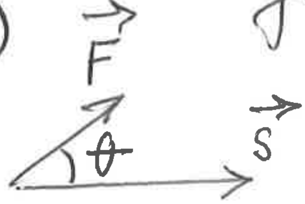
Mechanics 3 : Work and Energy

Work = a measure of energy transfer

$$W = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos\theta$$

- W: work (J)
- F: force (N)
- s: displacement (m)
- θ: angle between $\vec{F}$ and $\vec{s}$



$$1\,J = 1\,N \times 1\,m$$

Example 1

b) displacement $s = 0$, $W = 0$

c) $\theta = 90^\circ$, $\cos 90^\circ = 0$, $W = 0$

e) $Fd\cos\theta$

f) 

$\theta = 180^\circ$ $\cos\theta = -1$ $\therefore W = -Fd$

Example 2

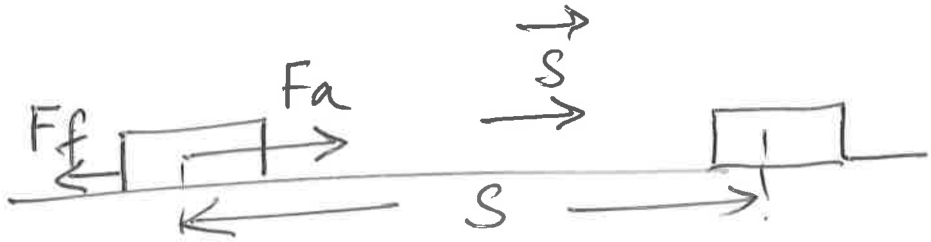
$W = F s \cos\theta = 800\ J$

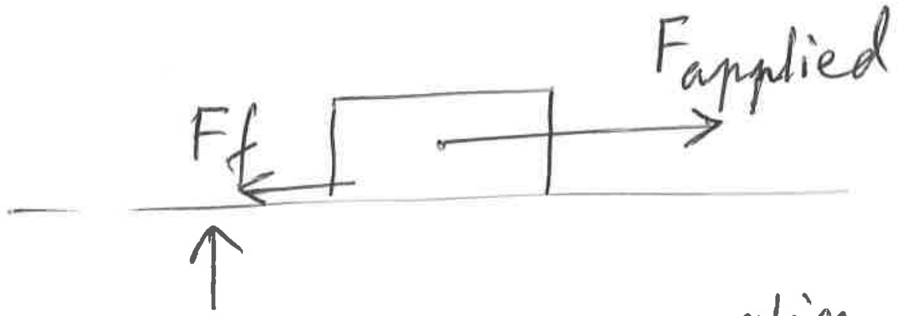
($F = 40N$, $s = 20m$, $\theta = 0^\circ$)

$W = F.s.\cos\theta = 690\ J$

($F = 40N$, $s = 20m$, $\theta = 30^\circ$)

Example 3





Friction opposes motion

a) $W_a = F_a s\cos\theta_1 = 500 \times 20 \times \cos 0^\circ = 10,000\ J$

$W_f = F_f s\cos\theta_2 = 100 \times 20 \times \cos 180^\circ = -2000\ J$

Network done: $W = W_a - W_f = 8000\ J$

b) Equal to $W_f = 2000\ J$

c) Total E required $= 10,000\ J$

$$KE = \frac{1}{2}mv^2$$

- KE: Kinetic E (J)
- m: mass (kg)
- v: velocity (m/s)

If $v = 0$, $KE = 0$

$$W = KE_2 - KE_1 = \frac{1}{2}m(v^2 - v_0^2)$$

KE_2: final KE

KE_1: initial KE

Example 4

a) $W = \Delta KE = \frac{1}{2}m(v^2 - v_0^2)$

m: 5kg, v: 4m/s, v_0: 2m/s

$\therefore W = 30\,J$

b) $v^2 = v_0^2 + 2as$

v: 4m/s, v_0: 2m/s, s: 20m

$\therefore a = 0.3 m/s^2$

$F = ma$

m: 5kg, a: $0.3 m/s^2$

General equation

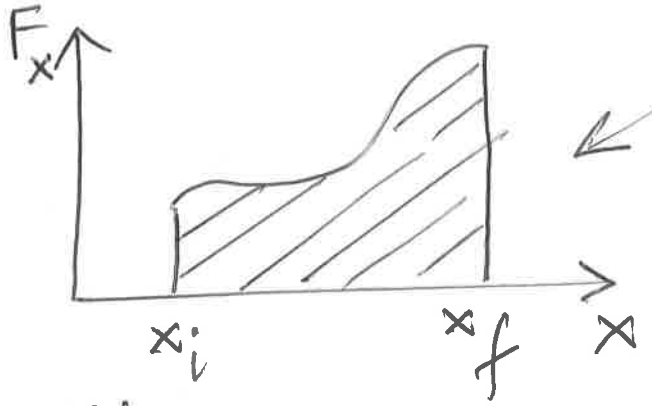
$$W = \int_{x_i}^{x_f} F_x \cdot dx$$

x_f — final position

x_i — initial position

F_x — force

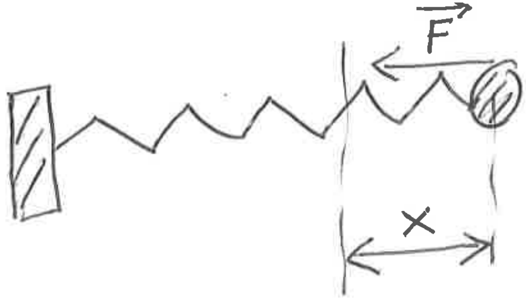
Area under the curve = Work



$$W = \int_{x_i}^{x_f} F \cdot dx = F \cdot \Delta x = F(x_f - x_i)$$

If $F =$ constant

Potential energy



Spring : Storing force

$$F_x = k \cdot x$$

Force (N) → F_x

Spring constant (N/m) → k

displacement (relative to equilibrium position) → x

$$W = \int_{x_i}^{x_f} F\,dx = \int_0^x kx.dx$$

$$\int x\,dx = \frac{1}{2}x^2$$

$$\therefore \quad W = \frac{1}{2}kx^2$$

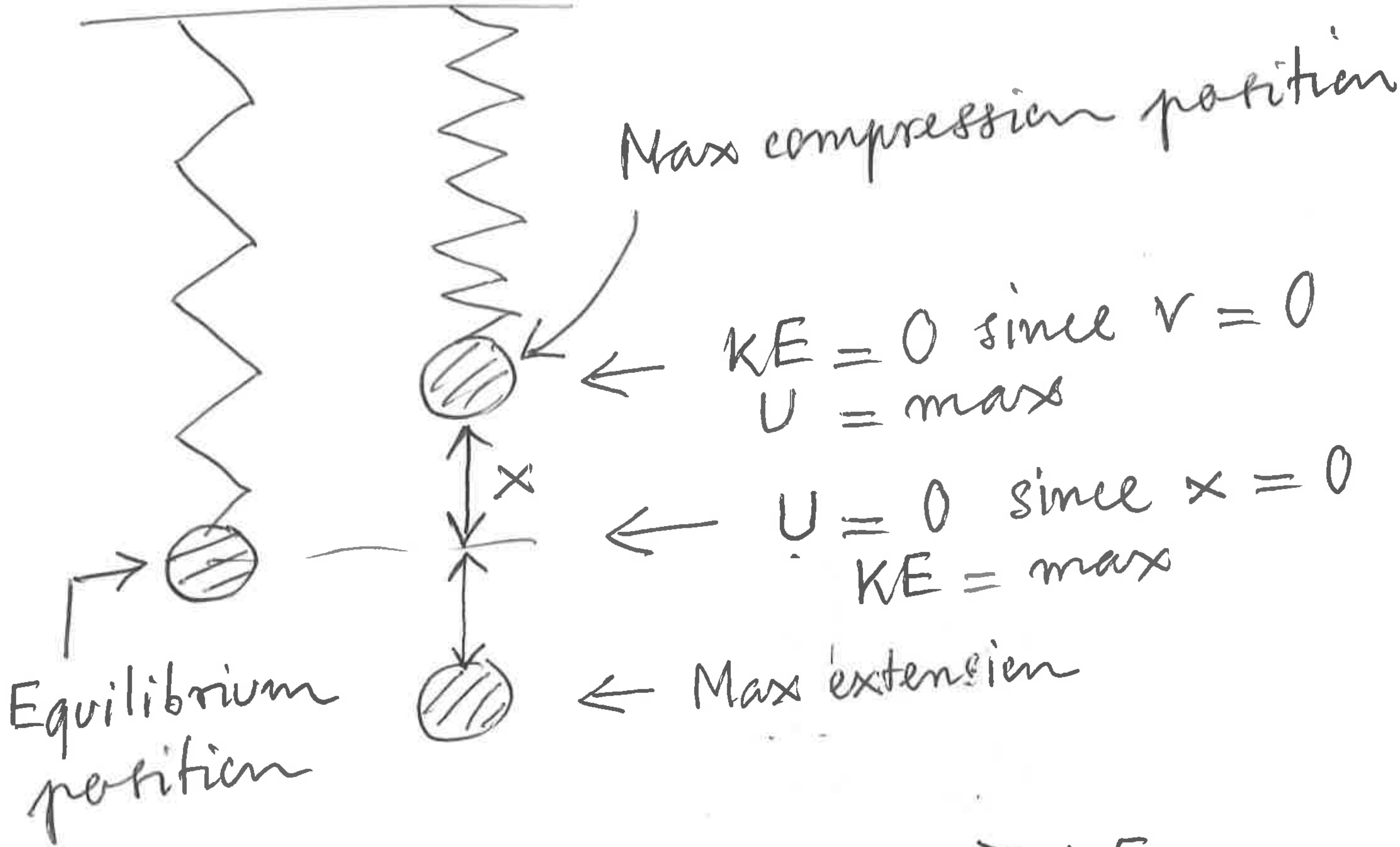
Work done by spring / elastic object

↳ This is stored in spring as potential energy

$$W = U = \frac{1}{2}kx^2$$

Example 5

Spring : energy oscillates between kinetic and potential energy



As object oscillates, $U \rightleftharpoons KE$

Energy at equlibrium = energy at max compression

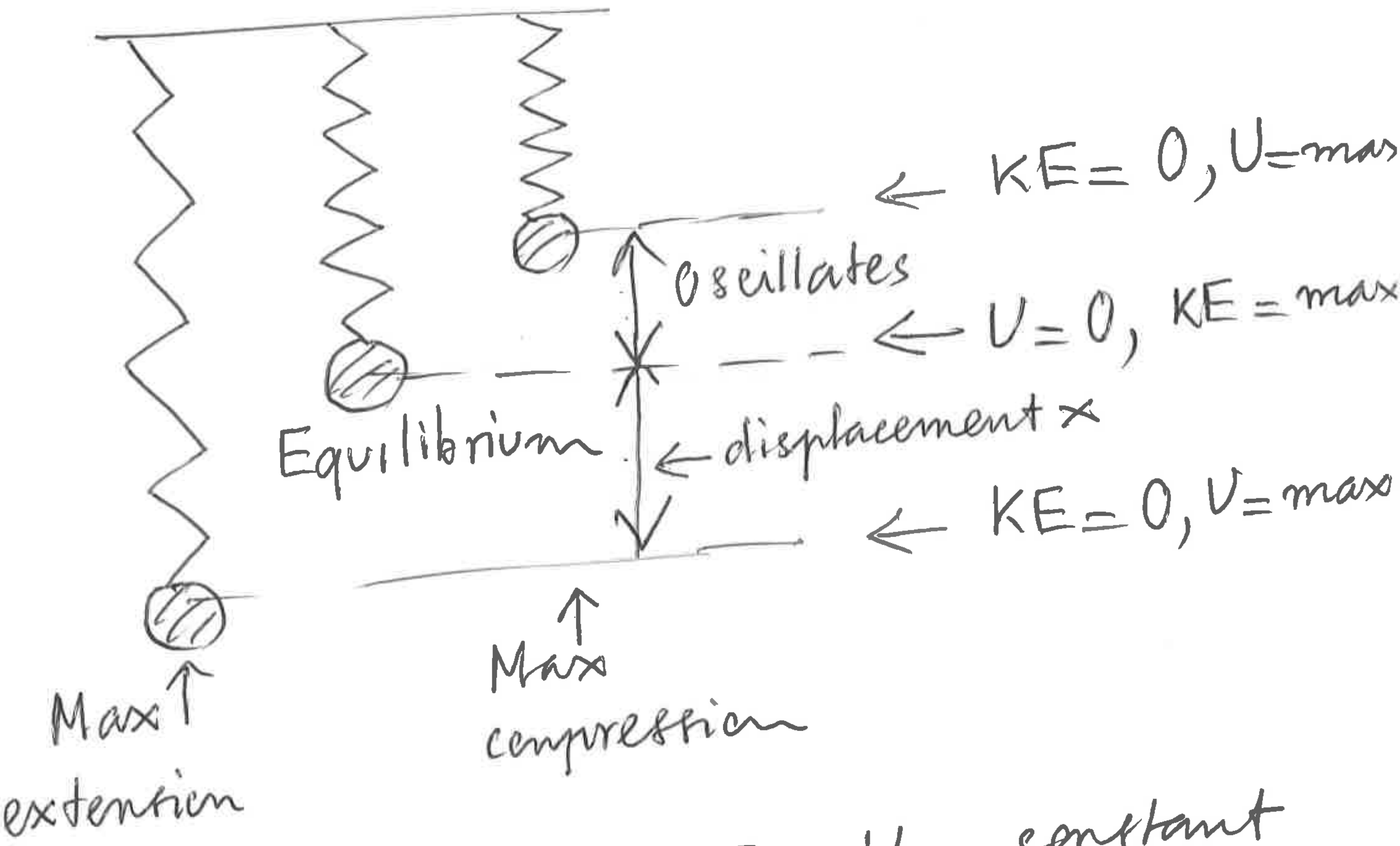
$$KE = U$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$ ← potential energy stored in spring

$$\therefore mv^2 = kx^2$$

m = 0.2 kg, k = 20 m/s, x = 0.1 m

$\therefore v = 1.0$ m/s



Total energy = KE + U = constant

$$KE_{equilibrium} = U_{max\ compression}$$

$KE_{equilibrium} \leftarrow \frac{1}{2}mv^2$

$U_{max\ compression} \leftarrow \frac{1}{2}kx^2$

Potential energy due to gravitational force

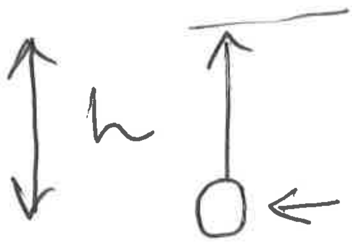
$W = F \cdot s$

- F → mg (weight)
- s → h

$\therefore$ $\boxed{W = mgh}$

- W → potential energy stored in object (J)
- m → kg
- g → $9.8 m/s^2$
- h → height

Example 6



max position / final position: $PE_f = max$, $KE_f = 0$

initial position: $PE_i = 0$, $KE_i = max$

As the ball travels upwards, KE is transferred to PE

$$KE_i = PE_f$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$\therefore h = \frac{v^2}{2g}$$

(v = 20m/s, g = $9.8 m/s^2$)

$\therefore h = 20.4m$

Mass is irrelevant!

Power : $P = \dfrac{W}{\Delta t}$

W ← work (J)

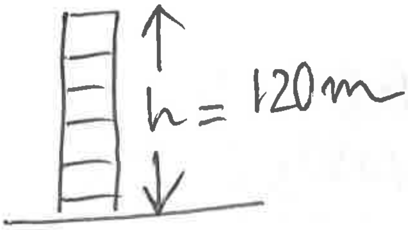
Δt ← time (s)

P: J/s = W (watt)

$1 \text{ horsepower} = \dfrac{W}{\Delta t} = \dfrac{mg\Delta h}{\Delta t} = 746\,W$

(m = 75kg, g = 9.8m/s^2, Δh = 1m, Δt = 1s)

<u>Example 7</u>

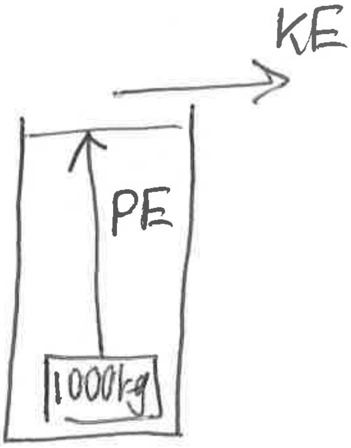


$P = \dfrac{W}{\Delta t} = \dfrac{mgh}{\Delta t}$

(m = 60kg, g = 9.8m/s^2, h = 120m, Δt = 8 × 60 = 480s)

$\therefore P = 147\,W$

<u>Example 8</u>



a) $W_{lift} = mgh$

(m = 1000kg, g = 9.8m/s^2, h = 8m)

$W_{lift} = 78400\,J$

b) $W_{kinetic} = \dfrac{1}{2} m v^2 = 12500\,J$

(m = 1000kg, v = 5m/s)

c) Power output

$P = \dfrac{W_{total}}{\Delta t} = \dfrac{78400 + 12500}{60\,(s)} = 1520\,W$