Mechanics 4:
Potential Energy, Momentum, Impulse and Collisions

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Resources:

Chapters 8 & 9 of your textbook:

Halliday, Resnick and Walker, “Fundamentals of Physics – 10th edition” Wiley.

Available free at the library.

https://ebookcentral.proquest.com/lib/uts/detail.action?docID=3059079&pq-origsite=primoLinks to an external site.

What is potential energy?

Potential energy is energy which results from **position** or **configuration**.

In any situation where the system position or configuration changes, the potential energy changes.

Examples are:

Gravitational potential energy $U=mgh$ (potential energy associated with gravitiational forces acting on a mass *m* at height *h*)
Elastic potential energy $U=kx^2$ (potential energy associated with extension/compression of an elastic object with spring constant *k*).

Potential energy and Work

You learnt in chapter 7 that work done by a variable force in one dimension is given by:

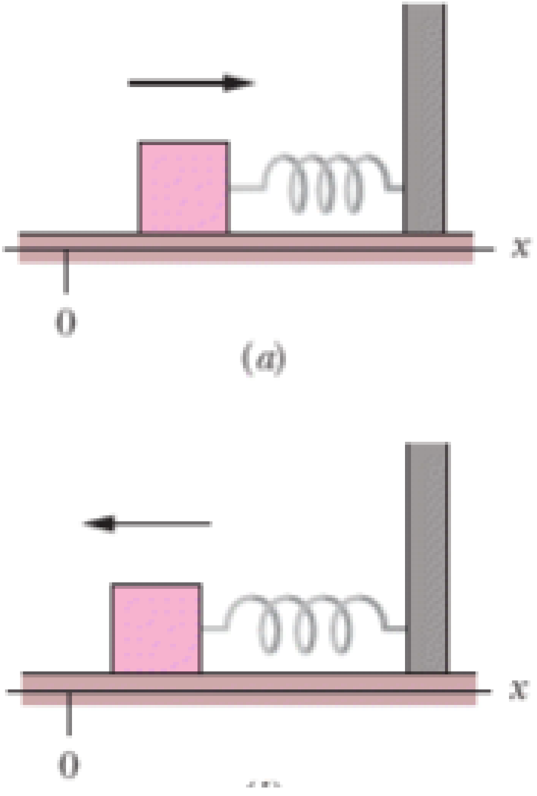
$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx,$$

And Work done is **equal to the change in kinetic energy** as the particle moves from position x_1 to position x_2:

$$W = K_f - K_i = \Delta K,$$

In any real scenario, there is always **potential energy** in addition to **kinetic energy, and we need to account for both when we calculate Work done.**

Conservative and nonconservative forces



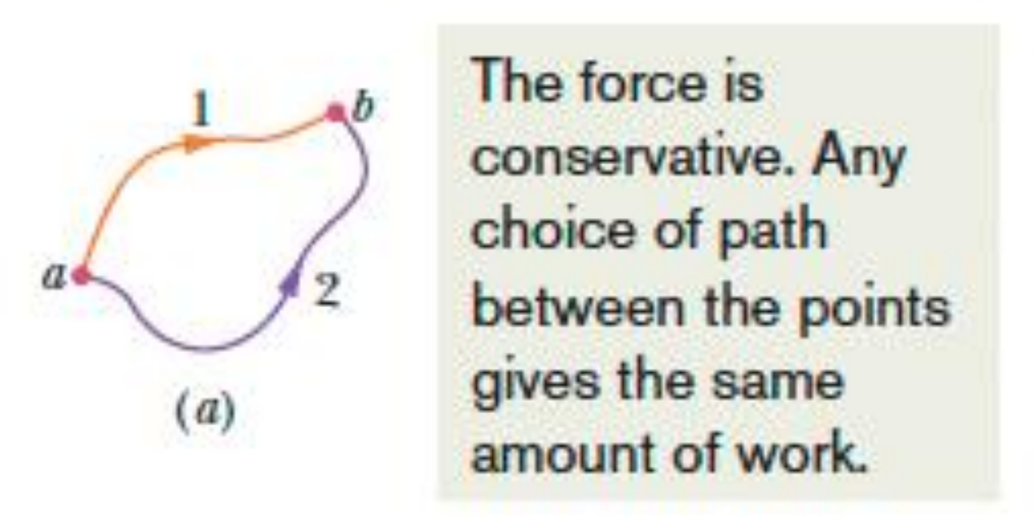
We can simplify any scenario in which work is done to the following:

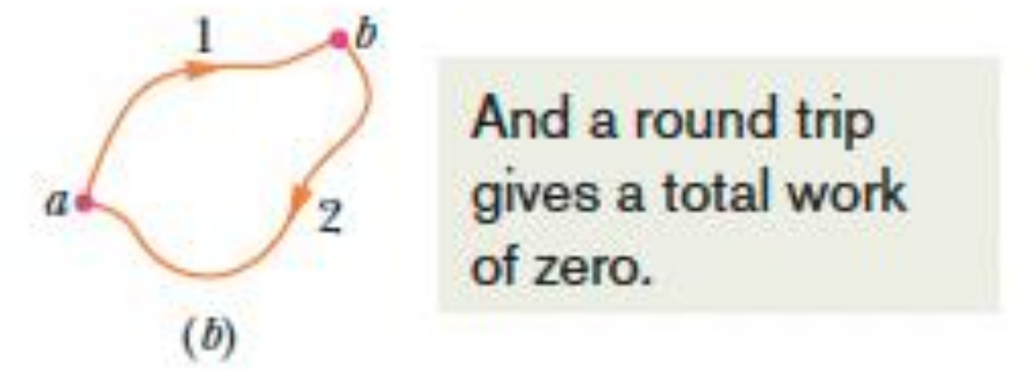
*A system containing 2 or more objects (eg. a block and a spring)
*A force acting between an object (eg. block) and the rest of the system
*Work which is done on the object when the system configuration changes from configuration 1 to 2 (eg. when the block attached to the spring moves).

When W_1=-W_2, the only other type of energy we need to consider besides kinetic energy is potential energy.

In this case, the force is called a **conservative force**

Work done by conservative forces





Conservative forces have special properties:

(1) The *work done* by a conservative force on a particle or object moving between two points is *independent of the path* taken.

(2) The *net work done* by a conservative force on a particle or object moving around any closed path is *zero*.

Conservation of energy

In an **isolated system where only conservative forces cause energy changes**, the kinetic energy and potential energy can change, but their sum, the mechanical
energy E_{mec} of the system, cannot change.

This is expressed in the following equation:

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

More generally, the total energy of a system is conserved. This means that the total energy can only change by amounts of mechanical, thermal or internal energy transferred to or from a system (eg. by doing work).
This is expressed as:

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}},$$

Calculating gravitational potential energy.

The change in gravitational potential energy is defined as being equal to the negative of the work done:

$$\Delta U = -W.$$

So using the definition of work, $W = \int_{x_i}^{x_f} F(x)\, dx.$

the **change in potential energy** due to a change in spatial configuration (position) is given by:

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$

$= - \text{mg}\ (x_f\text{-}x_i)$

(as F(x)=F=ma=mg)

Calculating gravitational potential energy.

Also, if U_i is a reference gravitational potential energy when the particle is at reference point y_i, the gravitational potential energy U at any other point can be calculated from:

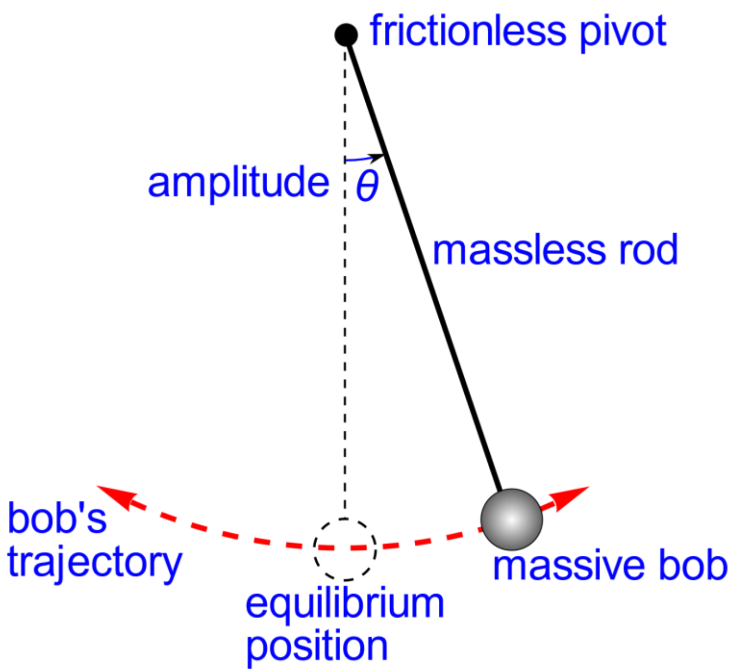
$$U - U_i = mg(y - y_i).$$

As only changes in potential energy are physically meaningful, we take $U_i=0$ and $y_i=0$.

The equation now becomes:

$$\boxed{U(y) = mgy}$$

Problem 1- Gravitational potential energy



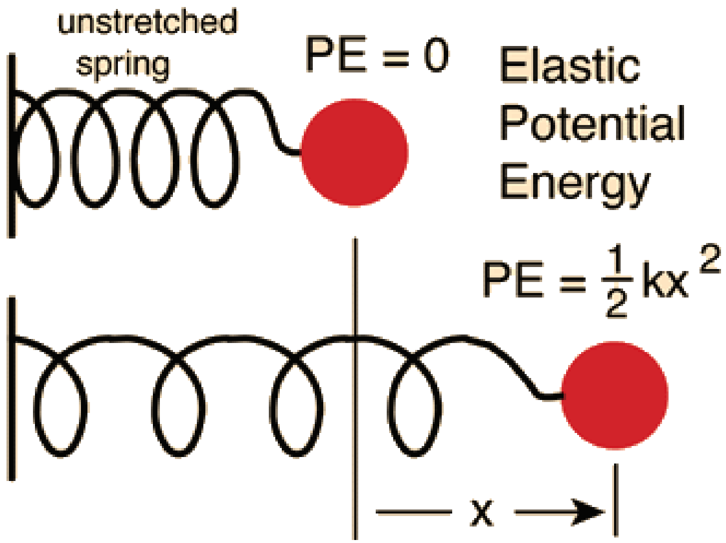
A pendulum is 1m long and has a 5.0 kg mass on its end.

(a) How much work is required to move the pendulum from the initial vertical position to a horizontal position?

Problem 1- Gravitational potential energy

(b) If the pendulum swings from a horizontal position, what will be the velocity and kinetic energy of the mass as it swings through its' lowest position?

Calculating elastic potential energy.



Now consider the mass on a spring with spring constant *k*.

As the mass moves from point x_i to x_f, the spring force $F_x=-kx$ does work on the block.

So to calculate the corresponding change in the elastic potential energy of the block-spring system, we substitute $-kx$ for F(x) in our equation:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\,dx = k\int_{x_i}^{x_f} x\,dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

Again, we put $U_i=0$ and $x_i=0$, giving:

$$U(x) = \tfrac{1}{2}kx^2$$

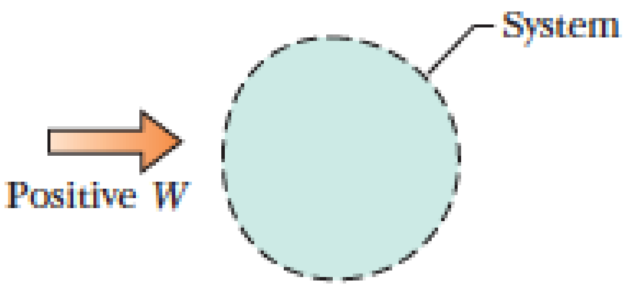
Problem 2- Conservation of energy

A pulley system is attached to 2 weights, a 20 kg mass suspended 0.4 m above the floor (mass 1), and a 5 kg mass resting on the floor (mass 2). The system is released from rest. Use the principle of conservation of energy to find the velocity with which the 20 kg block strikes the floor. Neglect friction and inertia of the pulley.

Work done by an external force

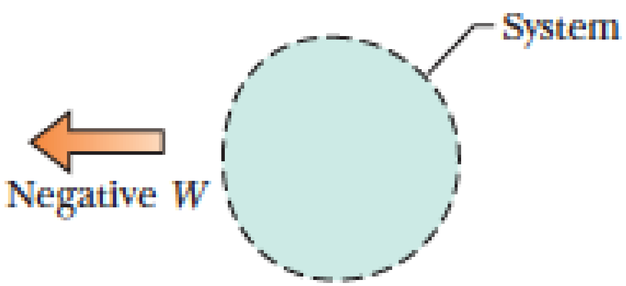
When more than one force acts on a system, their net work is the energy transferred to the system.

Positive W means that energy is transferred TO the system (eg. lifting a ball up against gravity).



(a)

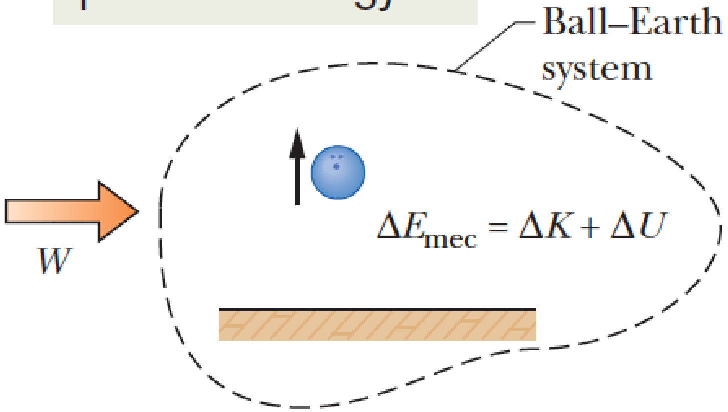
Negative W means that there is a transfer of energy FROM the system (eg. ball dropping to the earth).



(b)

Work done by an external force- no friction

Your lifting force transfers energy to kinetic energy and potential energy.



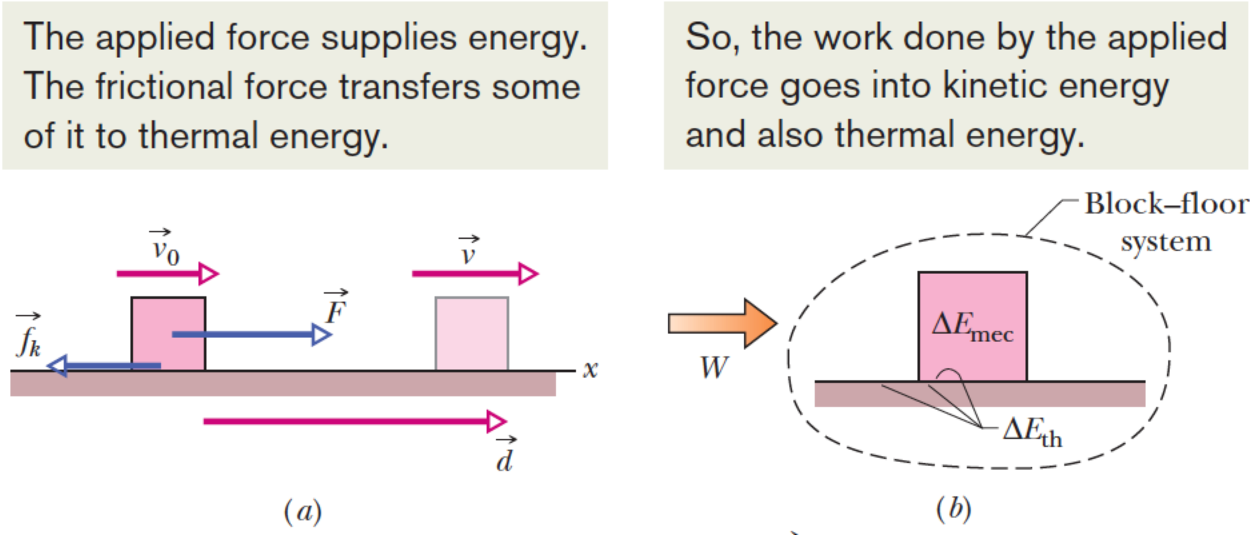
When there is no friction (a non-conservative force), all external work on a system done by an external force is converted to kinetic and gravitational potential energy:

$$W = \Delta K + \Delta U,$$

or

$$\boxed{W = \Delta E_{mec}}$$

Work done by an external force- with friction



Friction is a **non-conservative force**. When a block slides along a floor, frictional force $\boldsymbol{f_k}$ slows the block by transferring energy from its kinetic energy to thermal energy (generating heat).

From Newton's second law, $F - f_k = ma.$

Because the forces F and f_k are constant, acceleration a is constant too

Work done by an external force- with friction

Using $v^2 = v_0^2 + 2ad.$ and with $K=1/2mv^2$, we can **rewrite this equation as:**

$$Fd = \Delta K + f_k d.$$

More generally, there can also be a change in potential energy as well as kinetic energy:

$$Fd = \Delta E_{\text{mec}} + f_k d.$$

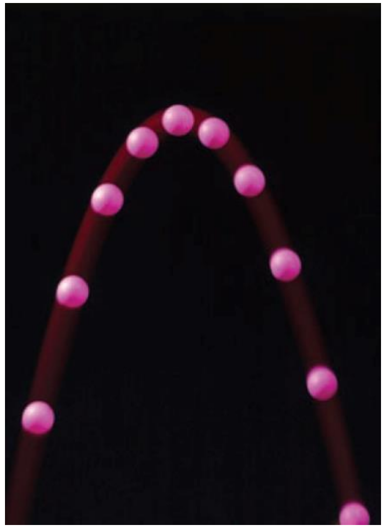
As the frictional energy $f_k d$ is transferred to thermal energy, we can also write this as

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}.$$

And

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \quad \text{(work done on system, friction involved).}$$

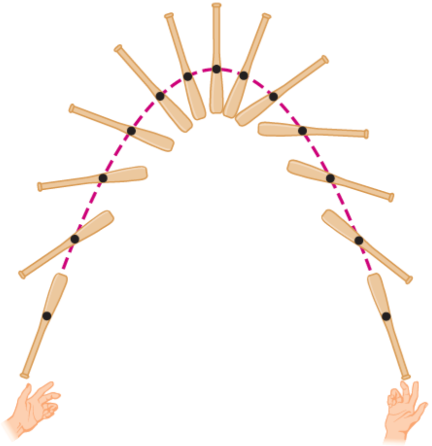
Centre of mass



Every physical system has a unique **centre of mass**. This is defined as the point that moves as though

(1) all of the system's mass was concentrated there, and

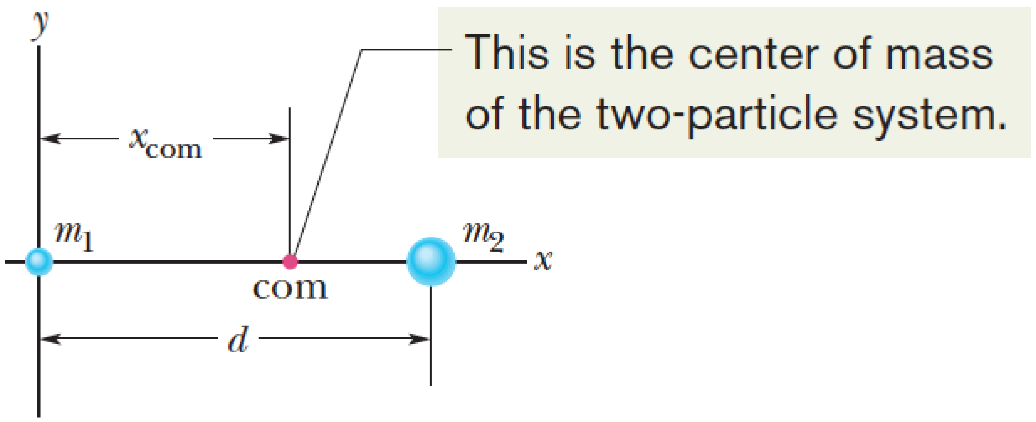
(2) all external forces were applied there.



In some cases, this is easy to find due to symmetry (eg, the centre of a ball).

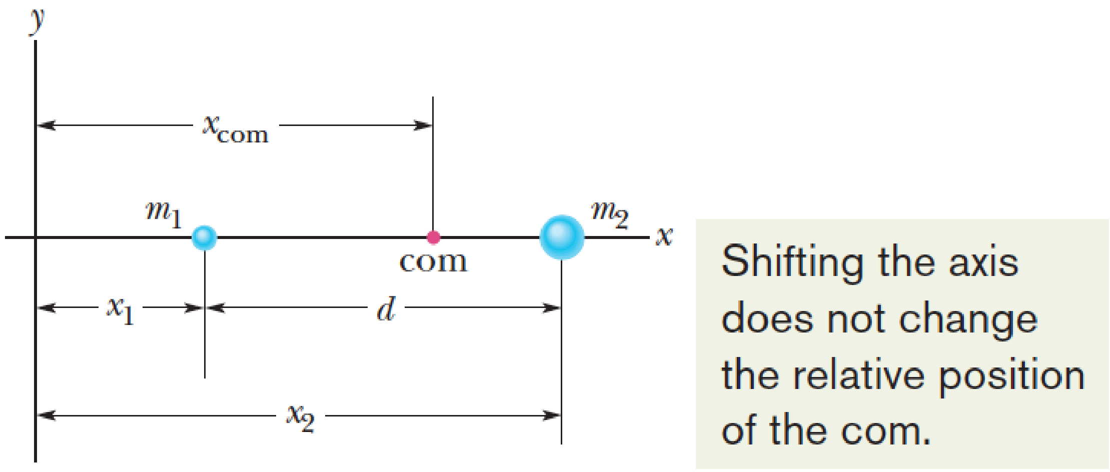
In other cases, we need to use geometrical and/or mathematical analysis in order to identify the centre of mass.

Centre of mass for 2 particle systems



For a 2 particle system consisting of masses m_1 and m_2 separated by distance d, we define the position of the centre of mass (x_{com}) to be:

$$x_{com} = \frac{m_2}{m_1 + m_2} d.$$



We can rewrite this in terms of he total mass $M=m_1+m_2$ and the distances x_1 and x_2 from the COM:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{M},$$

If we shift the axis, the relative position of the centre of mass doesn't change (it is still the same distance from each particle).

Centre of mass for many particles and in 3D

For a system of many (n) particles, we sum over all n particles:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_ix_i.$$

And in three dimensions, the centre of mass must be defined by three coordinates (relating to each coordinate axis x, y and z):

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i, \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i, \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iz_i.$$

Linear momentum $\vec{p}$

Newton's second law, ***F=ma***, can also be written in terms of a quantity called linear momentum.

Linear momentum is a vector quantity defined as:

$$\vec{p} = m\vec{v}$$

Expressed in terms of momentum, Newton's second law states that the time rate of change of momentum of a particle is equal to the net force acting on the particle, and has the same direction:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

As *a=dv/dt*, this can easily be shown to be equivalent to the equation *F=ma*

Conservation of linear momentum $\vec{p}$

The law of conservation of momentum states that if no external force acts on a system of particles, the total linear momentum cannot change:

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system)}.$$

Thus in a one-dimensional collision between two bodies 1 and 2:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f},$$

ie,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Problem 3- Linear momentum

A rifle weighing 2.00 kg fires a bullet weighing 0.005 kg at a muzzle speed of 500 m/s. What is the absolute recoil speed of the rifle?

Collision and Impulse

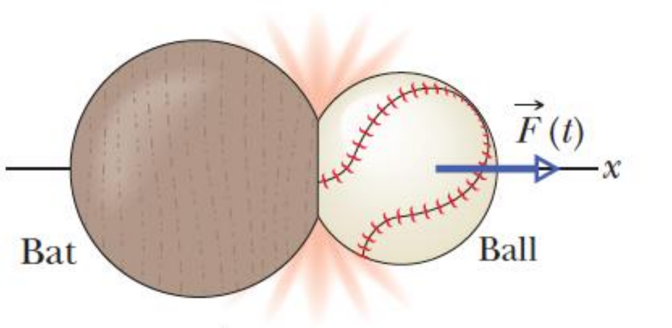


Figure 9-8 Force $\vec{F}(t)$ acts on a ball as the ball and a bat collide.

Newton's second law tells us that the linear momentum $\vec{p}$ of any particle-like body cannot change unless a net external force acts on it.

Rearranging the law, we have:

$d\vec{p} = \vec{F}(t).\ dt$

We can find the net change in the particle's momentum due to the collision by integrating both sides of this equation from a time t_i just before the collision to a time t_f just afterwards:

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\, dt.$$

Collision and Impulse

The left hand side of this equation gives us the change in momentum, and the right hand side is a measure of both the magnitude and duration of the collision force. This is called the **Impulse**:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

Thus, the **change in the object's momentum is equal to the impulse**:

$$\Delta\vec{p} = \vec{J} \qquad \text{(linear momentum–impulse theorem).}$$

Equivalently, we can find *J* by evaluating the average magnitude of the force F and multiplying by the time interval:

$$J = F_{\text{avg}}\,\Delta t. \qquad F_{\text{avg}} = -\frac{\Delta m}{\Delta t}\,\Delta v.$$

Problem 4- Impulse

A ball of mass 0.5 kg is thrown against a brick wall. When it strikes the wall it is moving horizontally to the left at 45 m/s and it rebounds horizontally to the right at 30 m/s. Find the impulse of the force exerted on the ball by the wall.