Fluid Statics & Dynamics

A fluid is any substance that can flow, and we use the term for both liquids and gases. Understanding the behaviour of fluids is fundamentally important to engineering students. An example is the utilisation of fluid principles in calculating forces on aircraft or the flow rate of petroleum through pipelines.

Fluid Statics

In the first part of the lecture, we will look at fluids at rest (fluid statics). We will consider:

- **Density and pressure of a fluid**
- **Variation of pressure with depth**
- **Pascal's principle**
- **Pressure measurements**
- **Archimedes' principle**

Density and pressure of a fluid

- An important property of any material (fluid or solid), is its density, defined as its mass per unit volume.

Density of a homogeneous material $\rho = \dfrac{m}{V}$ (Mass of material m; Volume occupied by material V)

- The SI unit of density is the kilogram per cubic meter (1 kg/m^3)

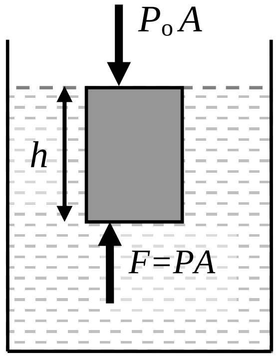
- The pressure, P, that anything exerts is simply the force, F, that it exerts per unit area over the area, A, that the force is exerted, i.e.

$$P = \frac{F}{A} \qquad \text{SI unit : N/m}^2 \equiv \text{pascal, Pa}$$

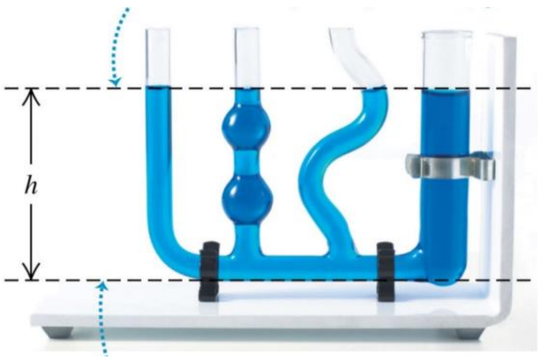
- We can think about the pressure that exits within a fluid in the following way: if a small volume of the liquid were removed we know that the remaining fluid would move (accelerate) into the volume that it occupied.
- We know then that there must be exerting a force outward onto the fluid that surrounds it. The pressure at the surface of the volume is then the average force exerted by the volume / the surface area of the volume.
- If we make the volume smaller and smaller then we can think of pressure at a point in a fluid (*hydrostatic pressure*).

Variation of fluid pressure with depth

- Consider a column of fluid with a height h, cross-sectional area A and a mass m and density ρ.



- The force at the base of the column is $F = mg$, then the pressure at the base of the column is
$$P = \frac{F}{A} = \frac{mg}{A}$$
- But the volume, V, of the column is $V = m/\rho = Ah$ so that
$$P = \rho g h$$
- If there is a pressure P_O at the top of the column then at any distance h below the surface the pressure is,
$$P = P_o + \rho g h$$
- This means that **the pressure at a depth *h* below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by *ρgh*.**
- The above expression says that the pressure in a fluid does not depend in any way on the shape of the container. **It depends only on the depth below the surface of fluid.**



The hydrostatic paradox: the pressure at the bottom of each fluid column has the same value P, regardless of the size and shape of the column.

- Remember that both gases and liquids are fluids, and thus follow the same rules of behaviour.
- The average atmospheric pressure at sea level is 1.013×10^5 Pa.

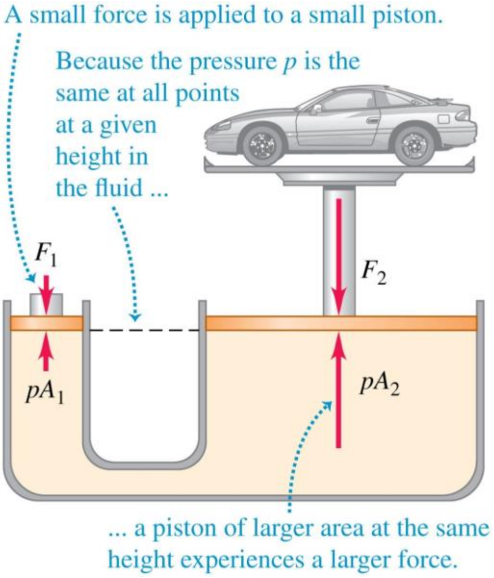
Example 1

a) Calculate the absolute pressure at an ocean depth of 500 m. Assume the density of water is 1000 kg/m^3 and the atmospheric pressure is 1.01×10^5 Pa.
b) Calculate the total force exerted on the outside of a 34 cm diameter circular submarine window at this depth.

Ans: 5.0×10^6 Pa, 4.5×10^5 N

Pascal's Principle

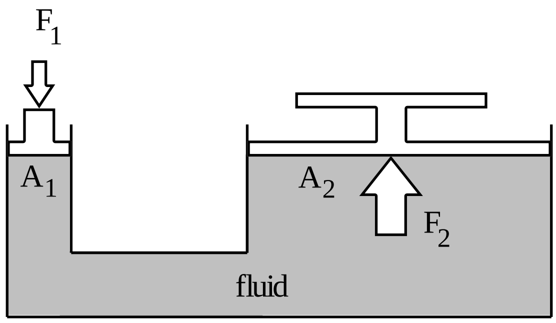
- Pascal's Principle states: **Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.**
- It should be noted that Pascal's Principle applies only to fluids of constant density.



In a hydraulic system, pressure transmitted to a fluid is identical to all parts of the container.

Example 2

(a) Derive an expression for the force applied to the larger surface in the hydraulic lift shown below.

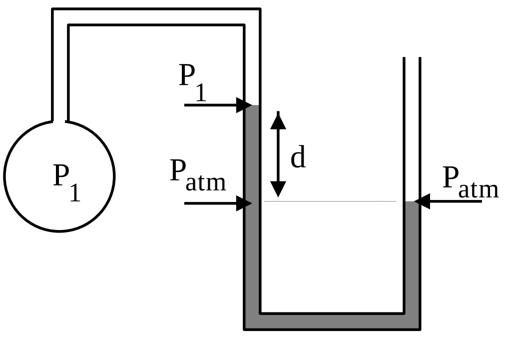


(b) Suppose the hydraulic lift shown above has a small cylindrical piston with radius 5.0 cm and a large piston with radius 20 cm. The mass of a car placed on the larger piston's platform is 1000 kg. What force must be applied to the small piston to lift the car?

ANS: $\boldsymbol{F_2 = (A_2/A_1)\,F_1}$**, 610 N**

Manometers & Barometers

- The expression $P = P_o + \rho gh$ can be used to measure pressure
- Numerous devices are available, the simplest being the *open-ended manometer*



- In this device a liquid of density ρ is put into a U-tube, one end of which is open to the atmospheric pressure, P_{atm}, and the other end of which is attached to a container which contains a gas at a pressure P_1 which is to be determined.
- From the diagram, we have

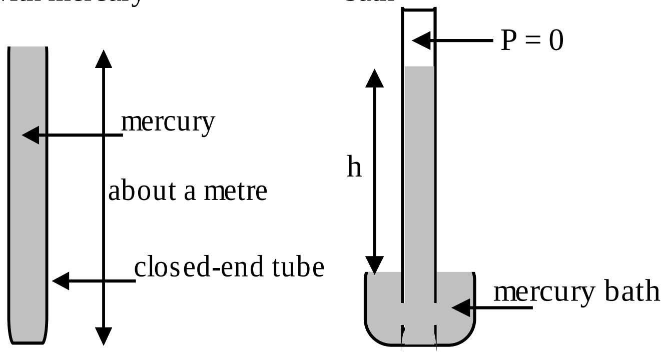
$$P_1 = P_{atm} - \rho gd$$

- $P_1 - P_{atm}$ is called the *gauge pressure*

A *barometer* is constructing in the following way;

1. fill a tube which has one open and one closed end with mercury

2. invert the tube and place the open end in a mercury bath



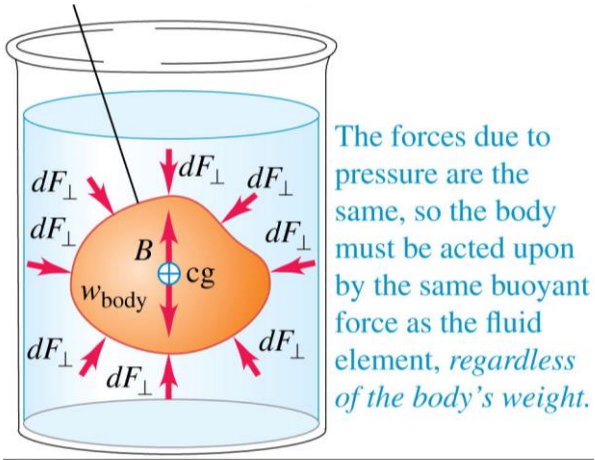
- The pressure at the top of the mercury bath is that of the atmosphere = the pressure at the same level inside the tube.
- The pressure at the top of the tube (vacuum) is zero, so we have

$$P_{atm} = \rho gh$$

- Knowing that the density of mercury is 13.6 × 10^3 kg/m^3 a measurement of *h* gives the pressure. This is typically measured in millimetres, and so we have a pressure unit of *millimetres of mercury*, or equivalently the *Torr*.
- This is a common method used to measure atmospheric pressure. **Atmospheric pressure is about 760 mmHg at sea level.**
- The readings can be corrected for the vapour pressure of Hg in the closed end of the column.

Archimedes' Principal (Buoyancy)

- **If a body is wholly or partially immersed in a fluid it experiences an upward force equal to the weight of the fluid displaced by the body**
- To see this consider an arbitrarily shaped volume of fluid in the same fluid; this volume has a weight, i.e. there is a gravitational force on it.



An object submersed in a fluid experiences buoyant force equal to the mass of the fluid it displaces.

- BUT this volume is at rest, and hence there must be an equal and opposite (in direction) force which is supporting this volume.
- This supporting force must be just the weight of this volume and directed upward; i.e.

$$B = \text{mass of volume} \times \text{acceleration due to gravity}$$

- Now what happens if we replace this imagined volume by an object made of different material? The force that is supporting this imagined volume come from the fluid that surrounds it - so that changing the material that our imagined volume is made of, does not change this force.
- So the upward force on any object of volume V_{object} in a fluid of density ρ_{fluid} is

$$B = \rho_{fluid} V_{object} g$$

- This force is the so-called *buoyant force.*
- If an object is floating in a fluid, the weight of the body *W* (pointing downwards) is equal to the buoyant force *B*.

Example 3

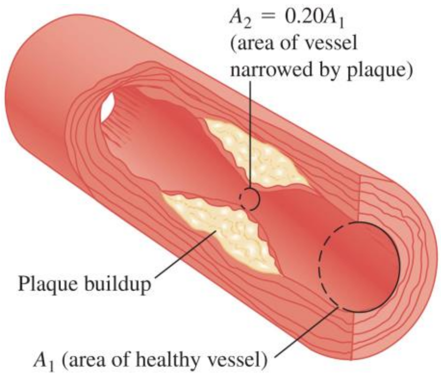
The density of ice is 920 kg/m^3 while that of seawater is 1025 kg/m^3. What fraction of an iceberg is submerged?

Ans: 90%

Fluid Dynamics

Fluid dynamics is concerned with the properties of a fluid in motion. In this lecture we will consider:

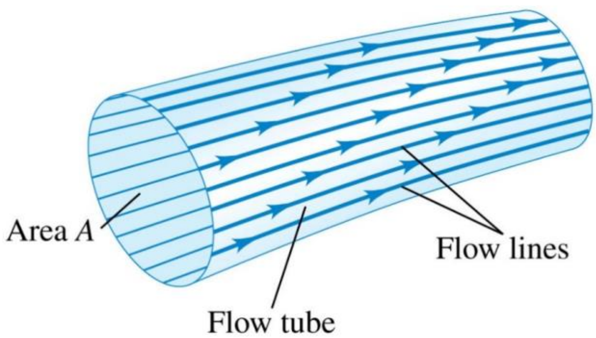
- **Behaviour of fluids in motion: laminar and turbulent flows**
- **Continuity equation for fluids flowing through a pipe of non-uniform size**
- **Bernoulli's equation for fluids flowing through a pipe of varying size and elevation**
- **Applications of fluid dynamics**



Blood flow characteristics are changed dramatically by plaque.

Laminar and turbulent flows

- The path of an individual particle in a moving fluid is a *flowline*.
- The flow is *laminar* if every particle passing through the same point has the same flowline. This means the fluid flows in a steady manner.
- In contrast, the flow is *turbulent* if flowlines are discontinuous, and there is no steady-state pattern of streamlines.
- We will not consider the situation of turbulent flows as it is beyond the scope of this subject.
- The flowlines which form the outer edge of the cross-sectional area form a *flowtube.*



- From the definition of a flowline (for steady flow) no fluid flows across the walls of a flow tube.

Ideal fluids

Properties of fluid motion can be understood by considering the behaviour of so-called *ideal* fluids, which:

(i) have no internal friction (i.e. no viscosity);

(ii) are not compressible;

(iii) have steady flows i.e. velocity and pressure at each point in fluids does not change over time;

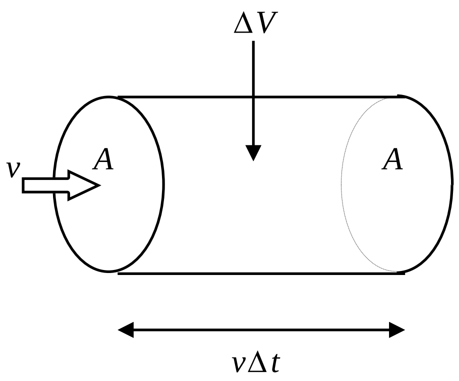
(iv) flow without turbulence.

- No fluid is ideal but it makes the treatment easier and valid in many cases.
- We will deal with viscous liquids later.

Equation of Continuity

- Consider a flow of fluid, at a speed v, through a pipe whose cross-sectional area is A.
- What volume of fluid, ΔV, crosses a plane of area A in a time interval Δt? Any fluid which is within a distance $\Delta x = v\Delta t$ of the area A will reach it within a time Δt. So that the volume flow rate, Q, across A is

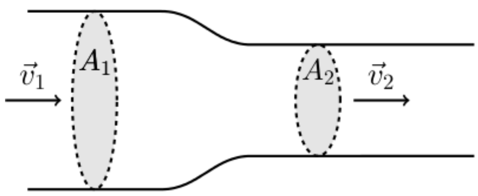
$$\text{volume flow rate, } Q = \frac{\Delta V}{\Delta t} = Av$$



Example 4. Determine the flow rate of water flowing at a velocity of 1.0 m/s through a pipe of 1.5 cm radius.

Ans: 7.1×10^{-4} m^3/s

- Now suppose that we have a fluid flowing through a pipe with a varying cross-section;

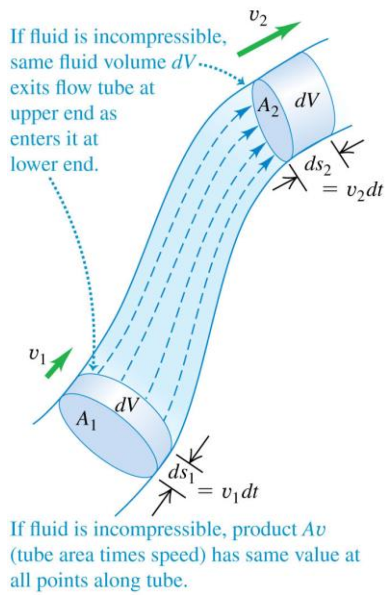


- Since we are dealing only with ideal fluids then conservation of mass implies that the volume flow rate across any two surfaces must be equal to each other i.e. $Q_1 = Q_2$, so that we have the *equation of continuity*,

$$A_1 v_1 = A_2 v_2$$

- **Product of the cross-sectional area and the fluid speed at that cross section is a constant.**
- We see from the equation of continuity that

$$A_1 > A_2 \quad \Leftrightarrow \quad v_1 < v_2$$



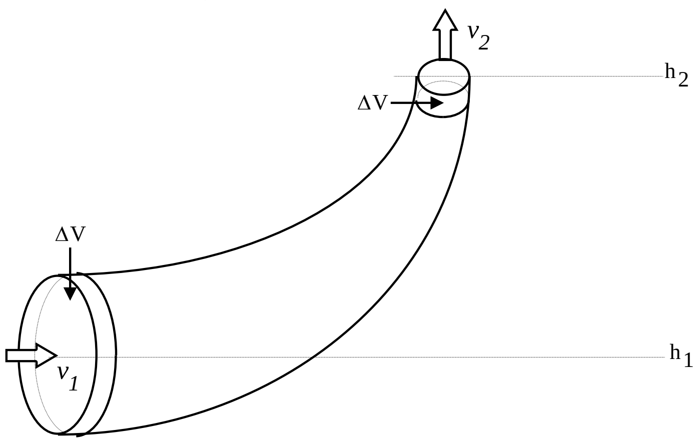
Example 5. Water is flowing in a pipe with a varying cross-sectional area. At point 1 the cross-sectional area of the pipe is 0.11 m^2, and the fluid velocity is 1.5 m/s. What is the fluid velocity at the point in the pipe where the cross-sectional area is 0.070 m^2 ?

Ans: 2.4 m/s

Bernoulli's Equation

The equation can be applied to steady flowing, incompressible fluids without viscosity. It is also a statement of the conservation of energy applied to fluid flow.

- Consider a small volume flowing along a pipe which is not at the one height and not of constant cross-sectional area.



- At point 1: pressure = P_1, potential energy = mgh_1, kinetic energy = $\frac{1}{2}mv_1^2$
- At point 2: pressure = P_2, potential energy = mgh_2, kinetic energy = $\frac{1}{2}mv_2^2$

- The fluid will not in general flow from point 1 to point 2 "by itself" but work needs to be done on it (by the surrounding fluid). For the volume ΔV this is equal to,

$$W = (P_1 - P_2)\Delta V$$

and this work goes into changing the height and/or the speed of the volume ΔV, so that

$$W = (P_1 - P_2)\Delta V = (\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2) + (mgh_2 - mgh_1)$$

i.e.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

- These last pair of equations are ways of writing *Bernoulli's equation*. It is valid for ideal fluids undergoing steady flow.
- Each term of the above equations has units of pressure (Pa or N/m^2).

Example 6. Flow velocity of blood in an artery is 0.30 m/s. If the arterial cross-sectional area is reduced by a factor of 3, what is its effect on the arterial blood pressure? Take the blood density to be 1050 kg/m^3.

Ans: P reduced by 380 Pa in the constricted region.

Example 7. Hurricane winds of 150 km/h are blowing over the flat roof of a well-sealed house.

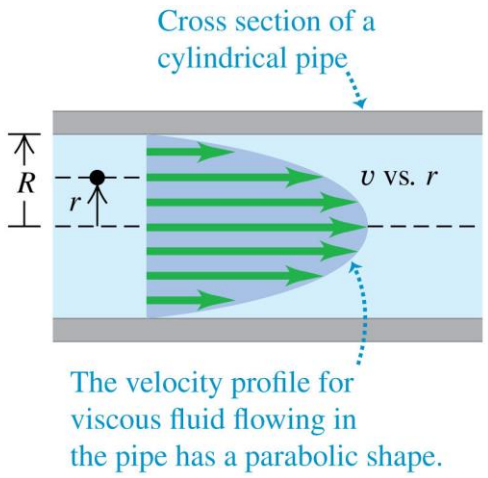
i) What is the difference in air pressure between the inside and outside of the house? Take the density of air to be 1.3 kg/m^3.

ii) What is the total force exerted by air on the roof if the roof area is 200 m^2.

Ans: 1130 Pa, 2.3×10^5 N

Viscosity

- Viscosity is internal friction in a fluid and viscous forces oppose the fluid motion.
- Fluids do have internal friction, which means that one section of a fluid exerts a force on adjacent sections.
- Due to viscosity, the speed is zero at the pipe walls (to which the fluid clings) and is greatest at the centre of the pipe.



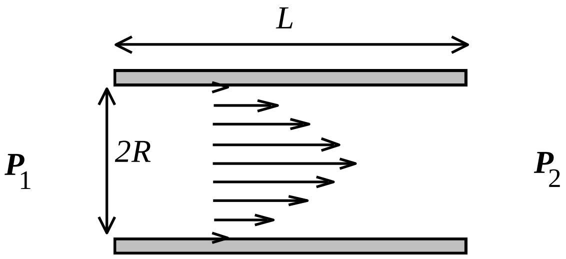
- The SI (standard) units of viscosity are N.s/m^2 or Pa.s. A non-SI unit called the *poise*, 1 *poise* = 10^{-1} Pa.s

Fluid	Coefficient of viscosity η (Pa.s)
Air (at 15ºC)	1.8×10^{-5}
Water (at 20ºC)	1.0×10^{-3}
Water (at 100ºC)	0.28×10^{-3}
Blood (at 37ºC)	2.7×10^{-3}
Motor oil (at 20ºC)	250×10^{-3}

Why does the viscosity of a liquid generally decrease with increasing temperature?

Flow of viscous liquids

- As described above, a fluid in contact with a solid "sticks" to the surface due to viscous effects. For fluid flowing down a tube of radius R and length L the velocity gradient is a parabola.



- To maintain a flow a pressure difference $P_1 - P_2$ is needed ($P_1 > P_2$)
- The rate of flow Q (volume per unit time) is dependent on the pressure difference ($P_1 - P_2$) and dimensions of the tube. The Poiseuille's law is,

$$Q = \frac{\pi}{8}\frac{R^4}{\eta}\frac{P_1 - P_2}{L}$$

  where η is the coefficient of viscosity. This equation is known as **Poiseuille's law**.

- Note the flow rate is proportional to the radius of the tube raised to the fourth power.

Example 8. The pulmonary artery, which carries blood from the heart to the lungs, has an inner radius of 2.5 mm and is 8.2 cm long. If the pressure drop between the heart and lungs is 420 Pa, what is the flow rate of blood in the pulmonary artery? Take the viscosity of blood to be 2.7×10^{-3} Pa.s.

Ans: 2.9×10^{-5} m^3/s