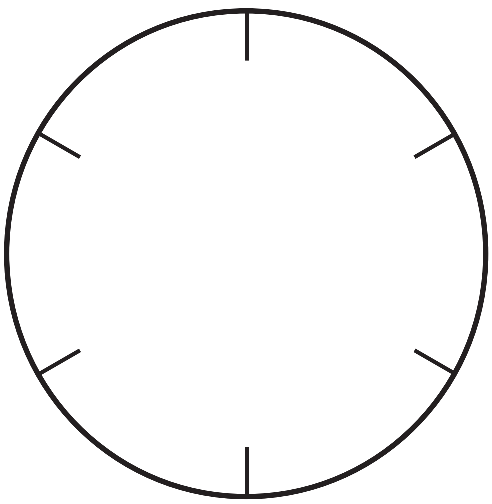


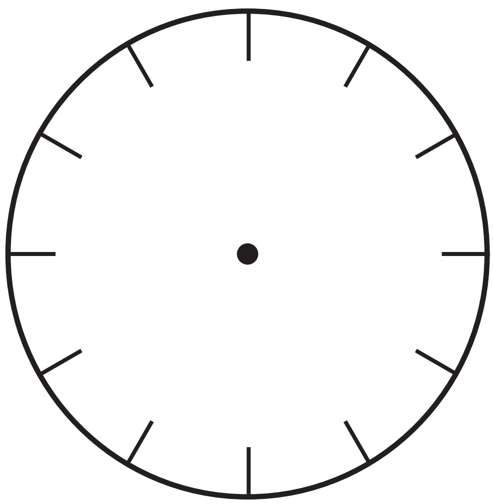


• Fraction Circles 2 •

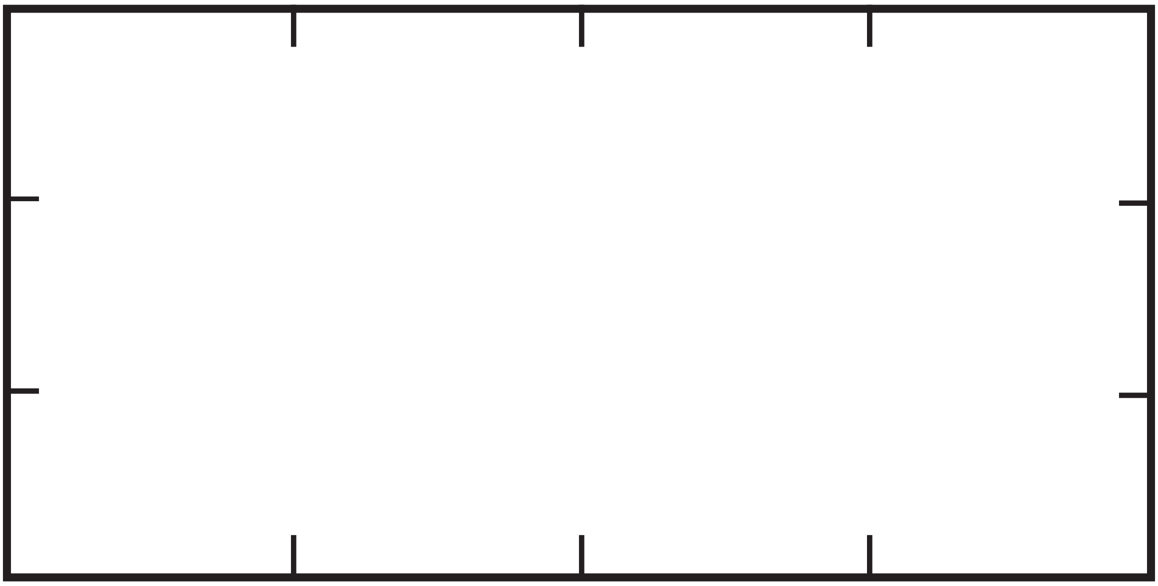


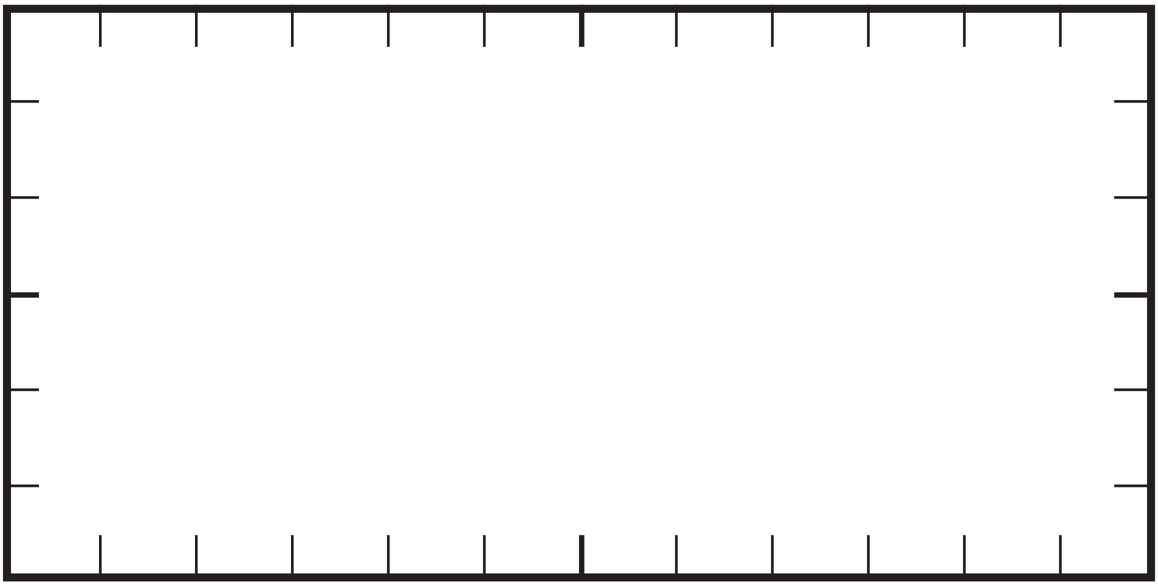


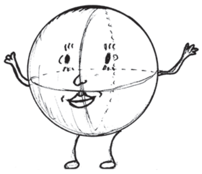




• Fraction Rectangles •







• Wholes and Parts (Unit Fractions) •

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

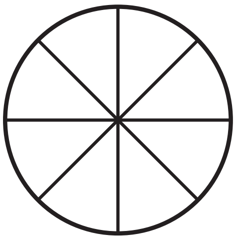
$\frac{1}{2}$	$\frac{1}{2}$

1

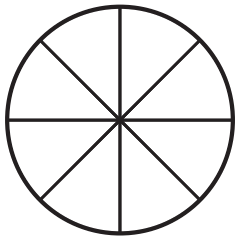
• Four Ways to Compare Fractions •

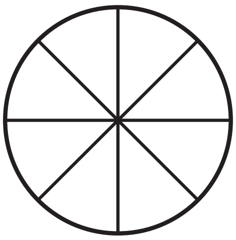
Shade in, then use these symbols to compare:

$>$ “greater than”
$<$ “less than”
$=$ “equal to”

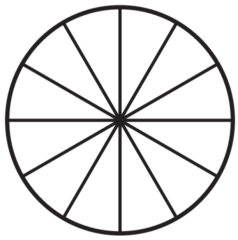
1) 

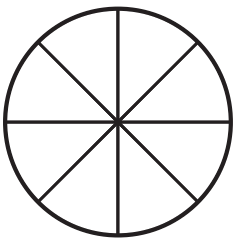
Common Denominators



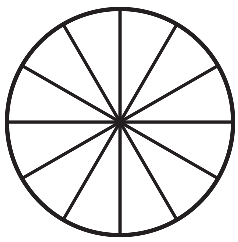
2) 

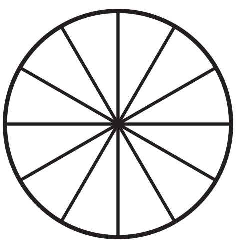
Same Numerators



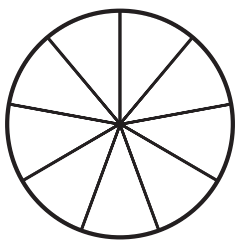
3) 

More Than Half, Less Than Half



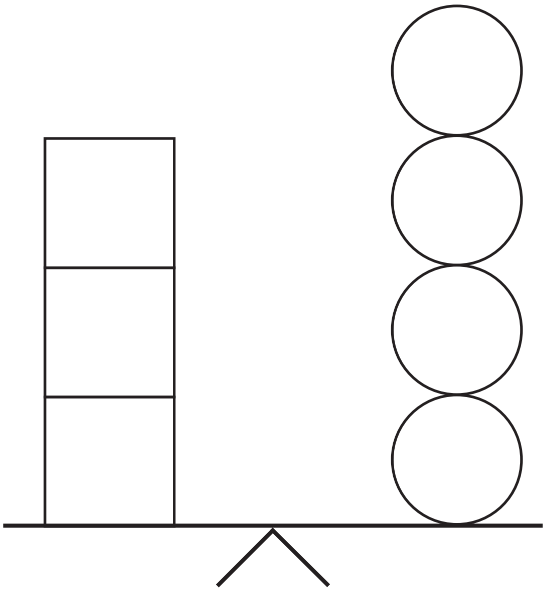
4) 

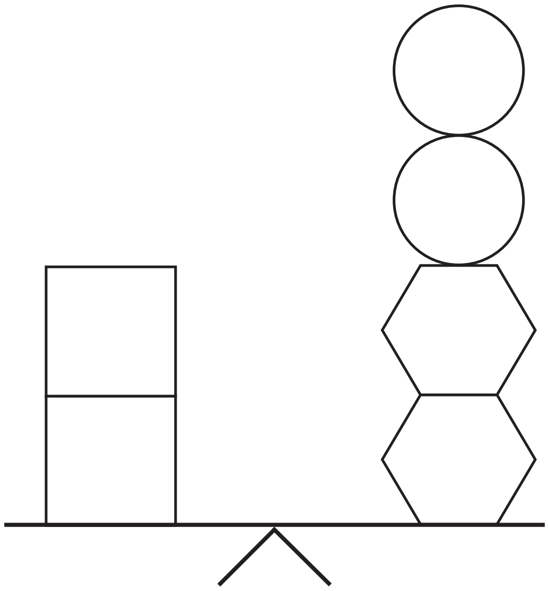
The Missing Piece

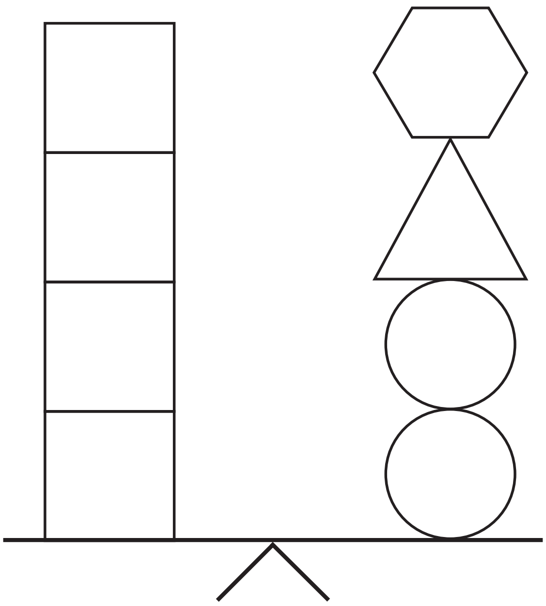


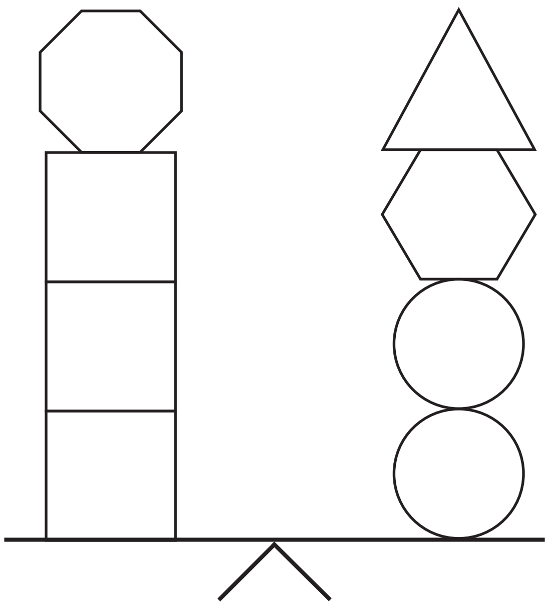
• Balancing Act •

In each exercise, same shapes must be the same number.



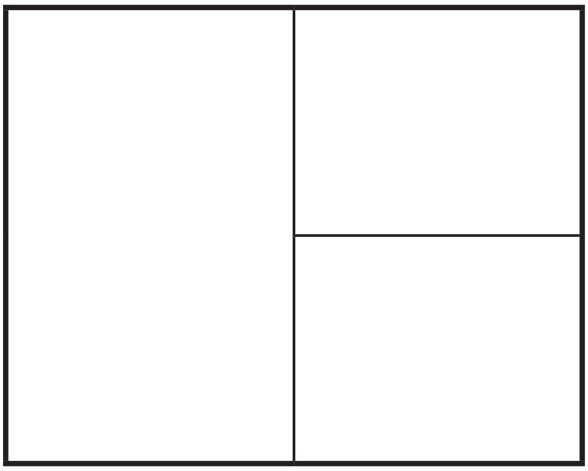


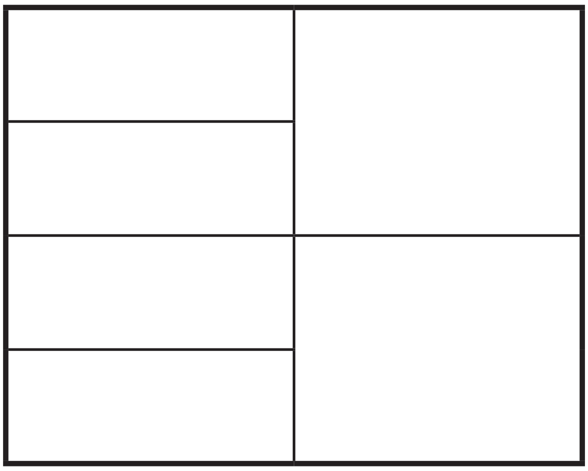


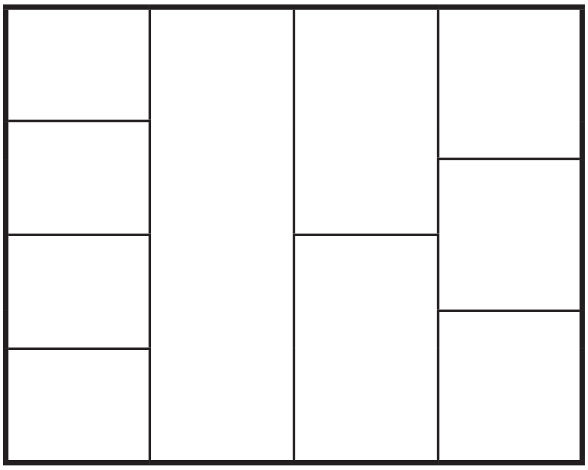


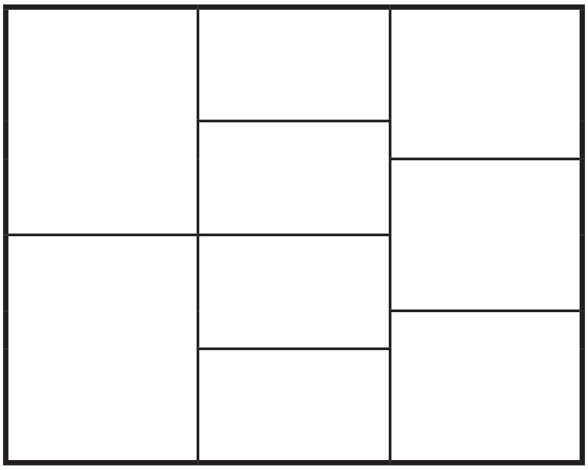
• Finding Missing Parts •

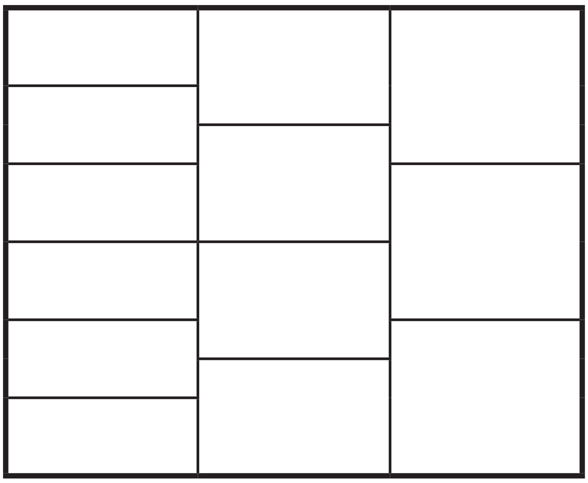
How much is the whole box worth?

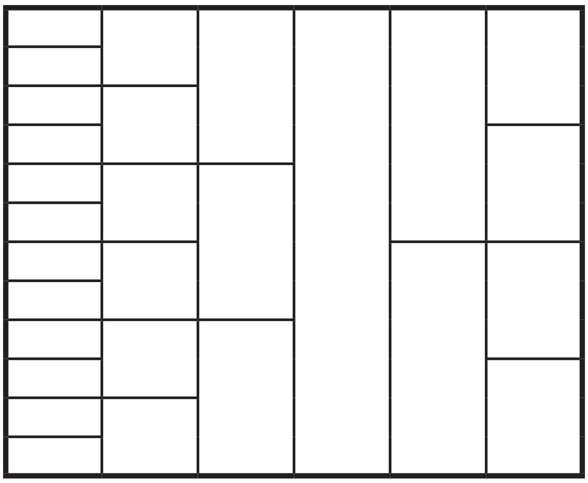








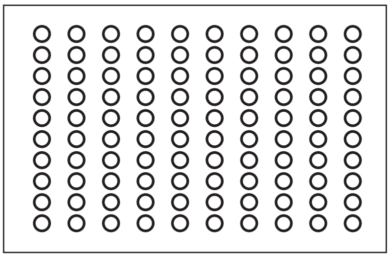




• Thinking in 1s, 10s, 100s & 1,000s •

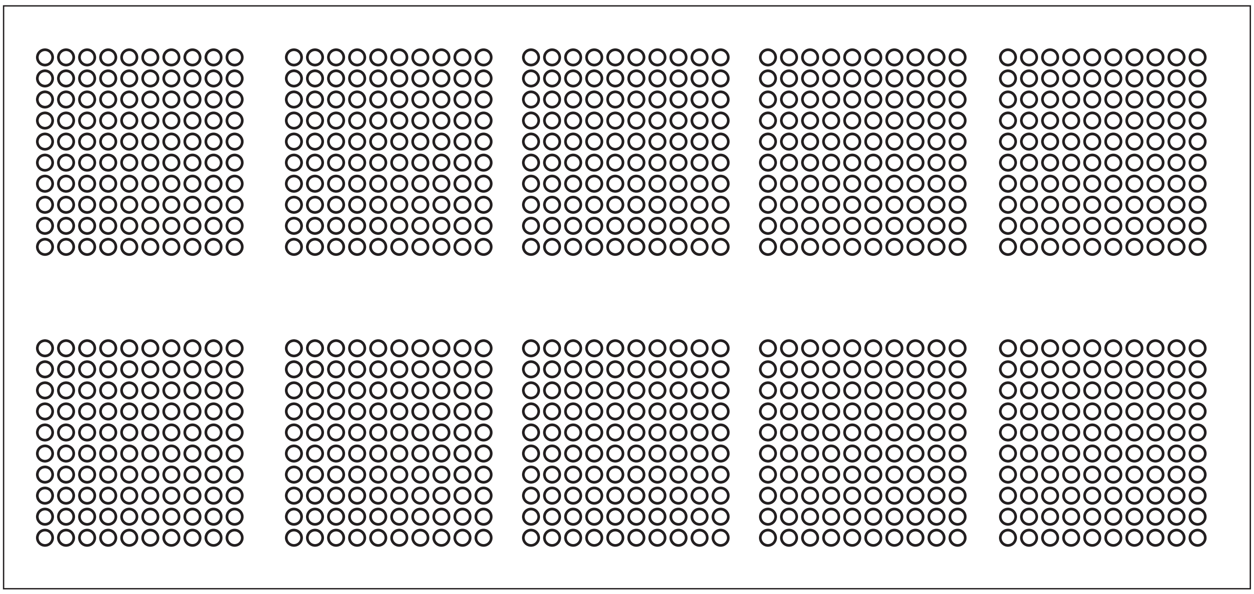
1) How many 1s make 10? ____________

2) How many 10s make 100? ____________



3) How many 100s make 1000? ____________

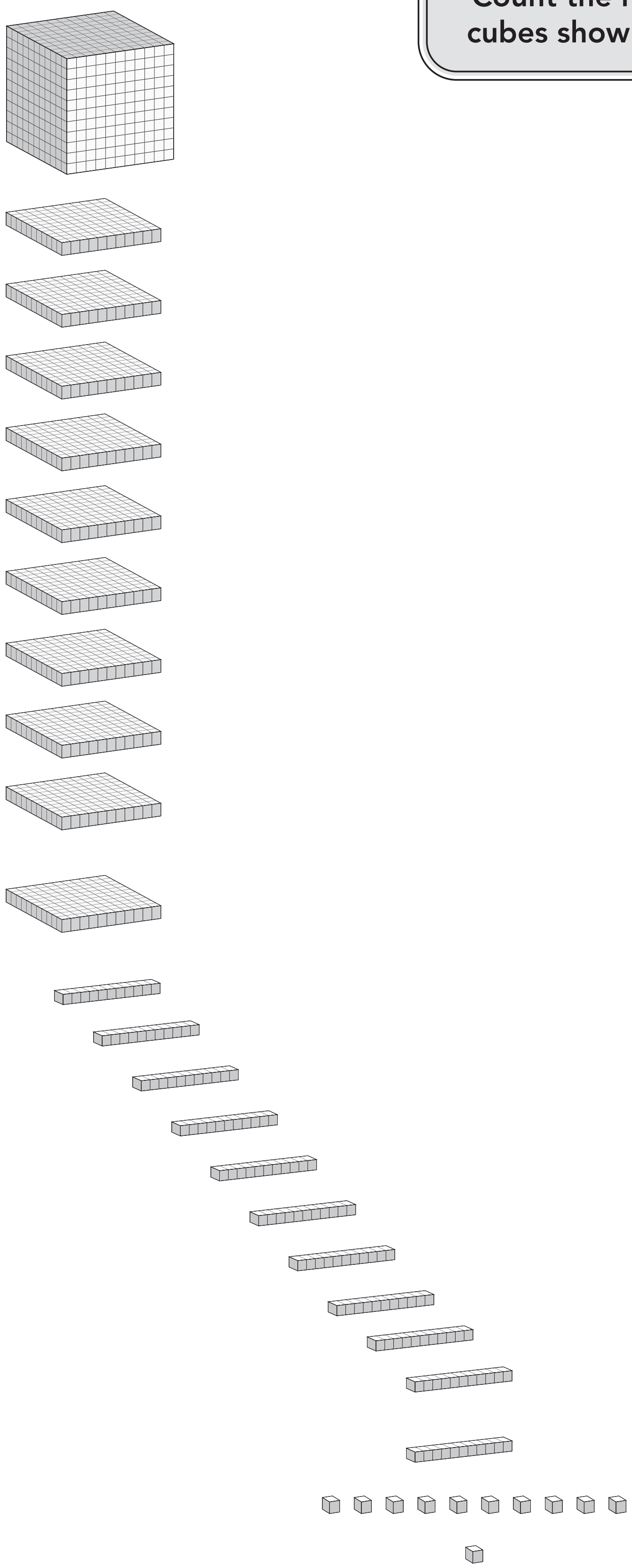
4) How many 10s make 1000? ____________



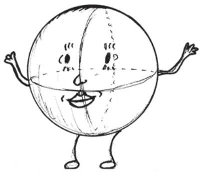
• Counting in Groups of 1, 10 & 100 •

Start here

Count the number of small cubes shown in the picture.



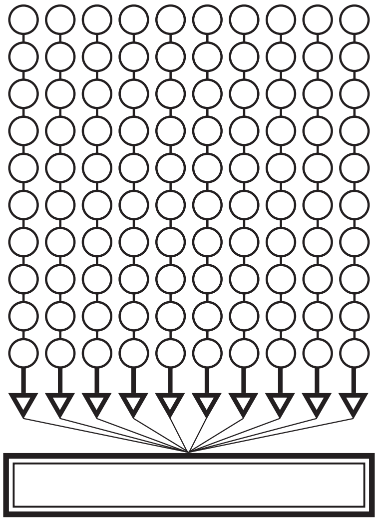
Start here



• Building Up by 10s •

Fill in the names of the missing symbols.

1)

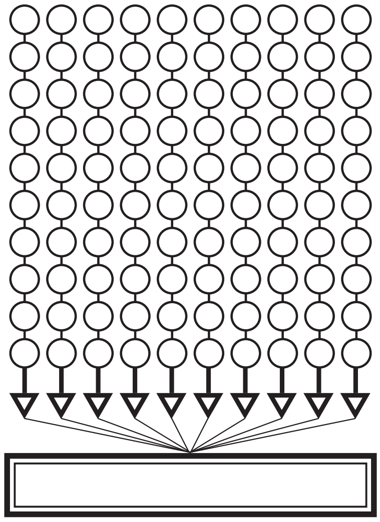


○ = ____________________

▽ = ____________________

▭ = ____________________

2)

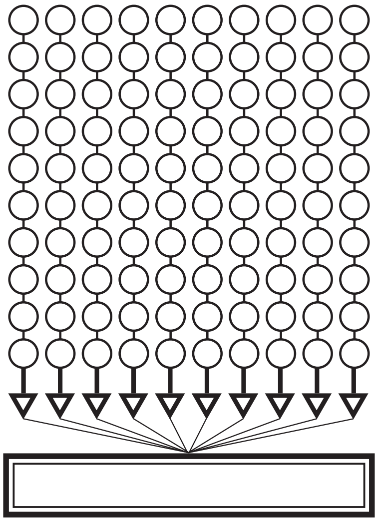


○ = ____________________

▽ = ____________________

▭ = ____________________

3)

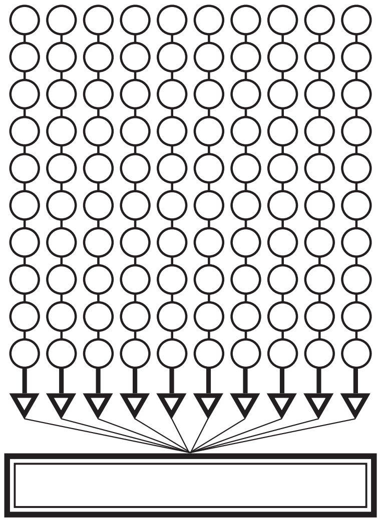


○ = ____________________

▽ = ____________________

▭ = ____________________

4)



○ = ____________________

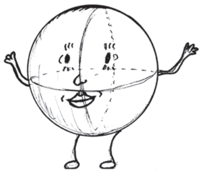
▽ = ____________________

▭ = ____________________

• Every Time You Count 10… •

100	10	1	,	100	10	1	,	100	10	1
	millions				thousands				ones	

100	10	1	,	100	10	1	,	100	10	1	,	100	10	1
	billions				millions				thousands				ones	



• The Decimal Number System •

General Rule: 10 of a smaller unit makes the next biggest unit.

____ ____ ____ , ____ ____ ____ , ____ ____ ____ . ____ ____ ____

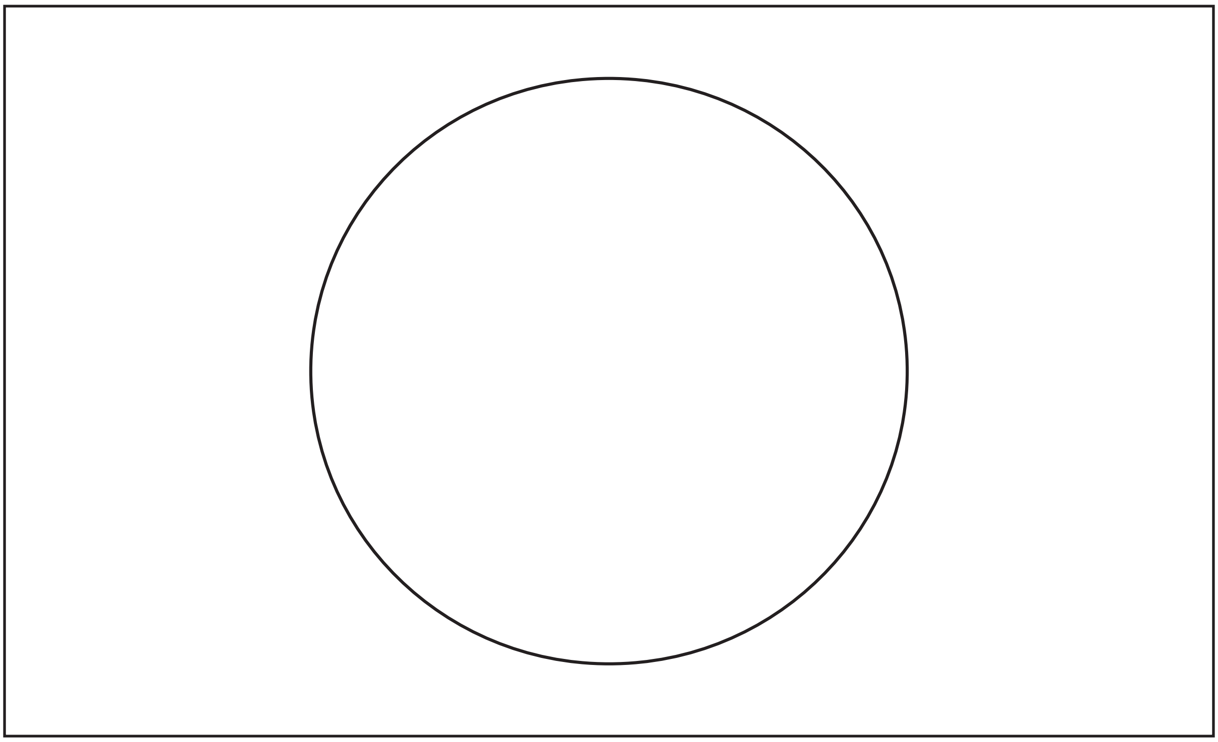
100	10	1	100	10	1	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	millions			thousands			ones		tenths	hundredths	thousandths

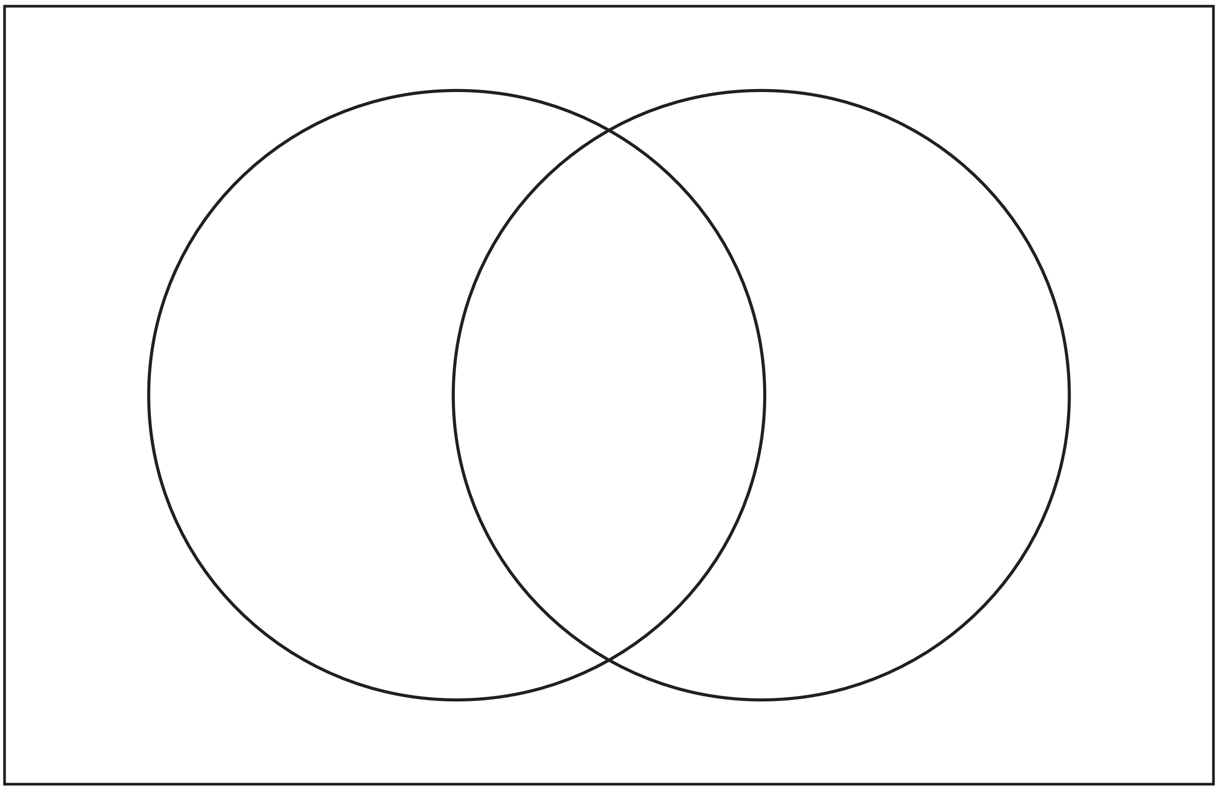
10 ones = 10
10 tens = 100
10 hundreds = 1000
(1000 ones = 1 thousand)

10 thousands = 10 000
10 ten thousands = 100 000
10 hundred thousands = 1 000 000
(1000 thousands = 1 million)

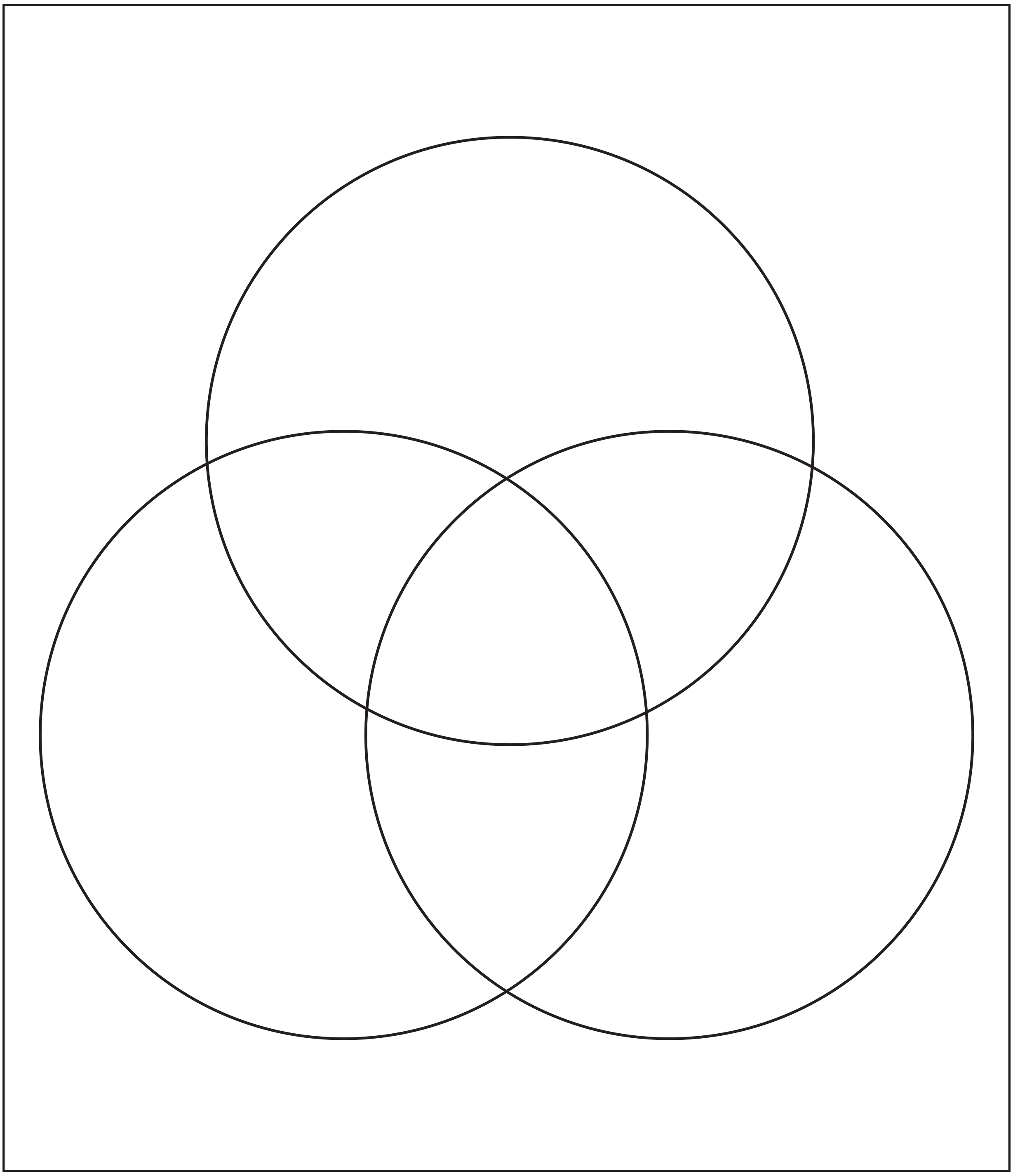
10 thousandths $(\frac{10}{1000}) = \frac{1}{100}$
10 hundredths $(\frac{10}{100}) = \frac{1}{10}$
10 tenths $(\frac{10}{10})$ = 1 whole
100 hundredths $(\frac{100}{100})$ = 1 whole
1000 thousandths $(\frac{1000}{1000})$ = 1 whole

• Sets and Venn Diagrams 1 •

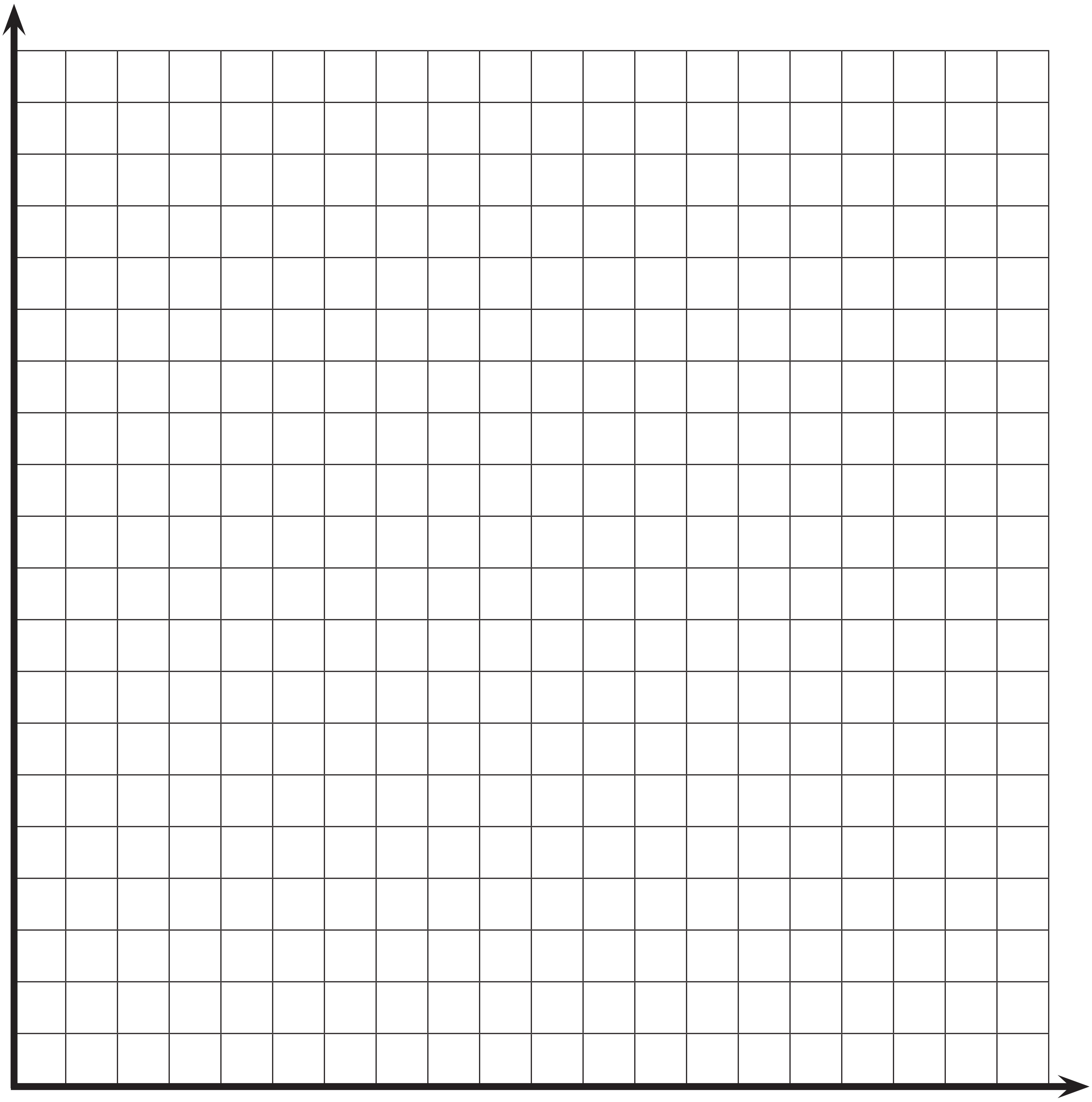


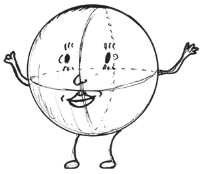


• Sets and Venn Diagrams 2 •

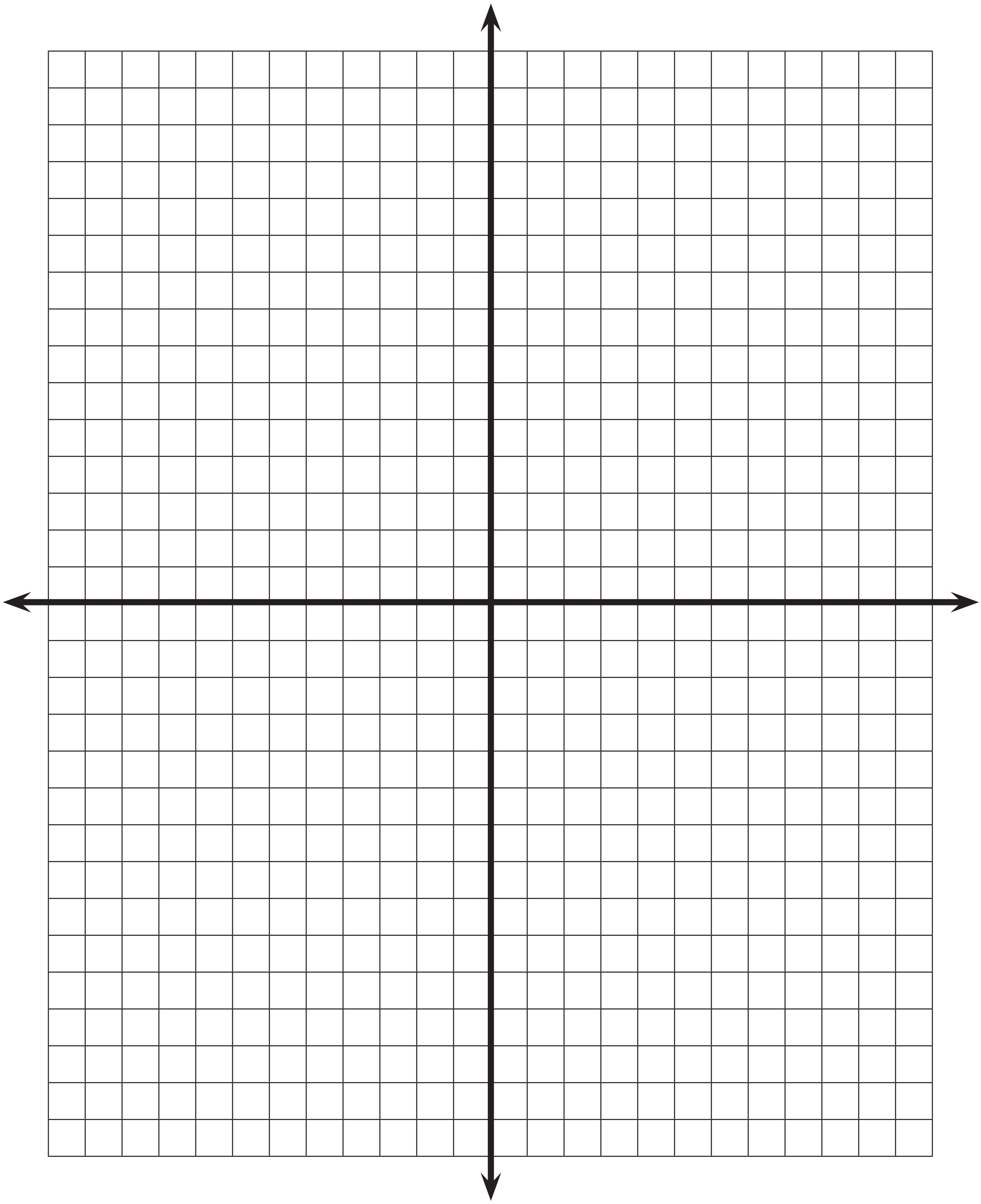


• First Quadrant Grid •

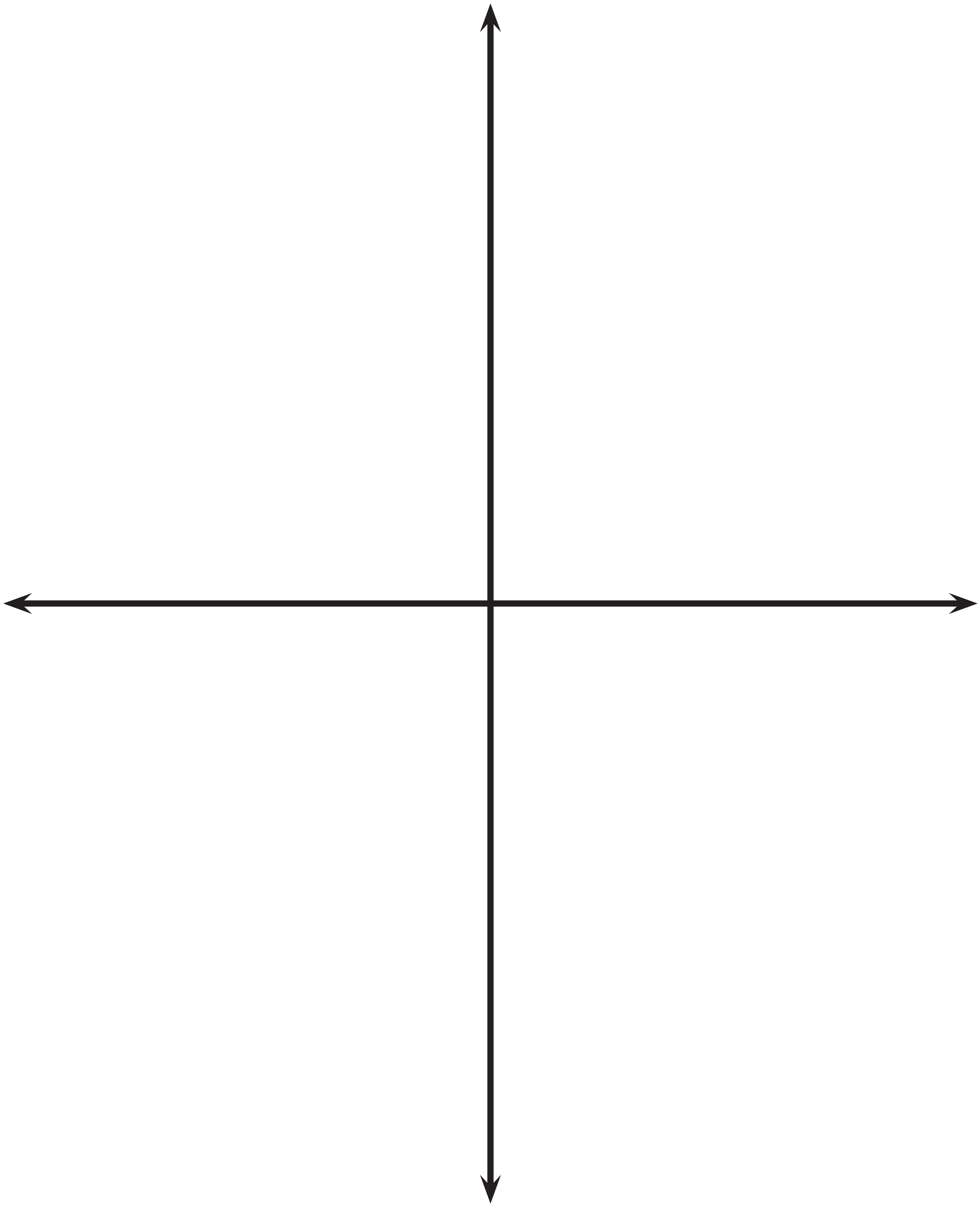


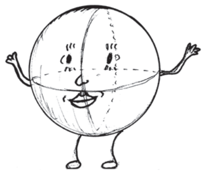


• X-Y Axis (with Grid) •

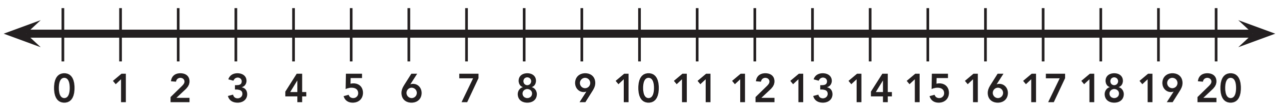


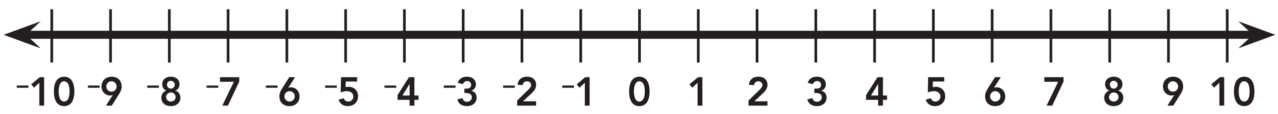
• X-Y Axis (No Grid) •



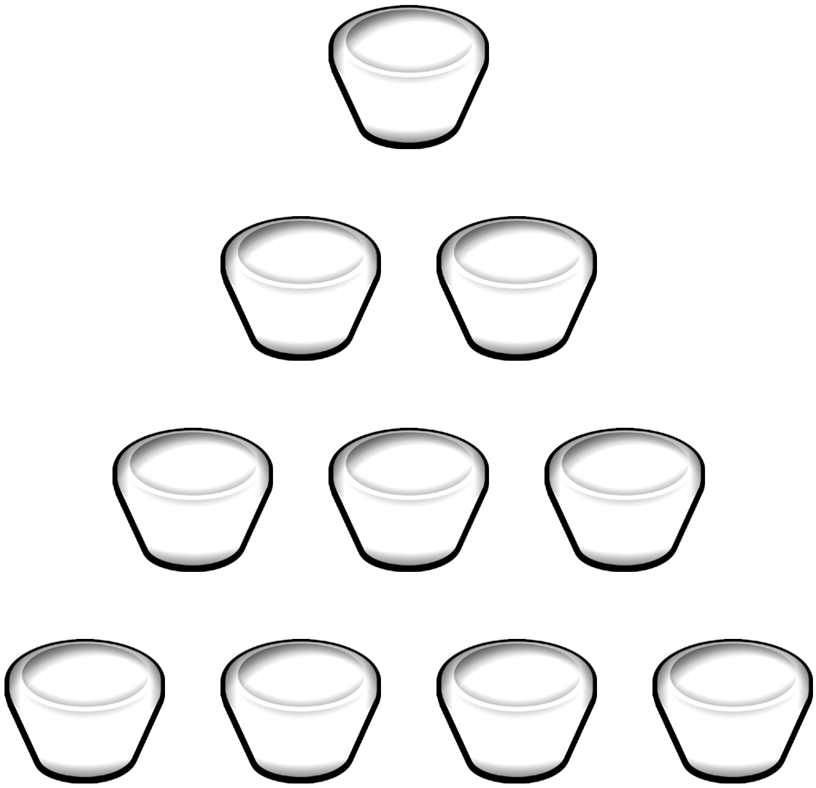


• Number Lines •



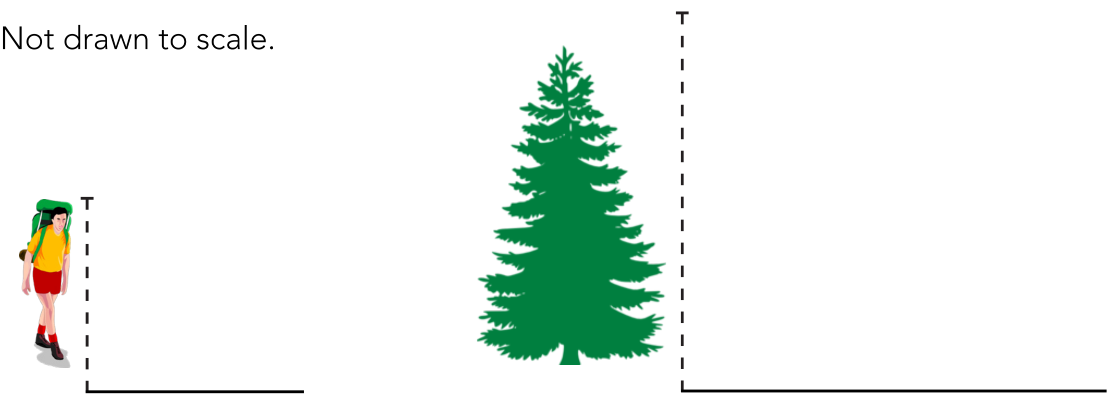


• Proportional Reasoning •

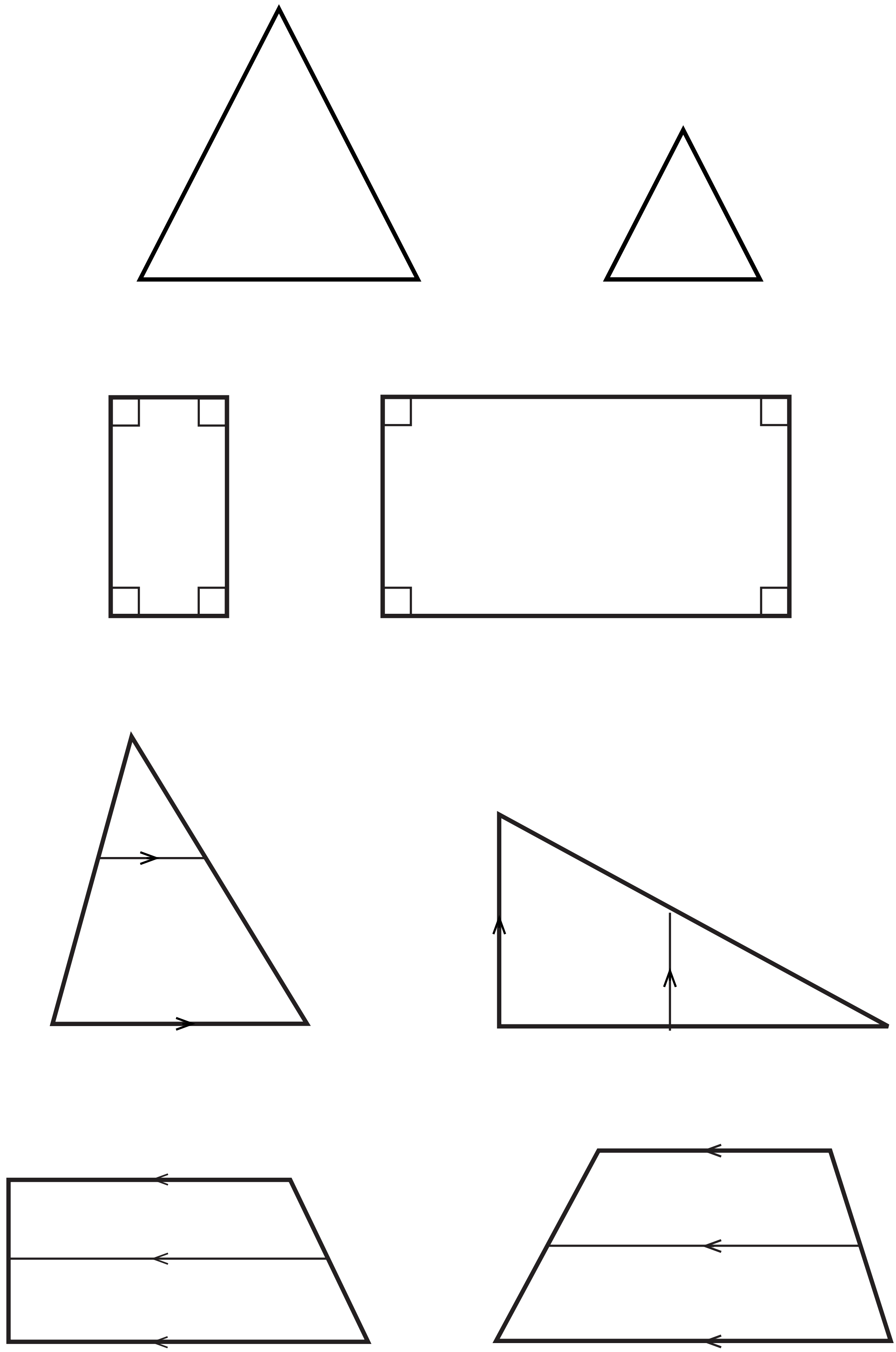


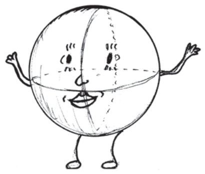
• Proportional Thinking 1 •

Not drawn to scale.

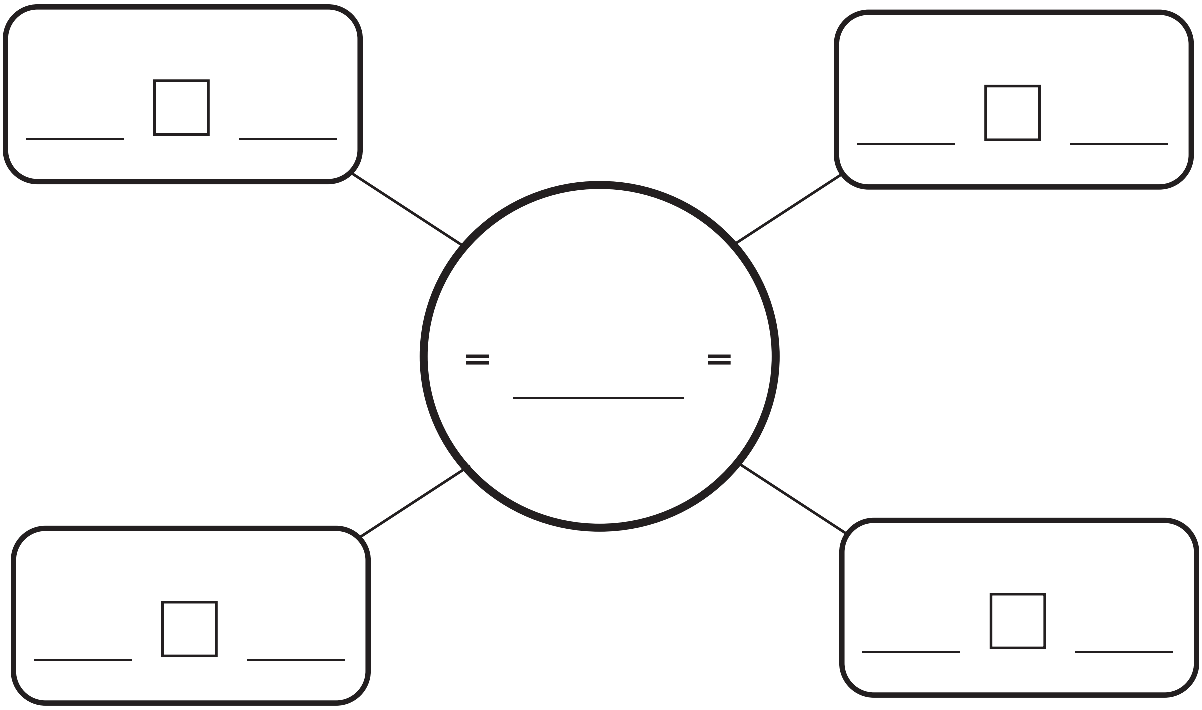


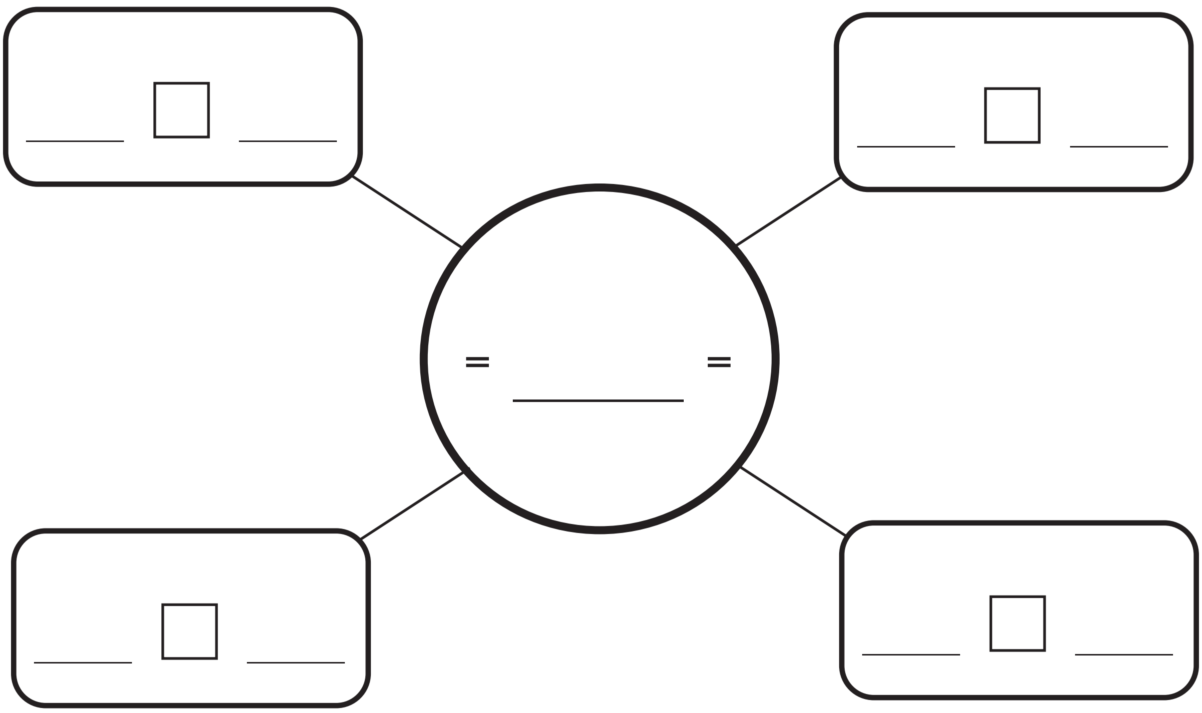
• Proportional Thinking 2 •



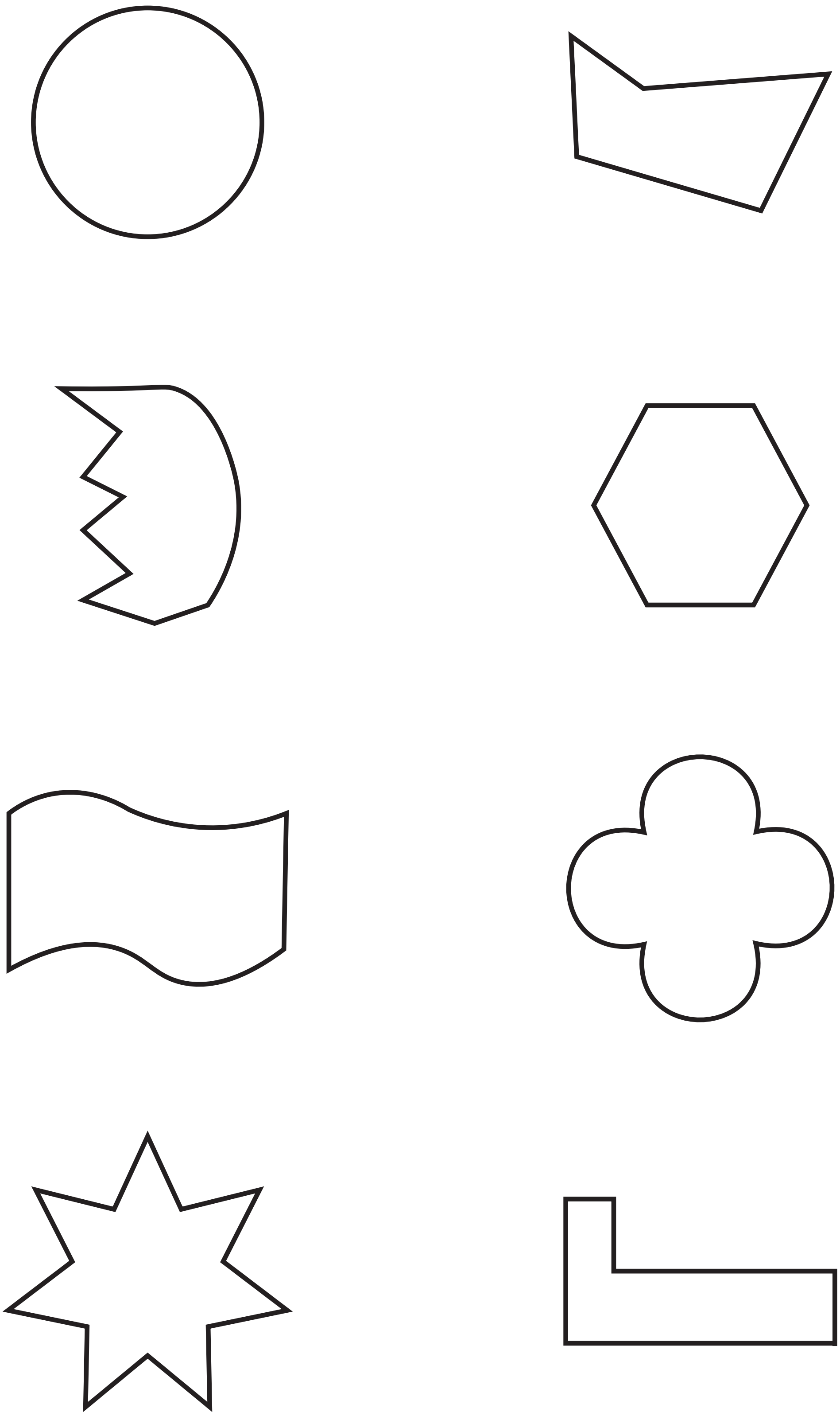


• Operation Shapes •



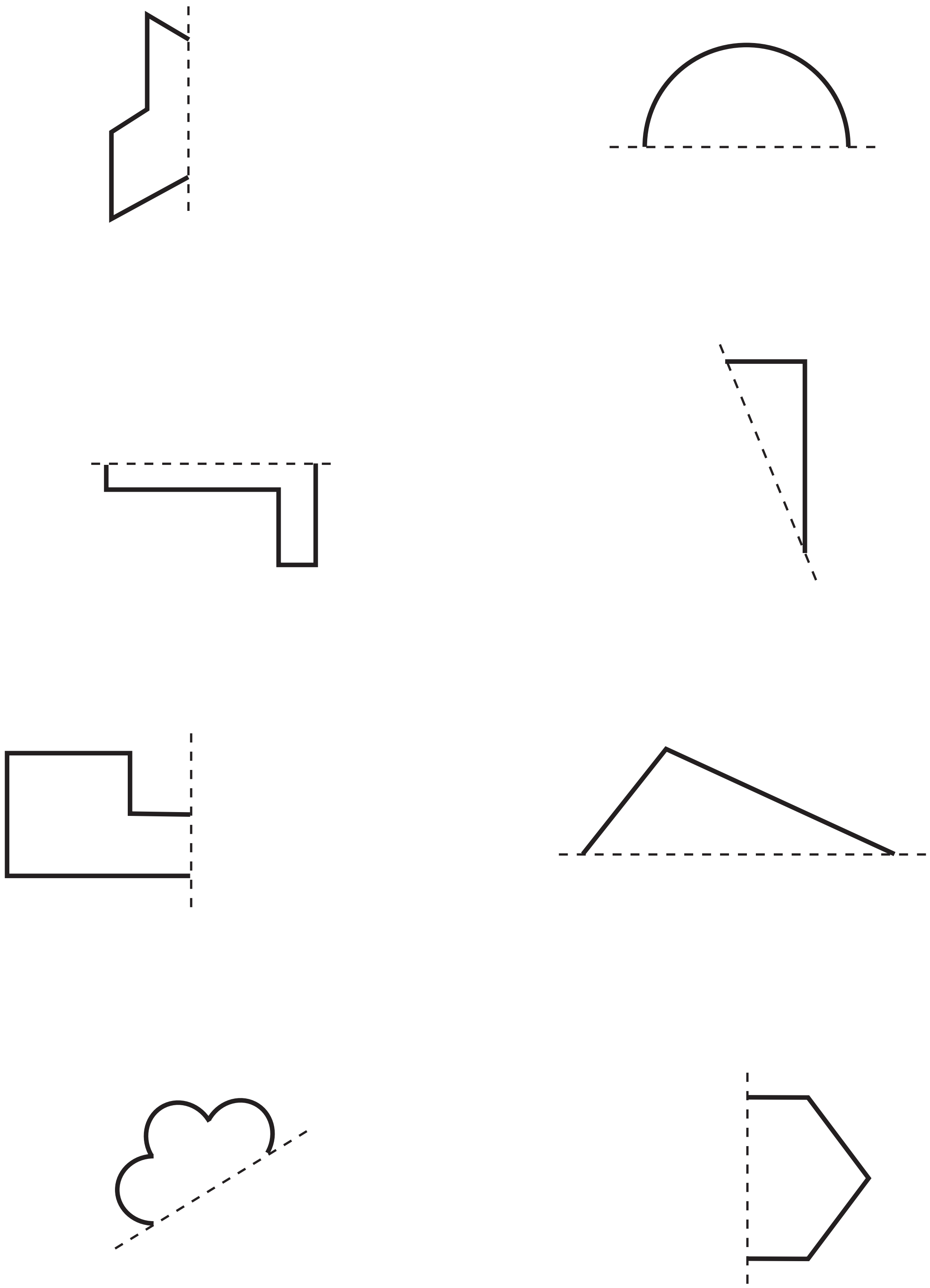


• Symmetry 1 •

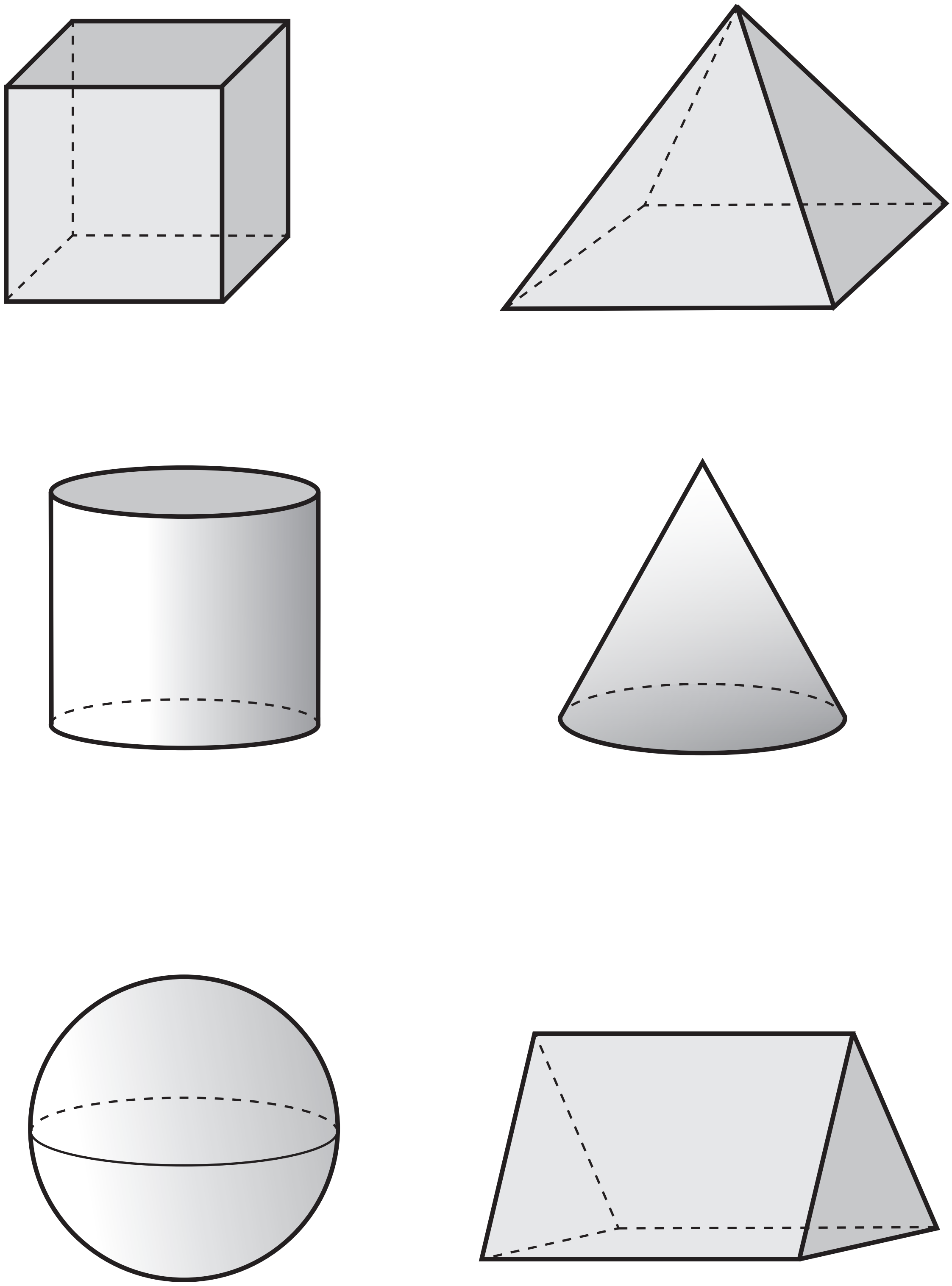


• Symmetry 2 •

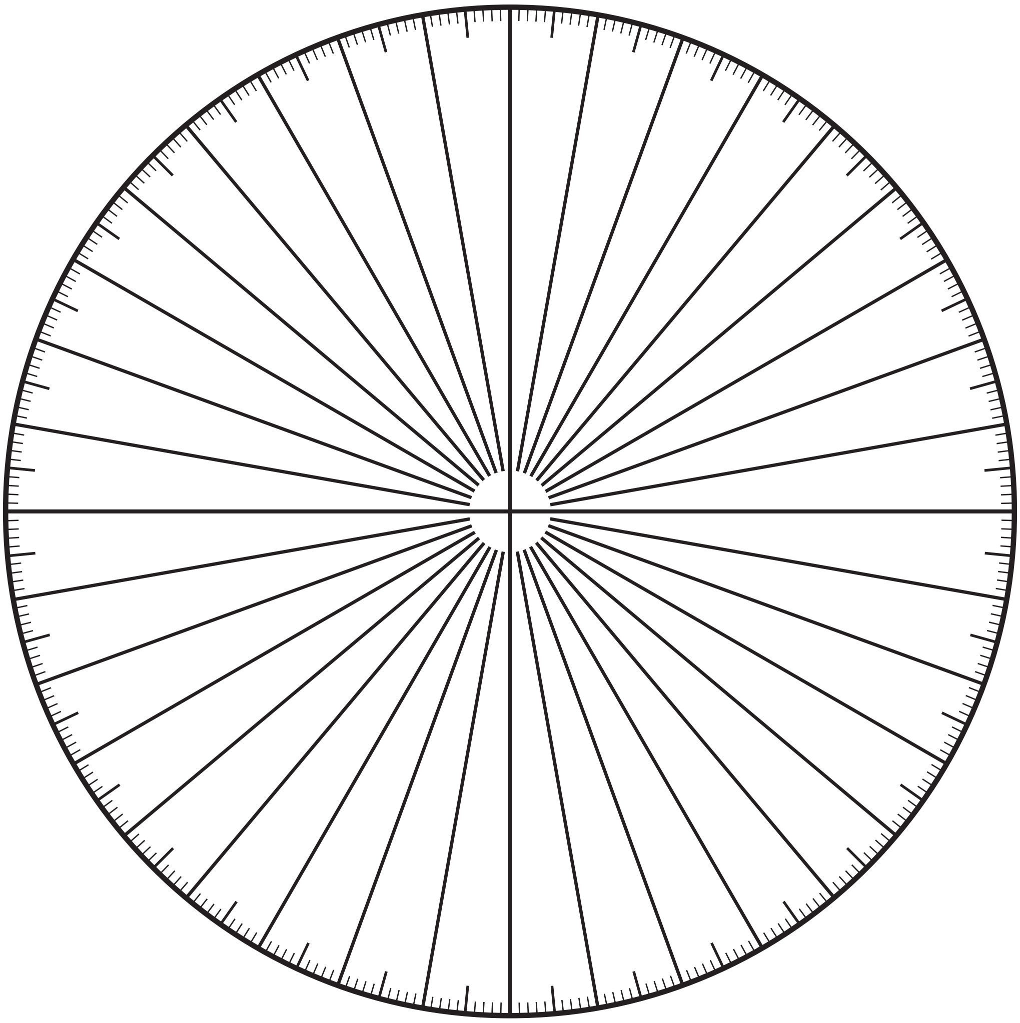
The dashed line is a LINE OF SYMMETRY. Draw the missing part of the shape.

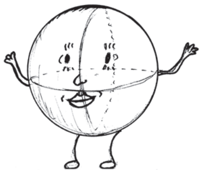


• Geometric Solids •



• Seeing the 360° of a Circle •





DeskTools and Manipulatives Activity

Identify appropriate uses of DeskTools and Manipulatives.

• List of Manipulatives •

Take an inventory of the manipulatives in the Centre. Check off the ones your Centre has and add any additional manipulatives below.

Manipulatives	Check
Mathnasium Cards	☐
Playing Cards	☐
6–, 12–, 30–sided dice	☐
Clock (analog)	☐
Coins	☐
Place value cubes/sticks	☐
Rainbow Fraction Tiles	☐
ManipuLite Geometric Solids	☐
Large Plastic Geometric Shapes	☐

Additional Manipulatives:

____

____

Try these: **Give an example of for how each manipulative could be used.**

1) Cards ______________________________

3) Dice ______________________________

4) Clock ______________________________

5) Coins ______________________________

6) Place value cubes/sticks ______________________________

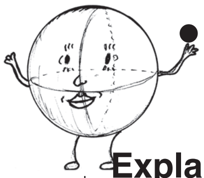
7) Fraction tiles ______________________________

8) Geometric Solids ______________________________

9) What is special about Mathnasium Cards?

____

____



• DeskTools and Manipulatives Activity •

Explain how you can use either the "100 Circles" (p. 11) or the "Hundred Chart" (p. 15) DeskTool to teach these questions.

3) 4, 9, 14, 19, 24, ______, ______, ______

4) 8, 11, 14, 17, 20, ______, ______, ______

5) 1, 11, 21, 31, 41, ______, ______, ______

PK-3053-00_Counting_Number_Patterns

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3) 52 + 11 = __________

4) 29 + 9 = __________

5) 66 + 8 = __________

PK-3106-00_Counting_by_Adding_10_First

____

____

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• DeskTools and Manipulatives Activity •

Decide whether a DeskTool or Manipulative would help you best teach each exercise. Then, explain how you would use it.

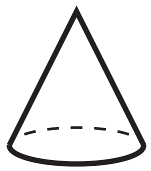
1) 8 **5¢ coins** = _______¢ = ______ **10¢ coins**

2) 25 **10¢ coins** = _______¢ = ______ **5¢ coins**

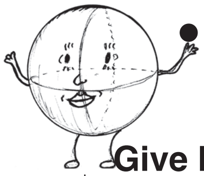
3) 5 **10¢ coins** = _______¢ = ______ **20¢ coins**

PK-3046-00_Coin_Equivalence

Write a description for each figure using as many properties as you know.

Figure	Figure name	Description
		

PK-3081-00_Properties_3D_Figures



• DeskTools and Manipulatives Activity •

Give both a Manipulative and a DeskTool example to help with the following questions.

3) half of 97 = __________

4) half of 81 = __________

5) half of 75 = __________

PK-3100-00_Half_of_Odd_Numbers

Manipulative	Desktool

• DeskTools and Manipulatives Activity •

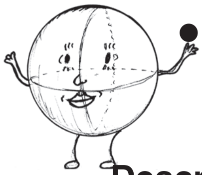
Give both a Manipulative and a DeskTool example to help with the following questions.

Arrange the numbers in ascending order.

2) $1,\ \frac{5}{12},\ \frac{4}{7},\ \frac{1}{8},\ \frac{8}{3},\ 0$ ______ ______ ______ ______ ______ ______

PK-3214-00_Ordering_Fractions

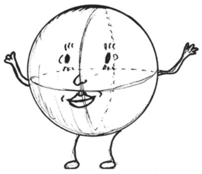
Manipulative	Desktool



• DeskTools and Manipulatives Activity •

Describe activities, either using Manipulatives, DeskTools or a combination of both, that can be used as a Multiplication Activity for the Play with Multiplication pages.

Give an example of how to use a Manipulative and a DeskTool to help a student who is struggling with equivalent fractions.



Reflection
&
Progress Check

• Reflection: DeskTools and Manipulatives •

Give an explanation for 7% of 250 using each of the five modalities (Mental, Visual, Verbal, Tactile, Written).

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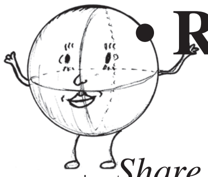
Use this space for any visuals.



• Reflection: DeskTools and Manipulatives •

Share a time where you used a DeskTool or Manipulative to facilitate a visual or tactile learning experience. What DeskTools and Manipulatives will you use the most? Why?

• Progress Check •

Evaluate your understanding of the following using this scale:

0 – No idea **1**– Disagree **2**–Unsure **3**–Agree

DeskTools and Manipulatives

I have a good understanding of:					
1.	what manipulatives are available in the Centre.	**0**	**1**	**2**	**3**
2.	how manipulatives and DeskTools to support a student's learning experience.	**0**	**1**	**2**	**3**
3.	how to check for transfer of knowledge after using a manipulative with a student.	**0**	**1**	**2**	**3**
4.	how to use Mathnasium cards for a quick activity.	**0**	**1**	**2**	**3**
5.	why students need maths instruction delivered in multiple ways (MVVTW).	**0**	**1**	**2**	**3**
6.	how I will implement DeskTools and Manipulatives in my instruction.	**0**	**1**	**2**	**3**

Meet with your facilitator to review all completed training since your last Progress Check. This may include:

- ☐ DeskTools and Manipulatives Video (section 19, p. 2)
- ☐ DeskTools and Manipulatives Reflective and Critical Questions (section 19, p. 3)
- ☐ The DeskTools and Manipulatives activity (section 19, p. 43)
- ☐ DeskTools and Manipulatives Reflection (section 20, p. 2, 3)

NOTES:

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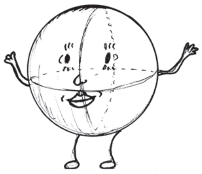


Practical Exam – Part 2

Practical Exam Prep

Practical Exam (see facilitator)

Self-Reflection

Practical Exam – Part 2 Prep

Final Practical Exam. Administered upon completion of the Right Instruction binder.

• Practical Exam – Part 2 Prep •

Upon completion of the Right Instruction Binder you will demonstrate what you have learned during the practical test. Your facilitator will evaluate you on your ability to deliver **Numerical Fluency Instruction, Multiplication Fact Fluency Instruction and use DeskTools and Manipulatives**. They will be using the skills outlined in the **10 Mathnasium Rules of Engagement with focus on #5, #7 and #8** as a guide for your evaluation. They will observe you while you teach multiple students. Below is a list of all 10 Rules and how you can prepare.

Review the Numerical Fluency (Section 15) and Multiplication Fact Fluency courses (Section 17) and manuals and the 10 Rules of engagement with a focus on #5, 7 and 8.

10 Mathnasium Rules of Engagement:

Rule #1

Use the Mathnasium teaching constructs. Use the Teaching Icons as guides.

To Prepare: Review the Mathnasium Teaching Constructs (Section 6 p. 11-18 and Section 8 p. 13-23).

Rule #2

Do not repeat an explanation if it is not working. Try another approach. Do not force students to keep working on material they have already mastered.

To Prepare: Review the Modes of Teaching document (Section 6 p. 5-9).

Rule #3

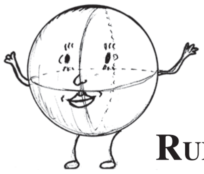
Fall back on prerequisite knowledge when the student is having trouble. Extend knowledge when the student understands the concept.

To Prepare: Review The Meaning of Percent: “Per Cent” document (Section 10 p. 13) and Units of Measure document (Section 10 p. 27).

Rule #4

Praise, encourage and constructively criticise when appropriate.

To Prepare: Review the Proactive Teaching document. (Section 8 p. 5).



• Practical Exam – Part 2 Prep •

Rule #5

Use Socratic Questioning when it's appropriate. Use Direct Teaching when it's appropriate.

To Prepare: Review and reflect on back on your "teach the facilitator" and student instruction interactions.

Rule #6

Use Mathnasium vocabulary because it makes sense. Avoid confusing nomenclature.

To Prepare: Review the Mathnasium Teaching Principle K.I.S.S (Section 8 p. 10). the Mathnasium Teaching Constructs (Section 6 p. 11-18 and Section 8 p. 13-23). and the Core Vocabulary (Section 10 p. 36).

Rule #7

Use drawings, diagrams, manipulatives and Desk Tools when appropriate to clarify and reinforce concepts visually.

To Prepare: Review the DeskTools and Manipulatives course and activity (Section 19 p. 39).

Rule #8

Enable students to achieve metacognition, an awareness of one's own thinking process.

To Prepare: Review your instructional experiences, feedback from your facilitator and instructional videos from the MU courses.

Rule #9

Require students to use mental maths. to enhance numerical fluency and limit reliance on pencil and paper.

To Prepare: Review the Addition, Subtraction and Division Tips documents (Section 6 p. 24-33).

Rule #10

Master Team Teaching.

To Prepare: Review the Team Teaching Checklist (Section 4 p. 6-8), Team Teaching Course and your teaching experiences.

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Practical Exam – Part 2

Meet with your facilitator, they will administer the Practical Exam.

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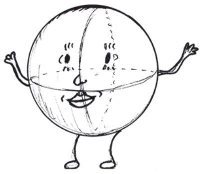

Self-Reflection

Complete after taking the Practical Exam

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• Self-Reflection •

Evaluate how well you feel you demonstrated the following:

1– Needs Improvement **2**– Met Expectations **3**–Exceeded Expectations

Practical Exam

Mathnasium's Rules of Engagement				
#1.	Use the Mathnasium teaching constructs. Use the Teaching Icons as guides.	**1**	**2**	**3**
#2.	Do not repeat an explanation if it is not working. Try another approach. Do not force students to keep working on material they have already mastered.	**1**	**2**	**3**
#3.	Fall back on prerequisite knowledge when the student is having trouble. Extend knowledge when the student understands the concept.	**1**	**2**	**3**
#4.	Praise, encourage and constructively criticise when appropriate.	**1**	**2**	**3**
#5.	Use Socratic Questioning when it's appropriate. Use Direct Teaching when it's appropriate.	**1**	**2**	**3**
#6.	Use Mathnasium vocabulary because it makes sense. Avoid confusing nomenclature.	**1**	**2**	**3**
#7.	Use drawings, diagrams, manipulatives and DeskTools when appropriate to clarify and reinforce concepts visually.	**1**	**2**	**3**
#8.	Enable students to achieve metacognition, an awareness of one's own thinking process.	**1**	**2**	**3**
#9.	Require students to use mental maths to enhance numerical fluency and limit reliance on pencil and paper.	**1**	**2**	**3**
#10.	Master Team Teaching.	**1**	**2**	**3**

Numerical Fluency and Multiplication Fact Fluency				
1.	Use Numerical Fluency techniques and vocabulary.	**1**	**2**	**3**
2.	Use Multiplication Fact Fluency strategies.	**1**	**2**	**3**
3.	Play MFF activities/games and modify when necessary.	**1**	**2**	**3**

Once you complete the Self-reflection meet with your facilitator to discuss your Practical, discuss continued training and an Improvement Plan (if necessary).

