- Electric fields

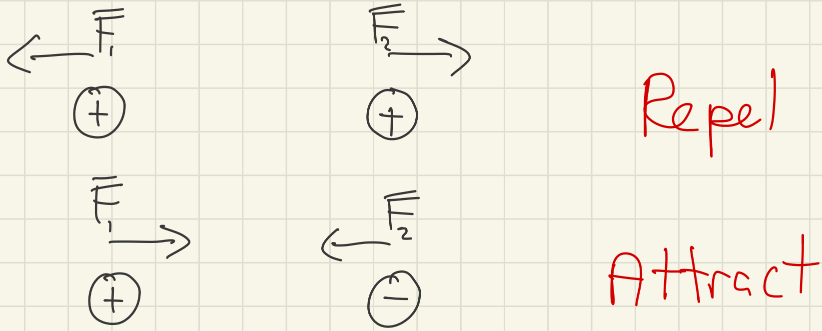
  * Charges
  * Forces
  * Field lines / Flux
  * Energy

- Capacitance

  * Parallel plates
  * Circuit combinations
  * Energy and dielectrics

What are charges?

- Forces between charges



- Charge conservation

Quiz

Three objects: A, B, C

Data:

A & B — attract

B & C — repel

Question: which statements are true

(a) A & C have same sign (of charge)

(b) A & C have opposite sign

(c) A, B, C have same sign

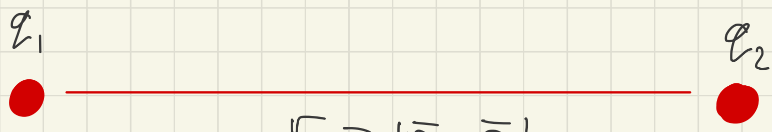
(d) One is neutral

(e) Need more experiments

Coloumb's Law

Force

$$F = k \frac{|q_1 q_2|}{r^2} \quad (= |\vec{F}|)$$



$$r = |\vec{r_1} - \vec{r_2}|$$

$$k = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \, N \cdot m^2 / C^2$$

$$\varepsilon_0 = 8.9 \times 10^{-12} \, C^2 / (N \cdot m^2)$$

$$e = 1.6 \times 10^{-19} \, C$$

hence

$$1\,C = \frac{e}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \, e$$

Example

$q_1 = 10 \text{ electrons} = -10\,e$

$q_2 = 2 \text{ protons} = +2\,e$

$\bar{r}_1 = (1, 1, 0)\ m$

$\bar{r}_2 = (1, 0, 0)\ m$

Find $|\bar{F}| = F$

$$F = k \frac{|q_1 q_2|}{|\bar{r}_1 - \bar{r}_2|^2} = k \frac{|-20e^2|}{1\,m^2}$$

$$= 9 \times 10^9 \frac{N \cdot m^2}{C^2} \times \frac{20e^2}{m^2}$$

$$= 9 \times 10^9 \frac{N}{C^2}\ 20 \times (1.6 \times 10^{-19}\,C)^2$$

$$= 4.6 \times 10^{-27}\ N$$

Example — Hydrogen Atom

q_1 = electron $= -e$

q_2 = proton $= +e$

$r = 5.3 \times 10^{-11}$

$$F_e = k \frac{|q_1 q_2|}{r^2} = \ldots = 8.2 \times 10^{-8} N$$

$$F_g = G \frac{m_1 m_2}{r^2} = \ldots = 3.6 \times 10^{-47} N$$

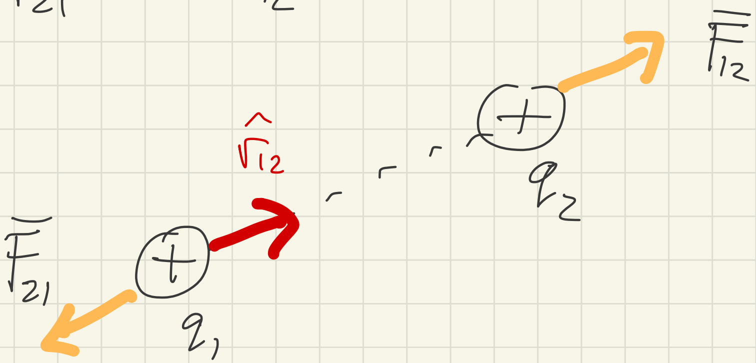
$$F_e / F_g \simeq 2 \times 10^{39}$$

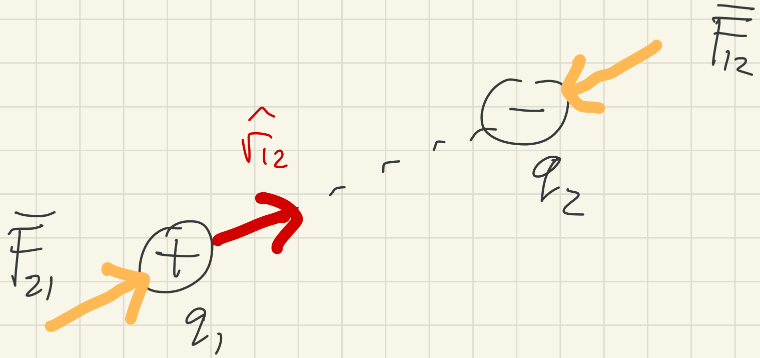
* Coulomb & gravity appear to follow the same 'inverse-square force' law
* How do they differ?

Coulomb Force Vector

$$\vec{F}_{1 \text{ on } 2} = \vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$





Quiz

Data

$A = +2\mu C$

$B = +6\mu C$

Question – which is true

(a) $\bar{F}_{AB} = -3\bar{F}_{BA}$

(b) $\bar{F}_{AB} = -\bar{F}_{BA}$

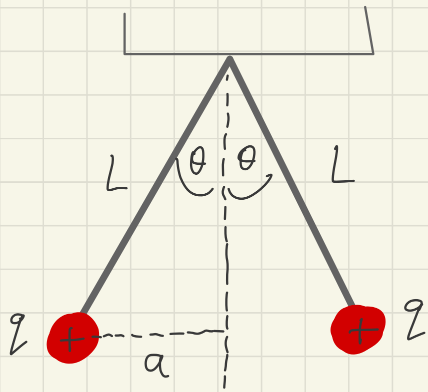
(c) $3\bar{F}_{AB} = -\bar{F}_{BA}$

(d) $\bar{F}_{AB} = 3\bar{F}_{BA}$

(e) $\bar{F}_{AB} = \bar{F}_{BA}$

(f) $3\bar{F}_{AB} = \bar{F}_{BA}$

Example



- Two identical charges are in equillibrium as shown
- Both mass $m = 3.00 \times 10^{-2}$ kg

  $L = 0.150$ m

  $\theta = 5.00°$

- Find charge, q

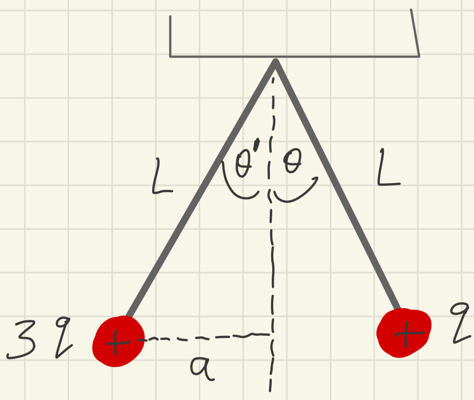
$$q = \pm 2L\sin\theta \sqrt{\frac{mg\tan\theta}{k}}$$

↑ which sign?

$$q = 4.42 \times 10^{-8}\ C$$

Questions

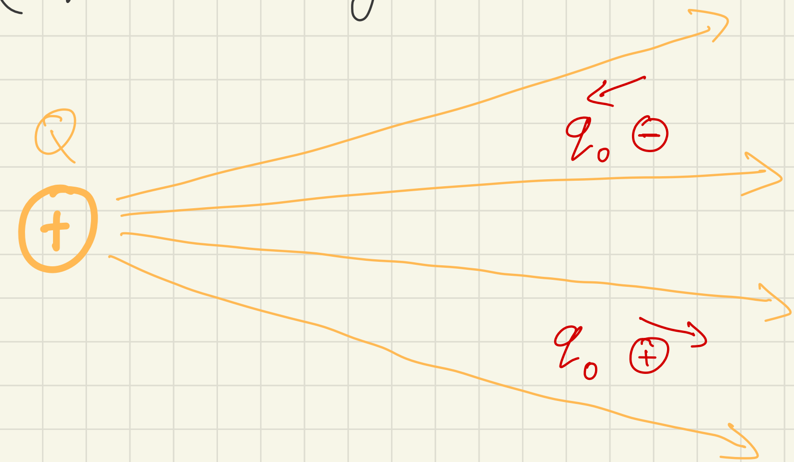
- What if one charge was $3q$, does $\theta' = \theta$?
- What if one mass was $3m$?



Electric Field Vector

$$\bar{F} = q_0 \bar{E} = k \frac{q_0 Q}{r^2} \hat{r}$$

$$\left(\bar{F} = m \bar{g} \right)$$

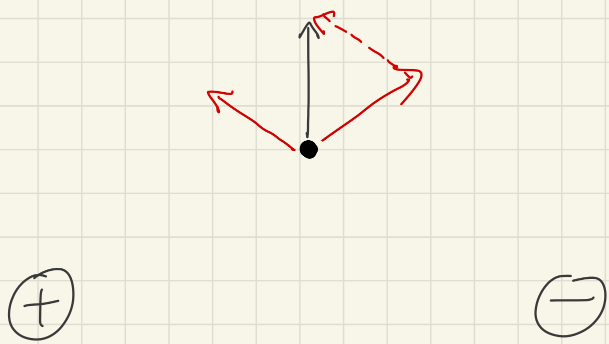


$$\bar{E} = k \frac{Q}{r^2} \hat{r}$$ (which 'r'?)

$\bar{E}$ from multiple charges

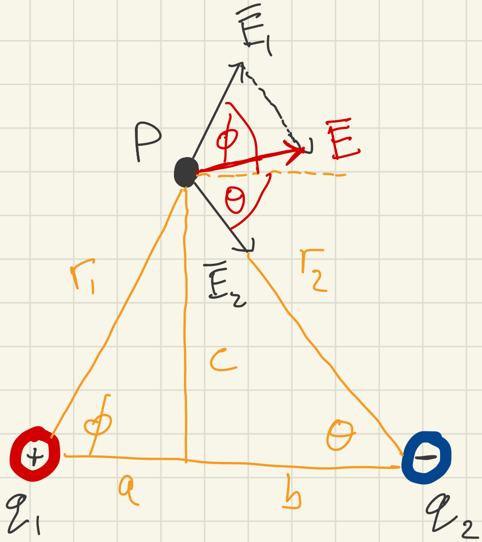
$$\bar{E}(\bar{r}) = \kappa \sum_i \frac{q_i}{|\bar{r}-\bar{r}_i|^2} \,\delta\hat{r}_i \qquad \left(\delta\hat{r}_i = \frac{\bar{r}-\bar{r}_i}{|\bar{r}-\bar{r}_i|}\right)$$

= superposition principle



Example

Find $\bar{E}$ at point $\bar{P}$



(1) Find $\bar{E}_1$, $\bar{E}_2$

$$\bar{E}_1 = \frac{k\, q_1}{r_1^{\,2}}\, \hat{r}_1$$

$$\bar{E}_2 = k\frac{q_2}{r_2^{\,2}}\, \hat{r}_2$$

$$\hat{r}_1 = \frac{a\,\hat{i} + c\,\hat{j}}{\sqrt{a^2 + c^2}}$$

$$\hat{r}_2 = \frac{-b\,\hat{i} + c\,\hat{j}}{\sqrt{b^2 + c^2}}$$

$$\bar{E}_1 = \frac{k q_1}{r_1^2} \hat{r}_1$$

$$\bar{E}_2 = k \frac{q_2}{r_2^2} \hat{r}_2$$

$$\hat{r}_1 = \frac{a\hat{i} + c\hat{j}}{\sqrt{a^2+c^2}} = \frac{a\hat{i} + c\hat{j}}{r_1}$$

$$\hat{r}_2 = \frac{-b\hat{i} + c\hat{j}}{\sqrt{b^2+c^2}} = \frac{-b\hat{i} + c\hat{j}}{r_2}$$

(2)

$$q_1 = |q_1|$$
$$q_2 = -|q_2|$$

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$$= k\left(\frac{|q_1| a}{r_1^3} - \frac{(-|q_2|) b}{r_2^3}\right)\hat{i}$$

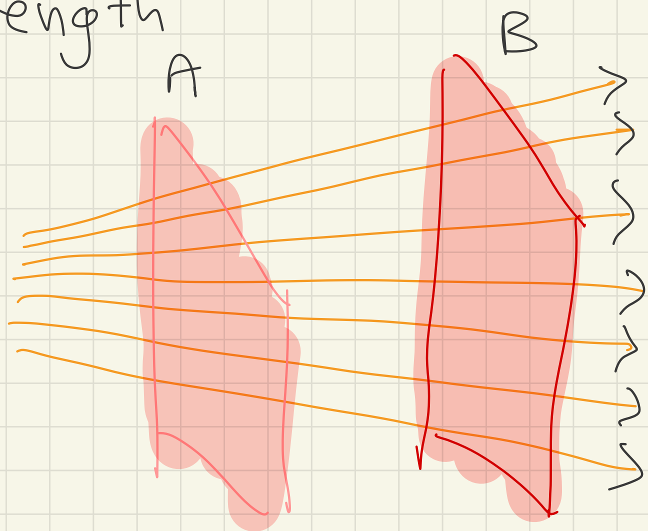
$$+ k\left(\frac{|q_1| c}{r_1^3} + \frac{(-|q_2|) c}{r_2^3}\right)\hat{j}$$

(3) Evaluate for $a = b$, $|q_1| = |q_2|$

$$\bar{E} = \frac{2k|q_1| a}{r_1^3}\hat{i}$$

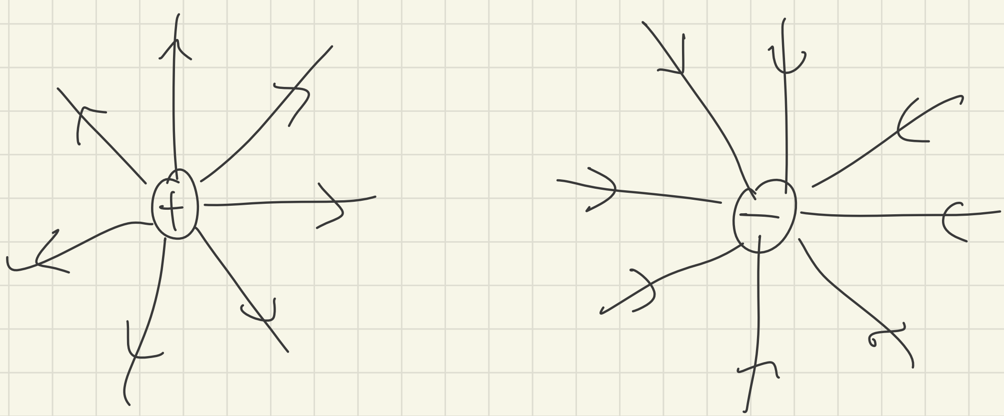
Electric Field Lines

- Strength

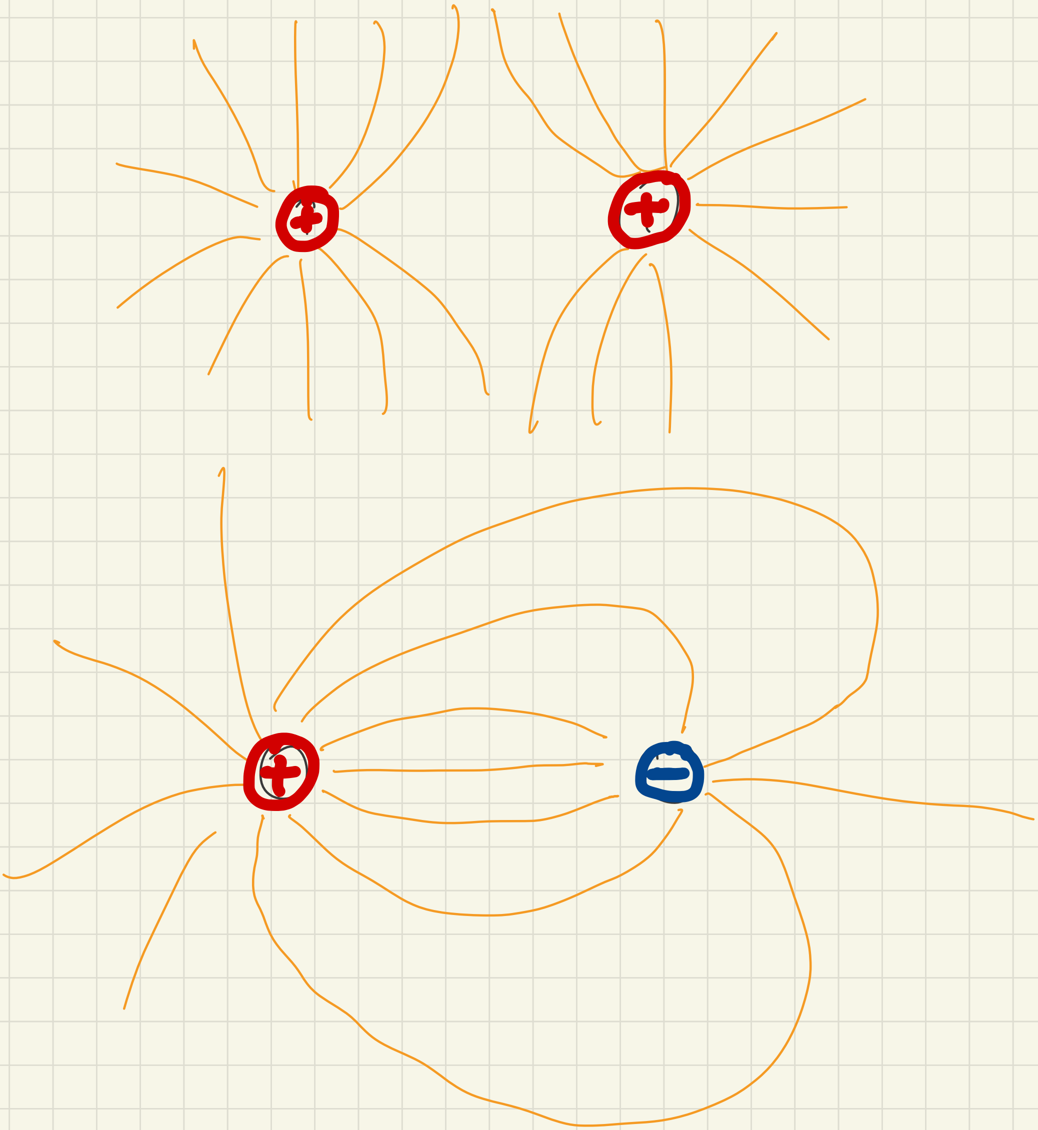


$|\vec{E}_A| > |\vec{E}_B|$

- Direction

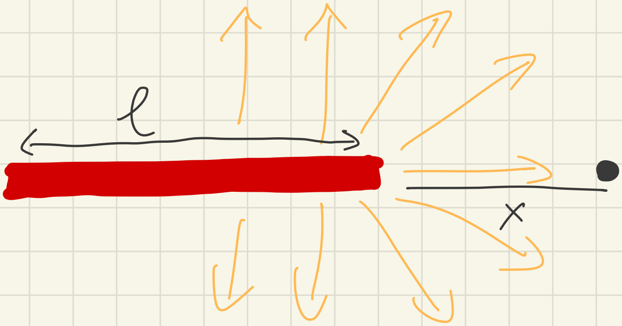


Adding Field Lines

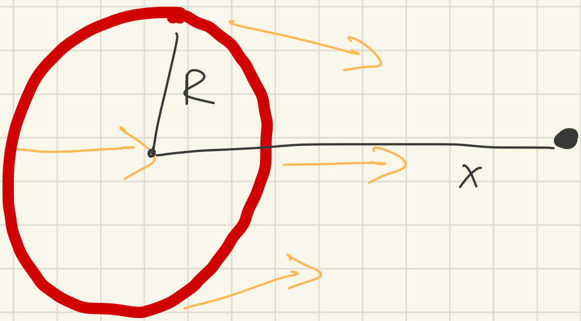


... Inverse square law?

E-field of various objects (Qualitative)

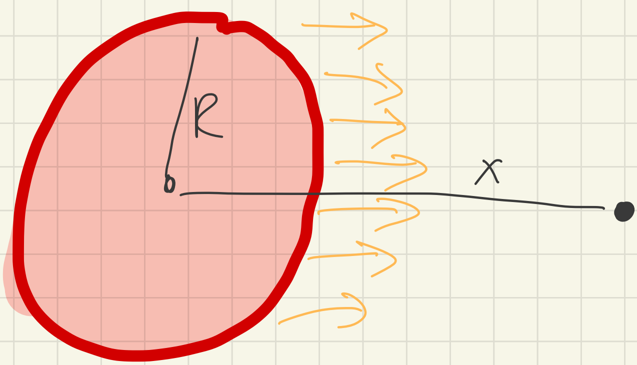


$$E = kQ \frac{1}{x(\ell + x)}$$



$$E = kQ \frac{x}{(R^2 + x^2)^{3/2}}$$

$$\sigma = \frac{Q}{\text{Area}}$$



$$E = 2\pi k \sigma \left[1 - \frac{x}{(R^2 + x^2)^{1/2}} \right]$$

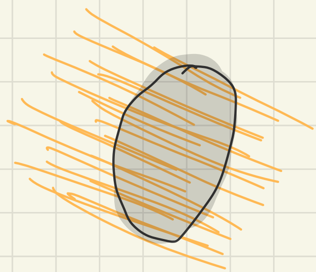
Question $R \to \infty$? $(\ell \to \infty ?)$

Gauss' Law [Brief]

Electric flux

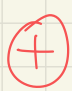
$$\Phi_E = EA_\perp = \vec{E}\cdot\vec{A}$$

$$= EA\cos\theta$$



Gauss Law

$$\Phi_E = \frac{q}{\varepsilon_0} \sim \# \text{ of field lines}$$



Mini Workshop

Coulomb's Law

(1) • Two protons in a nucleus are separated by a distance

$$r_{12} = 2 \times 10^{-15} \, m$$

• Find $|\vec{F}_{12}|$, the magnitude of the Coulomb force.

• Data: $q_1 = q_2 = 1.6 \times 10^{-19} \, C$

$$k = 8.99 \times 10^{9} \, N \cdot m / C^2$$

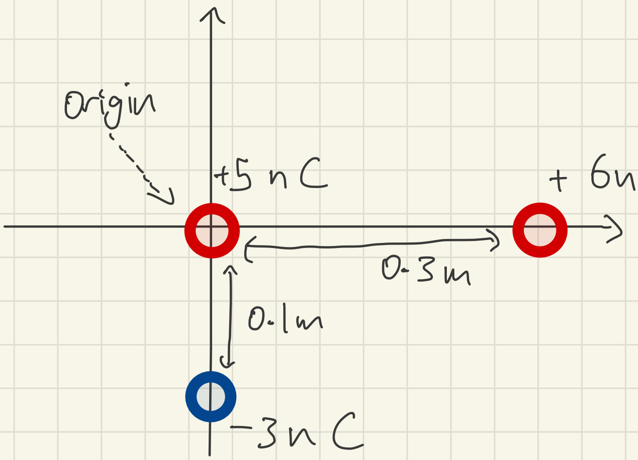
(2) Charge 'A' exerts a force of 2.62 μN to the right on charge 'B' when they are 13.7 mm apart. 'B' moves away from 'A' making the distance 17.7 mm. What vector force does it exert on 'A'?

$[\mu = 10^{-6}]$

(3) A molecule of DNA is 2.17 μm long. Each end becomes "ionized": negative charge on one end and positive the other, with charge $|q| = 1.6 \times 10^{-19}$ C.

The molecule is a spring-shape and compresses 1% of its length. Find the spring constant (Hint $F = kx$)

(4) Three point charges are arranged as shown



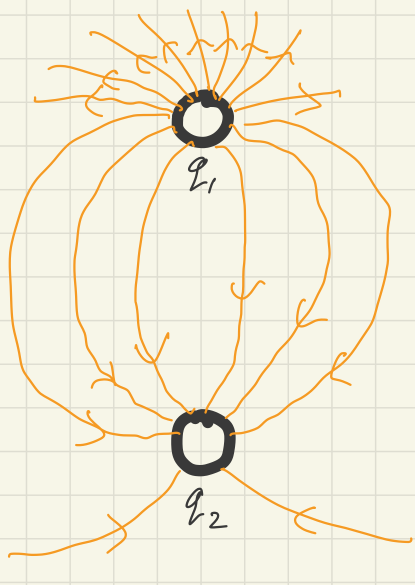
Find (a) magnitude, and (b) direction of the Coulomb force on the particle at the origin.

Electric Field Lines

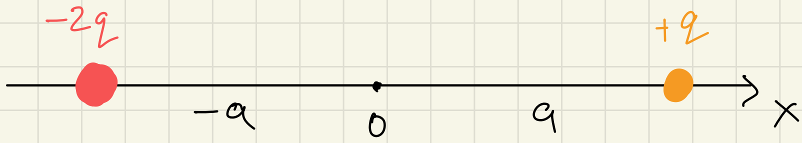
(5) The figure below shows the electric field lines for two charges q_1 and q_2.

(a) Is $|q_1/q_2| < 1$ or $|q_1/q_2| > 1$?

(b) What are the signs of q_1, q_2?



(6) Consider two charges as shown. Find the point where the electric field vanishes



Electric Energy
(voltage)

- Previously: **Force** of an electric field is $\bar{F} = q\bar{E}$

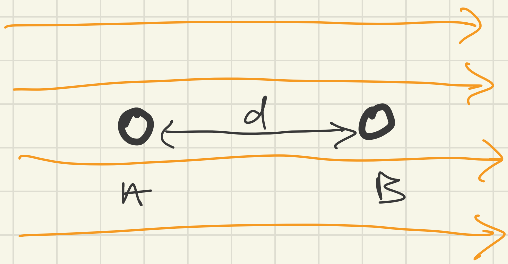
- Now: **Energy** of an electric field is

$$\Delta U = -\int_A^B \bar{F} \cdot d\bar{s} \quad (= -W)$$

$$= -q \int_A^B \bar{E} \cdot d\bar{s}$$

Example

$\bar{E} = \text{constant}$



$$\Delta U = -q \int_A^B \bar{E} \cdot d\bar{s}$$

Here $\bar{E} \parallel d\bar{s}$ $\therefore$ $\bar{E} \cdot d\bar{s} = E_0 \, dx$

$$\Delta U = -q \int_A^B E_0 \, dx$$

$$= -q E_0 (B - A) = -q E_0 d$$

Energy = charge x field x disp.

Force

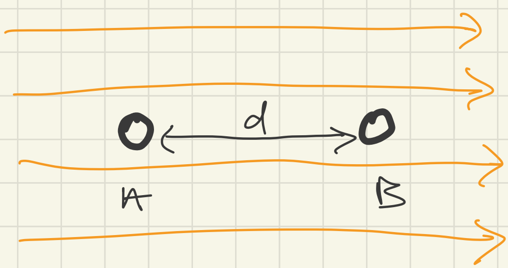
'Electric potential'
= voltage

- Electric potential

$$\Delta V \equiv \frac{\Delta U}{q}$$

Example

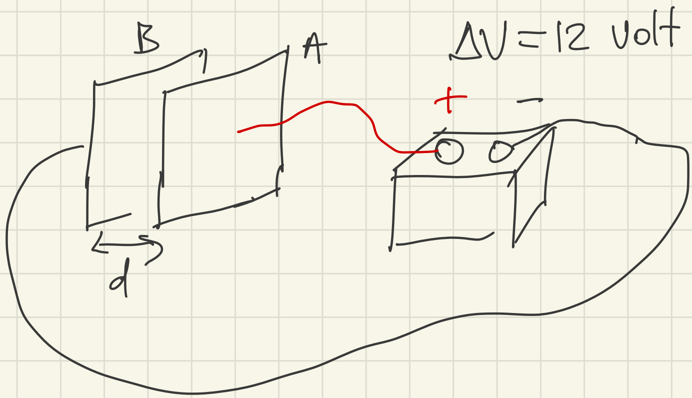
$\bar{E}$ = constant



- Proton released from rest at point A in a constant electric field

- Find an expression for its velocity at point B

Example



- Battery creates charged plates & therefore an electric field, $\bar{E}$
- what is the magnitude of $\bar{E}$?

Solution

$$\Delta V = \frac{\Delta U}{q} = \frac{-qEd}{q} = -Ed$$

$$E = -\frac{\Delta V}{d}$$

Magnitude $|E| = \left|\frac{\Delta V}{d}\right|$

Extra Comments

V = scalar

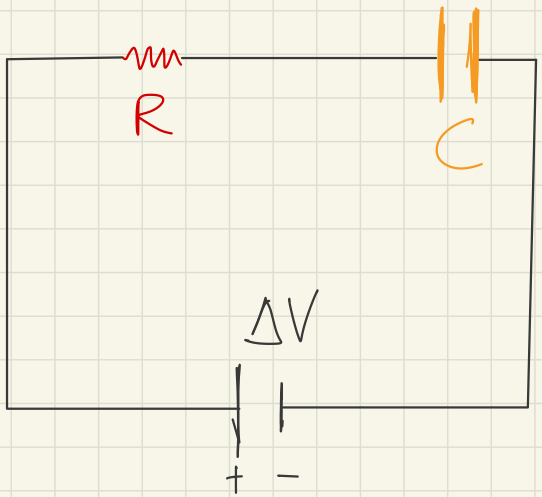
$\overline{E}$ = vector = (E_x, E_y, E_z)

$$E_x = -\frac{dV}{dx}$$

$$E_y = -\frac{dV}{dy}$$

$$E_z = \ldots$$

Electrical Circuits



Capacitance

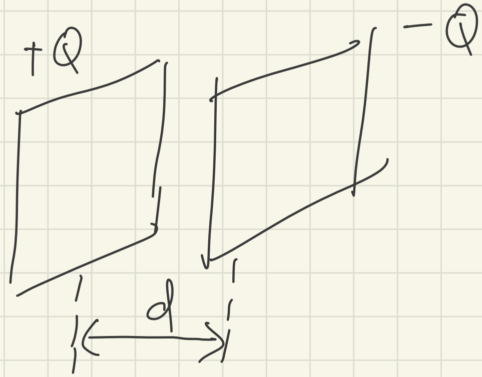
* Parallel plates

* Combinations (circuits)
  - parallel
  - series

* Energy stored

* Dielectrics vs Metal slab

Capacitance / Capacitor



- Parallel plate capacitor
  - stores energy
  - common circuit element

- Capacitance 'C'
  - Experiments found that

$$Q \propto \Delta V$$

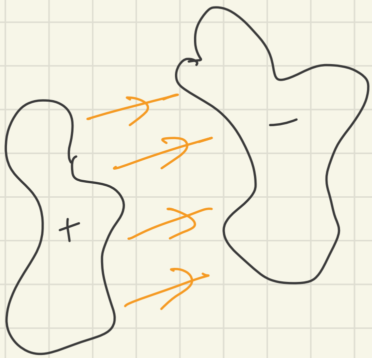
  - Given a ΔV, how much charge is on the plates?

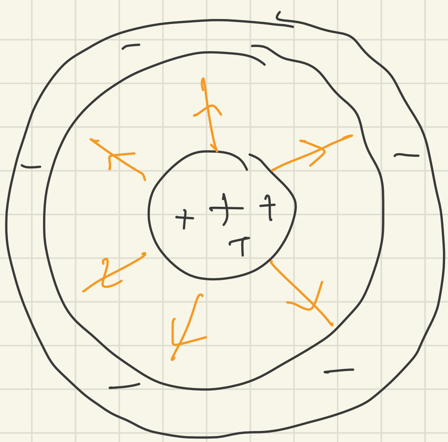
- Capacitance 'C'
  - Experiments found that

$$Q \propto \Delta V$$

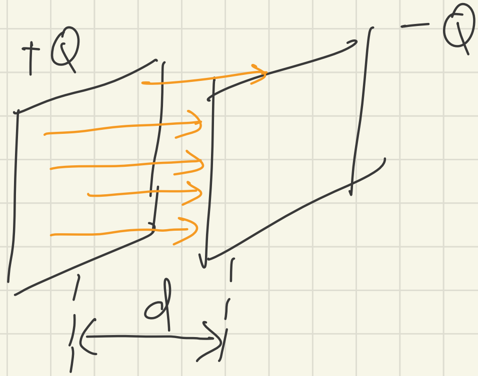
  - Given a ΔV, how much charge is on the plates?
  - $$C \equiv \frac{Q}{\Delta V}$$

- Other objects have capacitance





- Parallel Plate Capacitor



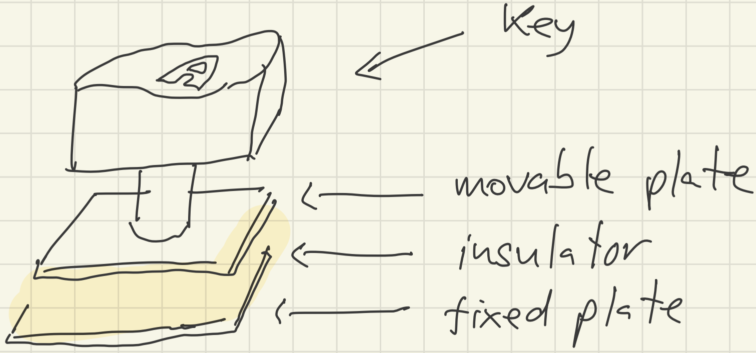
$$E = \frac{Q}{\epsilon_0 A} \quad (1)$$

$$\Delta V = Ed \quad (2)$$

$$C = \frac{Q}{\Delta V} \quad (3)$$

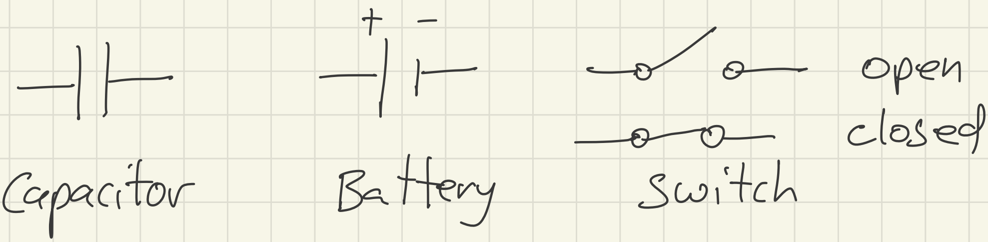
$$C = \frac{\epsilon_0 A}{d}$$

Application

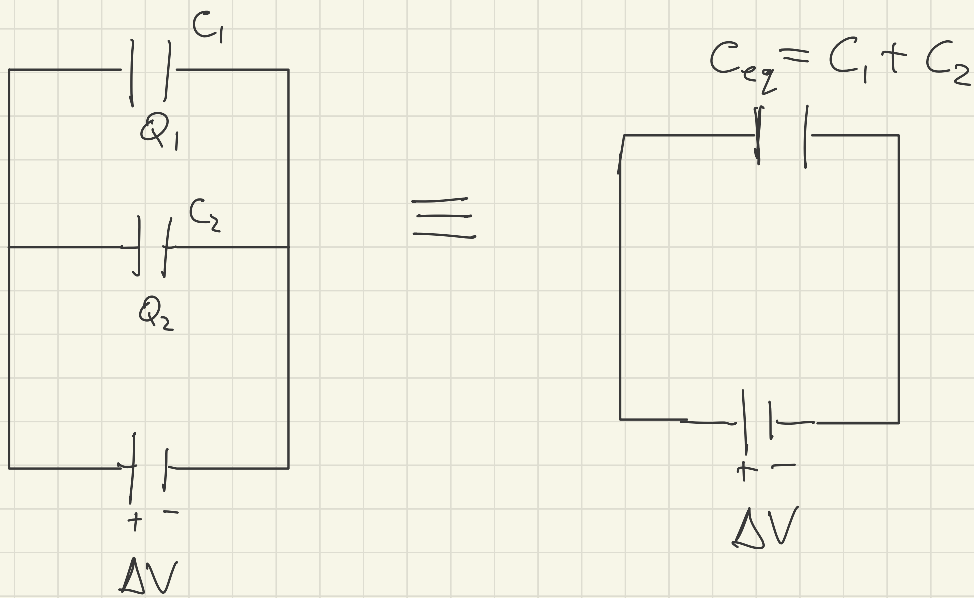


Combinations of Capacitors

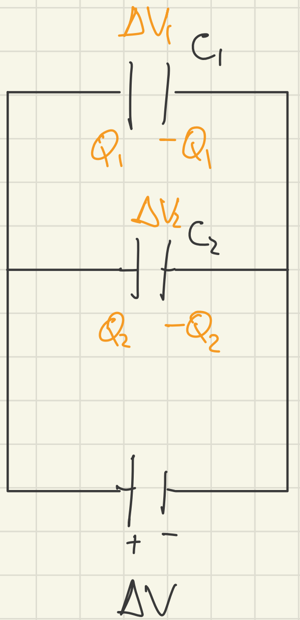
- Circuit diagrams/symbols



- Parallel Combination

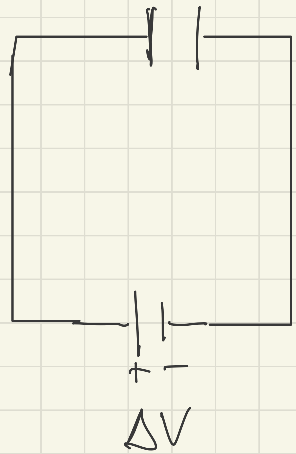


- Parallel Combination



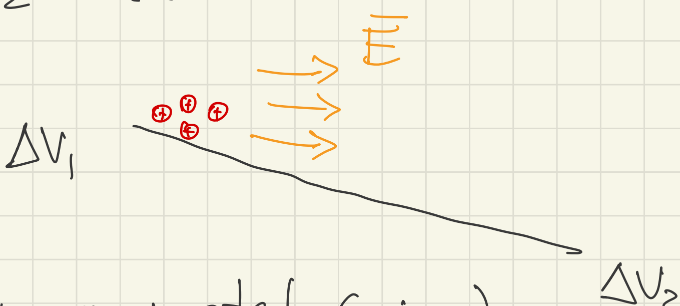
$\equiv$

$C_{eq} = C_1 + C_2$



(i) $\Delta V_1 = \Delta V_2 = \Delta V$

- Why?



- Charges in a metal (wire) redistribute (move) to cancel electric fields

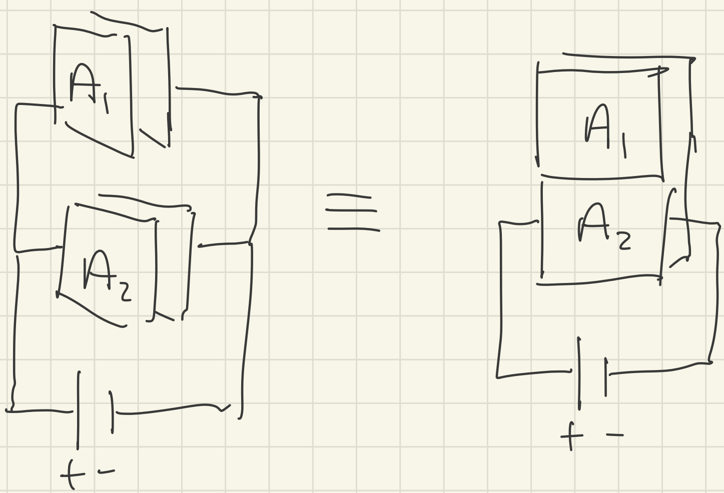
(iii) $$Q_{tot} = Q_1 + Q_2$$

$$= C_1 \Delta V_1 + C_2 \Delta V_2$$

$$= (C_1 + C_2) \Delta V$$

$$= C_{eq} \Delta V$$

- More intuitively

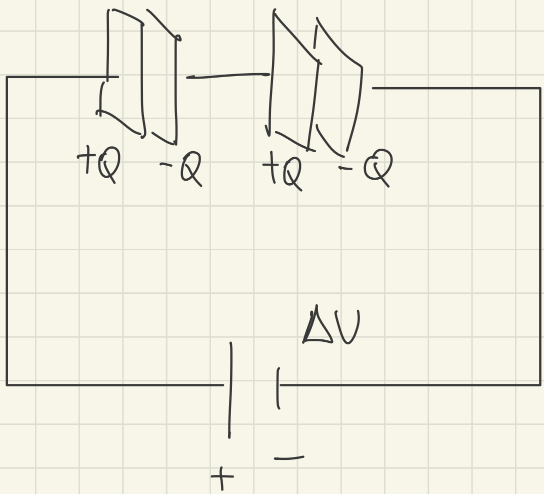


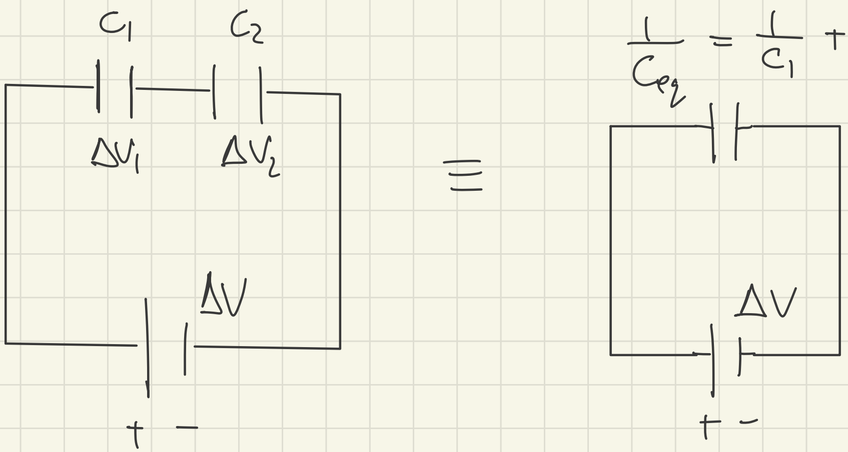
$$C_{eq} = \frac{\varepsilon_0}{d} (A_1 + A_2)$$

Parallel Combination

$$C_{eq} = C_1 + C_2 + \cdots$$

- Series Combination

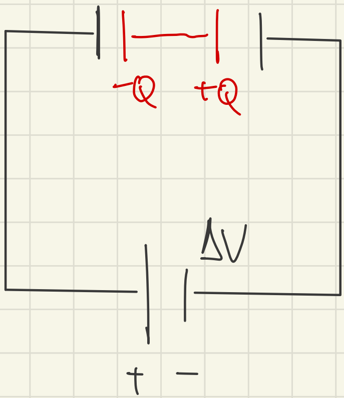




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

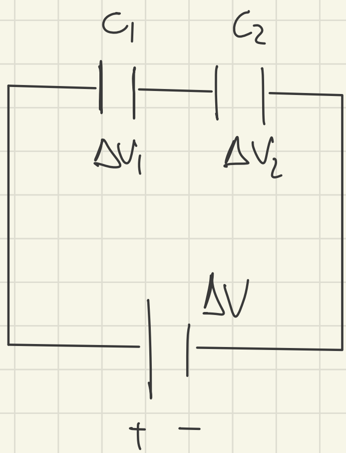
(i) charges

$\sum \text{charge} = 0$

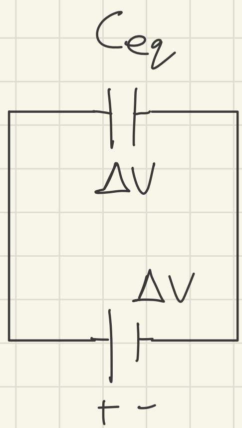


(ii) Voltages

$\Delta V = \Delta V_1 + \Delta V_2$



(iii) Equivalent capacitance



$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$= Q/C_{eq}$$

Note: $C_{eq} < C_1$

$C_{eq} < C_2$

Summary

Parallel Combination

$$C_{eq} = C_1 + C_2 + \cdots$$

$$C_{eq} > C_1, \; C_2$$

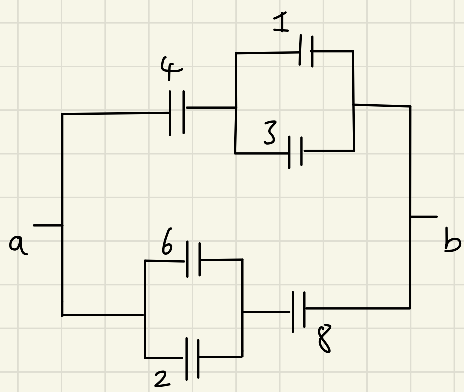
Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

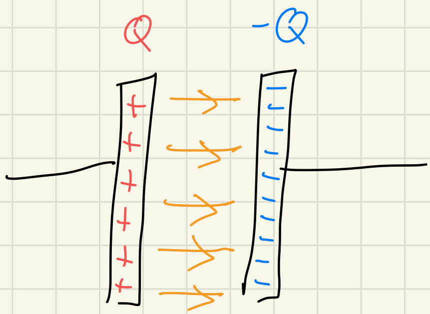
$$C_{eq} < C_1, \; C_2$$

Quiz

Find the equivalent capacitance between 'a' and 'b' for the combination of capacitors shown



Energy Stored in a Capacitor



Potential Energy

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Parallel Plate

$$\Delta V = E d$$

$$C = \epsilon_0 A / d$$

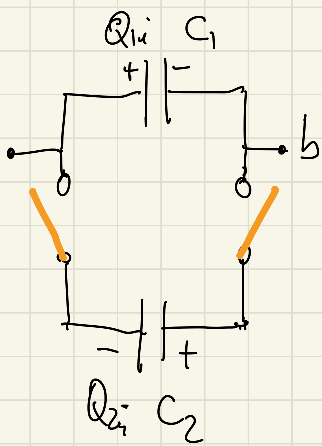
$$U = \frac{1}{2} \epsilon_0 E^2 (Ad)$$

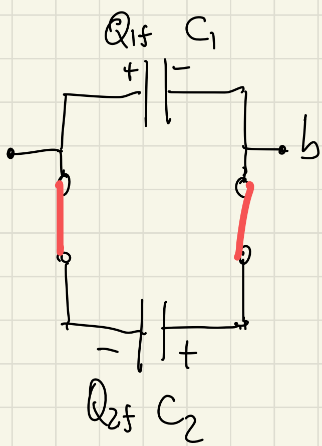
General

$$u = \frac{1}{2} \epsilon_0 E^2$$

Quiz

- Two capacitors C_1 and C_2 $(C_1 > C_2)$ are charged to the same initial potential difference ΔV_i. The charged capacitors are removed from the battery and their plates are connected with opposite polarity as shown.
- The switches 'S_1' and 'S_2' are then closed as shown (right).

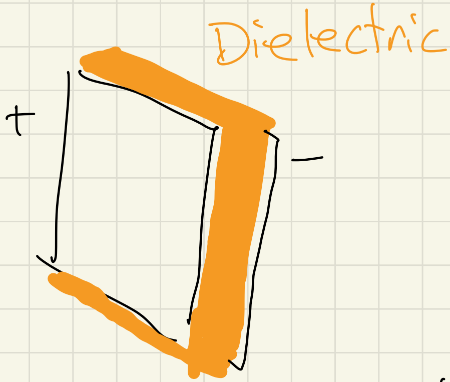


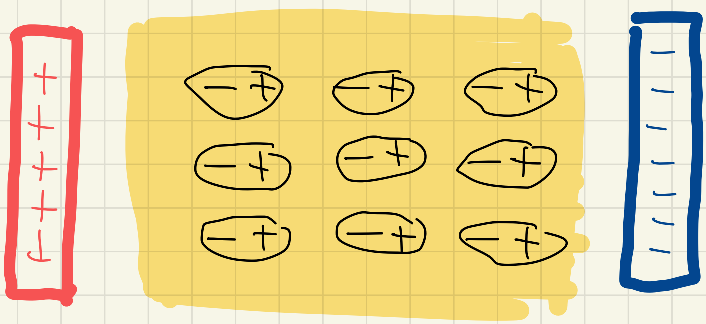


(a) Find ΔV between 'a' and 'b' after switch is closed

(b) Energy stored before and after

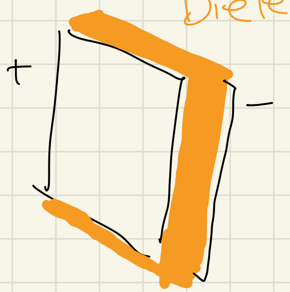
Capacitors with Dielectrics

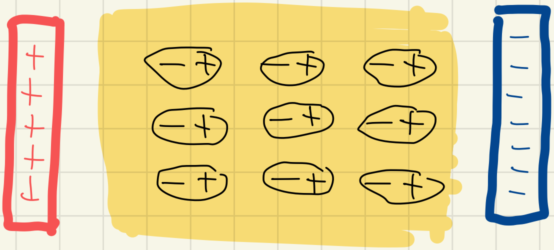




- Dielectric = insulating material (glass, rubber, hBN, air)
- Dielectric reduces the electric potential or field; "screens" Coulomb

Capacitors with Dielectrics





- $\Delta V_0 \longrightarrow \Delta V = \frac{\Delta V_0}{\epsilon} < \Delta V_0$

  ϵ = dielectric constant

- $C_0 = \frac{Q_0}{\Delta V_0} \longrightarrow C = \frac{Q_0}{\Delta V_0/\epsilon} = \epsilon C_0$

- $\epsilon \geq 1$ $\quad \Delta V < \Delta V_0$

  $C > C_0$