General Topology

Zac Zerafa

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Contents

Part I

Topological spaces

Chapter 1

Introduction to metric spaces

One major goal of topology is to define some notion of 'closeness' without resorting to measuring distance; this allows us to consider ideas like limits and continuous functions in very abstract spaces.

To develop the idea of a topological space, we first resort to the basics of metric spaces (spaces where distance is defined) and attempt to make abstractions. This idea is that metric spaces are much more intuitive, so we start with metric spaces and then slowly take away distance from the course of discourse.

1.1 Metric space

Definition 1.1. A *metric space*is an ordered pair (X, d).

- X is a set
- $d : X \times X \to \mathbb{R}^+$ is a *distance function* or *metricl* that defines the notion of distance.

The metric must satisfy the following properties.

- $d(x, y) \in [0, \infty)$
- $d(x, y) = 0 \iff x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq d(x, z) + d(z, y)$

This last condition for the distance function is known as *the triangle inequality*; it is an incredibly useful tool in proofs.

Here are some examples to familiarize us with metrics; the first of which is already known to us.

Example 1.1. The *Euclidean metric* is the following metric defined on $\mathbb{R}^n$.

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{y}_i)^2}$$

Notice that for $\mathbb{R}$, this is just $d(x, y) = |x - y|$, and higher dimensions are variations of the Pythagorean theorem.

Example 1.2. The *Chebyshev metric* is the following metric defined on $\mathbb{R}^n$.

$$d(\mathbf{x}, \mathbf{y}) = \max_i(|\mathbf{x}_i - \mathbf{y}_i|)$$

This metric represents the largest difference between two points on an axis.

Example 1.3. The *taxicab metric* is the following metric defined on $\mathbb{R}^n$.

$$d(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} |\mathbf{x}_i - \mathbf{y}_i|$$

This metric represents the distance of the smallest path between the points if one could only 'walk' along the axises.

1.2 Open balls

We now introduce a useful tool in the analysis of metric spaces.

Definition 1.2. Let (X, d) be a metric space, $p$be an element in X and $r \in (0, \infty)$ a nonnegative real number. An *open ball centered at p with radius r* is a set $B(p, r)$ defined as such.

$$B(p, r) = \{x \in X : d(x, p) < r\}$$

It is called a 'ball' because open balls made with the Euclidean metric look like a ball. The idea is that these sets cover any points strictly closer than r units away. The 'open' part of the name corresponds to the strict inequality $<$ rather than $\leq$, so that the ball doesn't contain the boundary points of the ball.

It is only because open balls exclude these boundary points that we can prove the following.

Proposition 1.1. If $x \in B(p, r)$, then there exists some $B(x, s)$ such that $B(x, s)$ is contained completely in $B(p, r)$. Elements in open balls of p have their own open balls completely contained in that open ball of p.

$$x \in B(p, r) \implies \exists s \in (0, \infty)[B(x, s) \subseteq B(p, r)]$$

1.3 Open and closed sets

Inspired by the properties of open balls, we would like to create classes for sets that contain none of their boundrary points, and all of their boundary points. We start by actually defining what boundary points are.

Definition 1.3. Let (X, d) be a metric space. The *boundary of* S is the set of all elements $p \in X$ such that all their open balls have intersections with S and $X \setminus S$. We denote the boundary of S as ∂S, and elements of ∂S are called boundary points of S.

Definition 1.4. Let (X, d) be a metric space. A *open set of* X is a set U that is disjoint to ∂U. We say that U *is open in* X.

Definition 1.5. Let (X, d) be a metric space. A *closed set of* X is a set F that completely contains ∂F. We say that F *is closed in* X.

1.4 Reducing open and closed sets to 'algebra of set' constructions

Noting that $\partial S = \partial(X \setminus S)$, we can prove the following curious proposition.

Proposition 1.2. F is closed iff $X \setminus F$ is open.

This gives us a nice set theoretic representation for closed sets that doesn't include a metric; we'll make this the prime definition once we've defined what topologies are!

Theorem 1.1. Let (X, d) be a metric space. If U is open in X then any element in U has an open ball completely contained in U.

This brings us 2 intuitive corollaries.

Corollary 1.1. Let (X, d) be a metric space. Open balls are open sets.

Corollary 1.2. Let (X, d) be a metric space. open sets are unions of open balls.

The following proposition is the key ingredient in defining what a topology is.

Proposition 1.3. Let (X, d) be a metric space.

- X and $\emptyset$ are open sets
- Open sets are closed under countable unions
- Open sets are closed under finite intersections

Since a family of open sets must obey these 3 properties, if we consider arbitrary sets that satisfy these 3 properties rather than the open balls, we have the definition of a topology.

Chapter 2

Topological spaces

2.1 Topology

We use the 3 set theoretic properties obeyed by open sets of metric spaces to define topologies in a way that are backwards compatible with metrics.

Definition 2.1. A *topology on a set* X is a set $\mathcal{T}$ of subsets of X such that:

- X and $\emptyset$ are in $\mathcal{T}$
- $\mathcal{T}$ is closed under finite intersections
- $\mathcal{T}$ is closed under countable unions

$$\mathcal{T} \subseteq \mathcal{P}(X) \text{ is a topology on } X \iff X, \emptyset \in \mathcal{T} \wedge [\bigcap_{i=0}^{n} U_i \in \mathcal{T}] \wedge [\bigcup_{i=0}^{\infty} U_i \in \mathcal{T}]$$

Definition 2.2. A *topological space* is an ordered pair $(X, \mathcal{T})$ of a set X and a topology $\mathcal{T}$ on X denoted as . Elements of X are referred to as *points*.

- X is a set
- $\mathcal{T}$ is a topology over X

The sets in $\mathcal{T}$ are called the *open sets*, and they are said to be *open in* X

$$(X, \mathcal{T}) \text{ is a topological space } \iff \mathcal{T} \text{ is a topology on } X$$

$$U \text{ is open in } X \iff U \in \mathcal{T}$$

We'll also port our definition of closed sets.

Definition 2.3.

$$(X, \mathcal{T})$$

$$F \text{ is closed in } X \iff X \setminus F \text{ is open in } X$$

We won't define boundaries for topological spaces just yet since we'll require the notion of 'neighborhoods and limit points', but we know from our study in metric spaces that we'll eventually prove that closed sets contain their whole boundary, and open sets contain none of it.

It's important to note that closed sets and open sets are not necessarily opposites in the sense of the English language; there exist sets which are both open and closed (sets with an empty boundary) or neither.

Definition 2.4.

$$U \text{ is clopen in } (X, \mathcal{T}) \iff U \text{ is closed and open in } (\mathcal{T}, X)$$

For any given set X, there are two 'obvious' topologies that could be made.

$$\mathcal{T} \text{ is the discrete topology on } X \iff \mathcal{T} = \mathcal{P}(X)$$

$$\mathcal{T} \text{ is the indiscrete topology on } X \iff \mathcal{T} = \{X, \emptyset\}$$

We introduce a topology that can be put upon infinite sets.

$$\mathcal{T} \text{ is the cofinite topology on } X \iff \mathcal{T} = \{U \subseteq X : U = \emptyset \vee |X \setminus U| < \aleph_0\}$$

$$X \text{ is finite } \wedge \mathcal{T} \text{ is the cofinite topology on } X \implies \mathcal{T} \text{ is the discrete topology on } X$$

2.2 Examples of topological spaces

To build some intuition for topological spaces, we offer some basic examples of what topological spaces look like (and what they don't look like).

Basic examamples

$$(\{1, 2, 3\}, \{\emptyset, \{1, 2, 3\}, \{2, 3\}, \{1, 2\}, \{2\}\})$$

$$(\{1,2,3,4\}, \{\emptyset, \{1,2,3,4\}, \{2,3\}, \{1,2,3\}, \{1,4\}, \{1\}\})$$

Examples that aren't topological spaces

$$(\{1,2,3,4\}, \{\emptyset, \{1,2,3,4\}, \{2,3,4\}, \{1,2,4\}\})$$

$$(\{1,2,3\}, \{\emptyset, \{2,3\}, \{1,2\}, \{2\}\})$$

Boring examamples

$$(X, \mathcal{P}(X))$$

$$(X, \{\emptyset, X\})$$

Topological spaces are not always so bland; some rather interesting topological spaces are out in the wild.

Weird example; cofinite topology on $\mathbb{N}$

$$(\mathbb{N}, \{U : |\mathbb{N} \setminus U| < \aleph_0\})$$

Though that last example was kind of cool, it's perhaps not entirely clear why we're doing topology in the first place. We now discuss a topological space that is quite familiar to us.

Useful example; Euclidean topology

$$(\mathbb{R}, \{U : U = \bigcup_{n \in \mathbb{N}} (a_n, b_n)\})$$

Chapter 3

Basis of a topological space

Those familiar with linear algebra are familiar with the idea of a *basis*; a set of elements that under some operation can generate an entire space. Topological spaces follow the same principle; often we can find some basis that can generate the topological space that helps our analysis of the space. Better yet, perhaps we want to define a topological space by means of a basis!

Definition 3.1. Let $(X, \mathcal{T})$ be a topological space. A *basis* of $\mathcal{T}$ is a set $\mathcal{B} \subseteq \mathcal{T}$ such that any set open in X is a union of sets in $\mathcal{B}$. Elements of $\mathcal{B}$ are called *basic sets*.

Every topological space can be represented by a basis since the topology itself forms a trivial basis for itself.

Proposition 3.1. Let $(X, \mathcal{T})$ be a topological space. $\mathcal{T}$ is a basis for $\mathcal{T}$

3.1 Generating a topology

- when does a basis generate a topology? Existence of topology for a basis

Like with open balls, a basis forms the 'ingredients' and the three conditions generate these ingredients into a topology. But unfortunately not every set of sets can be a basis; when can a set of sets acually generate some topology?

By ensuring our set of sets agrees with the 3 conditions of the algebra of sets, we can be sure that our set is basis for some topology on the space.

Definition 3.2. A set $\mathcal{B} \subset \mathcal{P}(X)$ generate some topology on X iff both of the following hold

- $X = \bigcup_{i \in \mathbb{N}} \mathcal{B}_i$
- For any sets $\mathcal{B}_i \mathcal{B}_j \in \mathcal{B}$, we have $\mathcal{B}_i \cap \mathcal{B}_j \in \mathcal{B}$

- Partitions on X are basis' for topologies on X -

3.2 Generating a given topology

- generating a basis for given topology

We can now check whether a set of sets can actually form a topology, but what if we want to check if our basis forms a *particular* topology in question?

We can refine our definition of a basis to be more 'constructive' in this sense.

Proposition 3.2. Let $(X, \mathcal{T})$ be a topological space. A set of sets $\mathcal{B}$ is a basis for $\mathcal{T}$ iff any of the following hold.

- If U is an open set, it is a union of sets in $\mathcal{B}$
- If U is an open set, for any $u \in U$ there is a set in $\mathcal{B}$ containing u that is completely contained in U

- The singletons are a basis for the discrete topology

3.3 Generating identical topologies

- identical topologies by comparing bases

A basis is a nice way to define a topology, however it is possible that different basis' can actually generate the same topology!

Proposition 3.3. Two basis $\mathcal{B}_1, \mathcal{B}_2$ generate the same topology iff all the following hold.

- For any $B_1 \in \mathcal{B}_1$ each $b_1 \in B_1$ has a set $B_2 \in \mathcal{B}_2$ such that $b_{\in} B_2$
- For any $B_2 \in \mathcal{B}_2$ each $b_2 \in B_2$ has a set $B_1 \in \mathcal{B}_1$ such that $b_2 \in B_1$

When we have two basis' for a topology, we can compare their *refinement*.

Definition 3.3. Let $(X, \mathcal{T})$ be a topological space. If $\mathcal{B}, \mathcal{C}$ are two basis' for $\mathcal{T}$, we say that $\mathcal{B}$ is a refinement of $\mathcal{C}$ iff $\mathcal{B} \subseteq \mathcal{C}$.

Chapter 4

Euclidean topology

One of the most interesting fundamental topologies is the Euclidean topological space, which characterizes open sets in a Euclidean space.

$$\mathcal{T}_{\mathbb{R}} = \{U : U = \bigcup_{i \in \mathbb{N}} (a_i, b_i)\}$$

Since we define open sets as unions of open intervals, we can see that the open intervals form a basis for the Euclidean topology.

$$\mathcal{B}_{\mathbb{R}} = \{(a, b) : a, b \in \mathbb{R} \wedge a < b\}$$

Noting that $(a, b) = (\frac{a+b}{2} - \frac{b-a}{2}, \frac{a+b}{2} + \frac{b-a}{2}) = B_{\mathbb{R}}(\frac{a+b}{2}, \frac{b-a}{2})$ we see that the open intervals are actually open balls of the single dimension Euclidean metric space. This alludes to the fact that this is actually a metric space; indeed this is true for any dimensional Euclidean topology, however we'll savour the details for later.

Due to the general familiarity that readers tend to have with the Euclidean topology, much of the theory developed for general topology will be applied to the Euclidean topology as a mode of demonstration.

Chapter 5

Neighborhoods

Now we will start developing more specific theory of topology that culminates to generalizing limits to topological spaces. We do this by means of beighborhoods.

Definition 5.1. A *neighborhood of p* is a set V containing some open set U containing p. An *open neighborhood of p* is a neighborhood of p that is an open set.

$$V \text{ is a neighborhood of } p \iff \exists U \subseteq V[U \text{ is an open set } \wedge p \in U]$$

$$V \text{ is an open neighborhood of } p \iff V \text{ is an open set } \wedge p \in V$$

Some authors define neighborhoods as open neighborhoods, however this book does not make that assumption. It might be interesting to note that neighborhoods of p can play a similar role to open balls of p in the sense that they both contain open sets containing p. Though there are some notable differences like open balls relying on a metric and neighborhoods being much more general, they can be used with similar functions in some circumstances.

Proposition 5.1. If p has a neighborhood V, then p has an open neighborhood $U \subseteq V$.

$$V \text{ is a neighborhood of } p \implies \exists U \subseteq V[U \text{ is an open neighborhood of } p]$$

- neighborhoods in Euclidean space In the Euclidean topology, any two points have disjoint neighborhoods. This actually goes for any topological space that is a metric space. The exact class of spaces where this occurs is called a Hausdorff space.

- power of neighborhioods in simplicying definitions on limit points - open cover - subcover

Chapter 6

Limit points

We may now define limits in topological spaces.

Definition 6.1. In a topological space $(X, \mathcal{T})$, a *limit* of a sequence is a point p where all its neighborhoods contain all remaining terms of a sequence. A *convergent sequence* is a sequence with a limit.

$$(X, \mathcal{T})$$

$$\lim_{n\to\infty} x_n = p \iff \forall V \subseteq X[V \text{ is a neighborhood of } p \implies \exists N \in \mathbb{N}[n > N \implies a_n \in V]]$$

If one restricts the terms of a sequence to some set, it may still be possible that the limit of the sequence lies *outside* this set. Consider the Euclidean topology on $\mathbb{R}$, the open set $(0, 1)$ and the sequence $a_n = \frac{1}{n+1}, n \geq 1$. Though we have $a_n \in (0, 1)$, we also have $\lim_{n\to\infty} a_n = 0$, which is out of the set!

The phenomenon where limits can exceed the set their terms are chosen from is interesting indeed; any point that is the limit of some sequence of terms within a set is called a *limit point* of that set.

Definition 6.2. A *limit point of a set S* is a point p such that all neighborhoods of p include another point in S that isn't p.

$$p \text{ is a limit point of } S \iff \forall V[V \text{ is a neighborhood of } p \implies V \cap S \neq \emptyset]$$

6.1 Closure and interior

Definition 6.3. The *closure of a set S* is the union of S and the set of all limit points of S. Given that the topological space is $(X, \mathcal{T})$, the closure of

S is denoted as $\text{cl}_X(S)$, and its elements are called points of closure of S.

$$\text{cl}_X(S) := S \cup \{p : p \text{ is a limit point of } S\}$$

Closures are very useful constructs in topology with many important properties.

Proposition 6.1. Let $(X, \mathcal{T})$ be a topological space. For any set S, $\text{cl}_X(S)$ is closed.

$$(X, \mathcal{T})$$

$$\forall S \in X[\text{cl}_X(S) \text{ is closed in } (X, \mathcal{T})]$$

Proposition 6.2. Let $(X, \mathcal{T})$ be a topological space. $\text{cl}_X(S)$ is the smallest possible closed set containing S.

$$\forall T[T \text{ is closed in } (X, \mathcal{T}) \wedge S \subseteq T \implies \text{cl}_X(S) \subseteq T]$$

Here's one intuitive way to think about that proposition; think of a swarm of all the open sets disjoint to S, all making a union around S. This swarm of open sets is trying to engulf everything around S, so any points that 'survive' are in the smallest possible closed superset of S.

Proposition 6.3. Let $(X, \mathcal{T})$ be a topological space. For any set S, $\text{cl}_X(S)$ is closed.

$$(X, \mathcal{T})$$

$$\forall S \in X[\text{cl}_X(S) \text{ is closed in } (X, \mathcal{T})]$$

Limit points follow a 'transitive property', that is, if the limit points of S have limit points themselves, they are also limit points of S; we've had them the entire time. This leads to the following proposition.

Proposition 6.4. Let $(X, \mathcal{T})$ be a topological space. For any S, $\text{cl}_X(S) \setminus S$ is closed.

Definition 6.4. The *interior of a set* S is the union of all subsets $U \subseteq S$ that are open. Given that the topological space is $(X, \mathcal{T})$, the interior of S is denoted as $\text{int}_X(S)$.

$$U \text{ is open in } X \iff \text{int}_X(U) = U$$

6.2 Boundary

-in metric space, all neighborhoods fully contained in an open ball

We can leverage the fact that all neighborhoods in metric spaces can be fully contained by an open ball to almost directly port our definition of boundaries to topological spaces!

Definition 6.5. Let $(X, \mathcal{T})$ be a topological space. The *boundary of* S is the set of all points $p \in X$ such that all their neighborhoods have intersections with S and $X \setminus S$. We denote the boundary of S as ∂S, and elements of ∂S are called boundary points of S.

Using the language of closures and interiors, we can prove the following equivalent definitions.

Theorem 6.1 (Equivalent definitions of a boundary)**.** Let $(X, \mathcal{T})$ be a topological space. ∂S is the boundary of S iff any of the following hold.

- For each point in ∂S, all the points neighborhoods have intersections with S and $X \setminus S$.
- $\partial S = \mathrm{cl}_X(S) \setminus \mathrm{int}_X(S)$
- $\partial S = \mathrm{cl}_X(S) \cap \mathrm{cl}_X(X \setminus S)$

Now that we've defined boundaries for topological spaces, we can come full circle to rediscover our definition of closed sets that we defined on metric spaces!

Theorem 6.2 (Equivalent definitions of a closed set)**.** Let $(X, \mathcal{T})$ be a topological space. A set F is closed in X iff any of the following hold.

- $X \setminus F$ is open.
- $\mathrm{cl}_X(F) = F$ (F contains all its limit points)
- $\partial F \subseteq F$

$$F \text{ is closed in } X \iff [X \setminus F \text{ is open in } X] \vee [\mathrm{cl}_X(F) = F] \vee [\partial F \subseteq F]$$

Corollary 6.1. Closure is idempotent.

$$\mathrm{cl}_X(\mathrm{cl}_X(S)) = \mathrm{cl}_X(S)$$

Corollary 6.2. Let $(X, \mathcal{T})$ be a topological space. For any S, the set of its limit points is closed.

$$\forall S[\{p : p \text{ is a limit point of } S\} \text{ is closed in } (X, \mathcal{T})]$$

Theorem 6.3 (Equivalent definitions of an open set)**.** Let $(X, \mathcal{T})$ be a topological space. A set U is open in X iff any of the following hold.

- $X \setminus U$ is closed.
- $\text{int}_X(U) = U$
- $\partial U \cap U = \emptyset$

$$F \text{ is closed in } X \iff [X \setminus F \text{ is open in } X] \vee [\text{cl}_X(F) = F] \vee [\partial F \subseteq F]$$

Definition 6.6. Let $(X, \mathcal{T})$ be a topological space, a set S is *dense in* X iff its closure equals X.

Here's an example demonstrating this idea in the Euclidean topological space.

Proposition 6.5. Let $(\mathbb{R}, \mathcal{T}_{\mathbb{R}})$ be the Euclidean topological space, then $\mathbb{Q}$ is dense in $\mathbb{R}$.

Part II

Continuous functions

Chapter 7

Continuous function (Topology)

Like limits, a topological space is also sufficient grounds to develop a general definition of a continuous function!

Definition 7.1. A function f is *continuous at p* iff for any neighborhood $V \subseteq \mathrm{im}(f)$ of $f(p)$, $f^{-1}(V)$ is a neighborhood of p.

Notably, we can recylce our fact that neighborhoods are always within open balls to reverse engineer the definition of a continuous function from real analysis to obrain the topological definition.

Chapter 8

Intermediate Value Theorem (Topology)

- IVT - special case of Brouwer fixed-point theorem for [0,1] continuous operations.

Chapter 9

Homeomorphisms

Like how group homomorphishms preserve a groups structure, continuous functions preserve neighborhood structure of a topology. This leads to the question; like how group isomorphisms show that two groups are 'algebraically equivalent', is there some class of function to show that two topological spaces are 'topologically equivalent' (in the sense that they have the 'isomorphic neighborhoods')?

Definition 9.1. A *homeomorphism* between two topological spaces T and U is a bijective function $f : T \to U$ such that both f and f^{-1} are continuous.

Part III

Constructions of topological spaces

Chapter 10

Topological subspaces

A topological subspace of $(X, \mathcal{T})$ is a topological space $(Y, \mathcal{T}_Y)$.

- $Y \subseteq X$ is a subset of X
- $\mathcal{T}_Y = \{Y \cap U : U \in \mathcal{T}\}$ is the induced topology for the topological subspace

Chapter 11

Product topological spaces

Chapter 12

Quotient topological spaces

Part IV

Metric spaces

We now return to metric spaces using our new fancy knowledge of topological spaces.

Theorem 12.1. A metric space (X, d) is a topological space $(X, \mathcal{T})$ generated by its open balls as a basis.

Definition 12.1. A *metrizable topological space* is a topological space $(X, \mathcal{T})$ such that there exists a metric $d : X \times X \to [0, \infty)$ such thatthe topology induced by d is $\mathcal{T}$.

- topologies induced by metric space are hausdorff

Chapter 13

Limits in metric spaces

Though convergence of sequences may be established without a metric space, metric spaces provide a more intuitive definition for convergence.

Definition 13.1. In a metric space (X, d), a *limit* of a sequence is a point p that is arbitrarily close all remaining terms of a sequence. A *convergent sequence* is a sequence with a limit.

$$(X, d)$$

$$\lim_{n\to\infty} x_n = p \iff \forall\varepsilon \in (0,\infty)[\exists N \in \mathbb{N}[n > N \implies d(x_n, p) < \varepsilon]]$$

$$\lim_{n\to\infty} x_n = p \iff \forall\varepsilon \in (0,\infty)[\exists N \in \mathbb{N}[n > N \implies x_n \in B_X(p, \varepsilon)]]$$

Definition 13.2.

- same sequence converging to x and y means x=y
-cauchy sequence -subsequence -bolzano weierstrass therem topology

Chapter 14

Complete metric space

Chapter 15

Banach fixed-point theorem

- Contraction mapping - Banach fixed point theorem - Allude to applications in numerical analysis

Chapter 16

Baire spaces

Part V

Compact sets

- compact iff every open cover has finite subcover

Part VI

Connected sets

- Allude to the importance in complex analysis

Definition 16.1. A *connected space* $(X, \mathcal{T})$ is a topological space where the only clopen sets are X and $\emptyset$.

Euclidean topological space is connected.

- simply connected space

¡h3¿Simply connected space¡/h3¿ ¡p¿Topological space with no holes, characterized by the ability to continuously transform any loop around a point¡/p¿ ¡p¿X is simply connected $\iff$ ¡/p¿ ¡p¿X is path-connected¡/p¿ ¡p¿$\forall \mathcal{C} : [t_0, t_1] \to X(\mathcal{C}$ is a closed simple curve $\implies \exists f \ell(f(\mathcal{C}) = 0)$¡/p¿

Part VII

Separation properties

- T_0 space (Kolmogorov space)

Definition 16.2. A T_0 *space (Kolmogorov space)* is a topological space such that for every distinct pair of points, at least 1 point in the pair has a neighborhood not containing the other point.

Definition 16.3. A T_1 *space (Topological Fréchet space)* is a topological space such that for every distinct pair of points, both points in the pair has a neighborhood not containing the other point.

Definition 16.4. A T_2 *space (Hausdorff space)* is a topological space such that for every distinct pair of points, there exists a pair of neighborhoods of both points which are disjoint.

- finite frechet spaces are discrete topological spaces

Part VIII

Topological groups

- Jordan curve theorem (JCT) - Urysohn's lemma