This Lecture

- Superposition, interference and boundary conditions
- Multidimensional waves
- Power & Intensity of waves

Superposition Principle

- Definition: Two waves travelling through a medium — resultant wave is the sum of the wave-functions of each wave

- $$y = y_1 + y_2$$

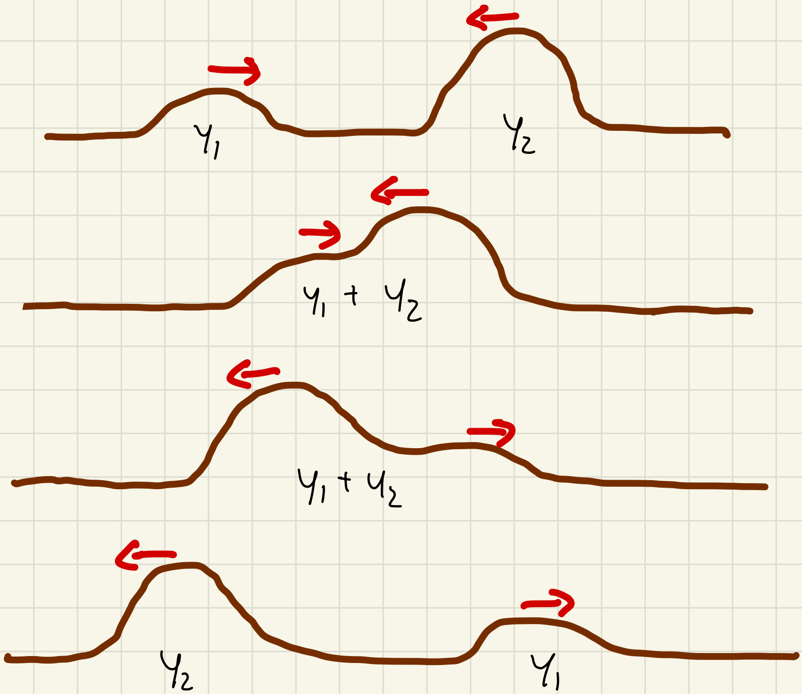
e.g.

$$y(t) = y_1(t) + y_2(t)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

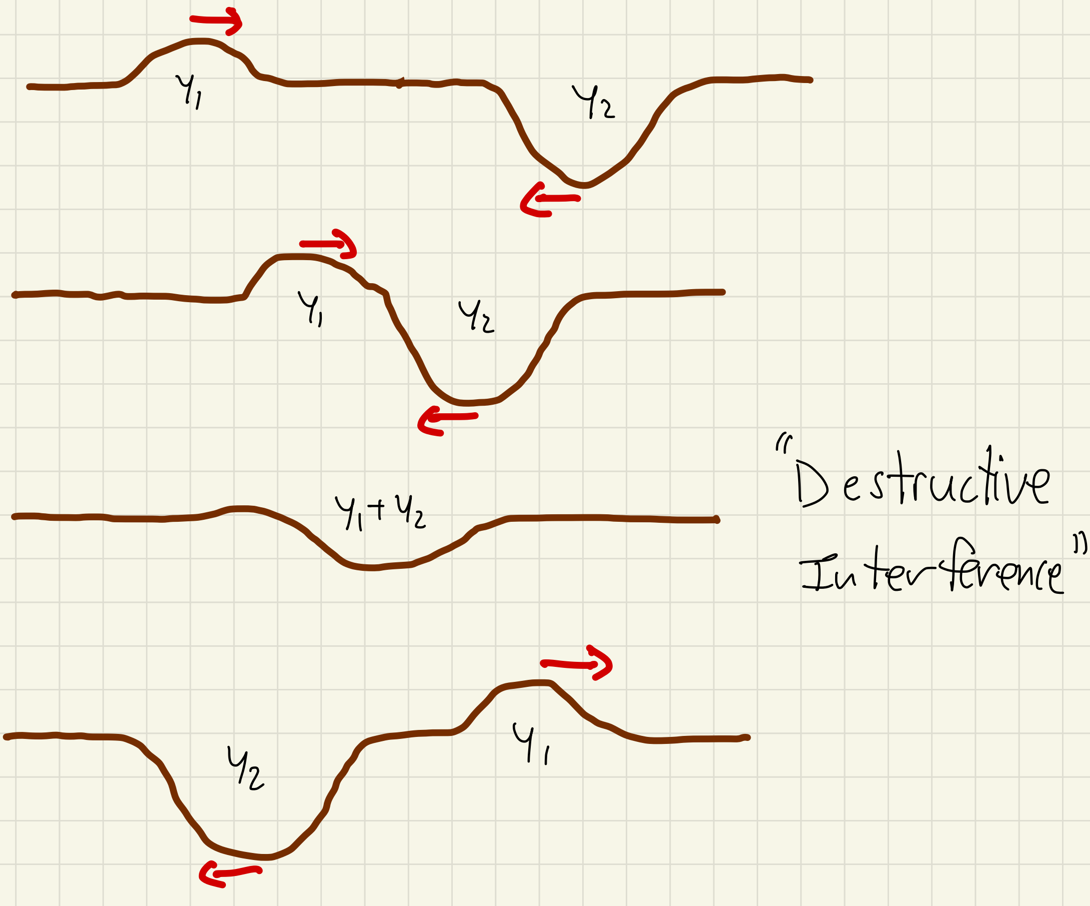
$$\vdots$$

Superposition of Pulses



"Constructive Interference"

Superposition of Pulses



Conceptual Question:

Two pulses move in opposite directions and have same shape except one has positive displacement, the other negative. At the moment the pulses overlap, what happens?

(a) The energy associated with the pulses vanishes

(b) The string is not moving

(c) The string form a straight line

(d) Pulses have vanished and will not reappear

Superposition of sinusoidal Waves

- Consider two wavefunctions

$$y_1 = A \sin(kx - \omega t)$$
$$y_2 = A \sin(kx - \omega t + \phi)$$

- Is there a 'simple' expression for the superposition?

$$y = y_1 + y_2$$

- Useful result from Trigonometry:

$$\sin(a) + \sin(b) = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

sinusoidal

Superposition of sinusoidal Waves

- $y = A\left[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)\right]$

  $\sin(a) + \sin(b)$

  $= 2A\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$

  $= 2A\cos\left(+\frac{\phi}{2}\right)\sin\left(kx-\omega t+\phi/2\right)$

- Question: why is this a 'simple' expression?

- Question: why is this a 'simple' expression?

$$y = \underbrace{\left(2A\cos\left(\frac{\phi}{2}\right)\right)}_{B} \sin\left(kx - \omega t + \phi/2\right)$$

$$= B \sin\left(kx - \omega t + \phi/2\right)$$

Properties of the superposed wave:

Amplitude: $B = 2A\cos(\phi/2)$

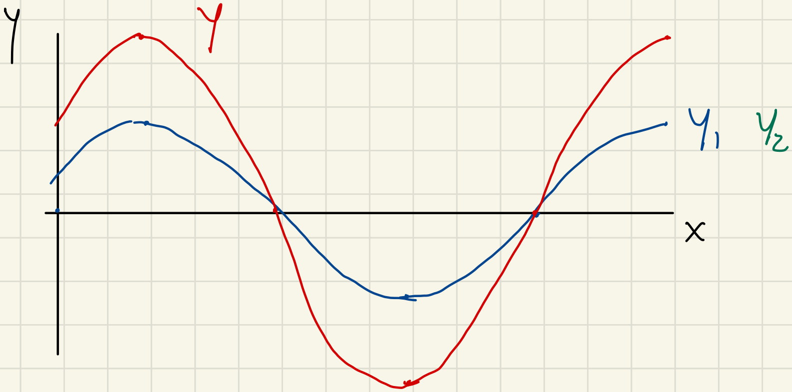
Wavelength: $\lambda = \frac{2\pi}{k}$ = unchanged

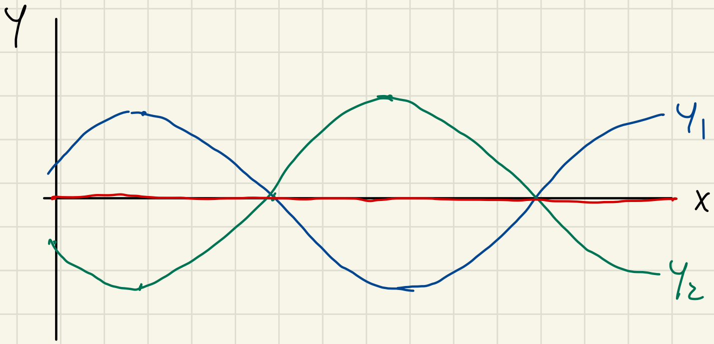
frequency: $f = \frac{\omega}{2\pi}$ = unchanged

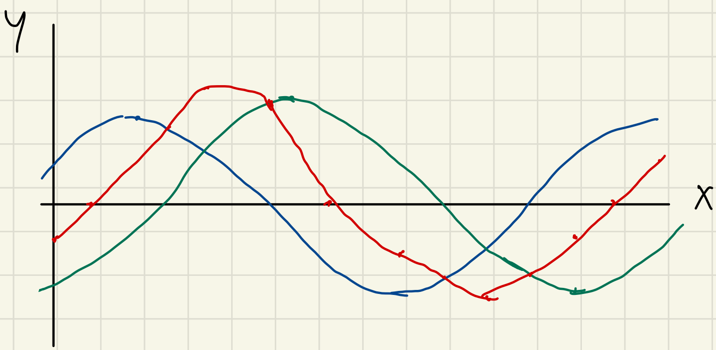
Amplitude: $B = 2A\cos(\phi/2)$

- $\phi = 0$: $B = 2A$ Constructive
- $\phi = \pi$: $B = 0$ Destructive

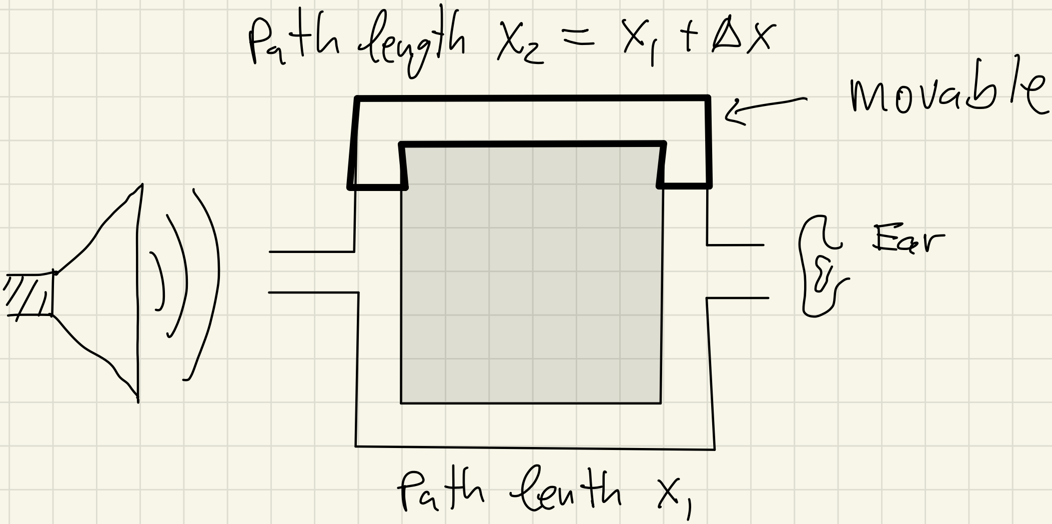
Superposition of sinusoidal Waves







Interference Experiment



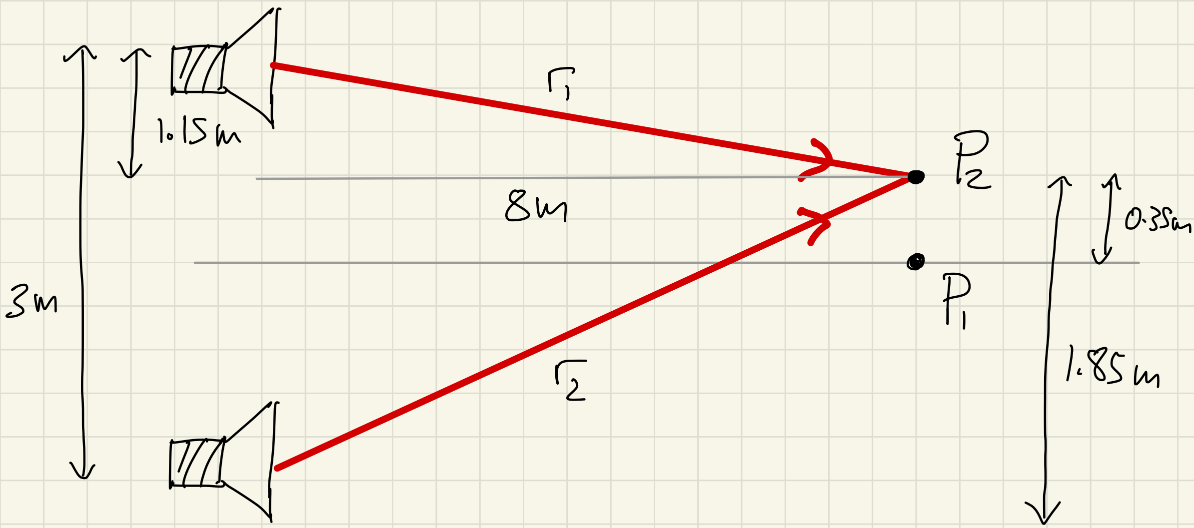
Ear hears

$$y = A\left[\sin(kx_1 - \omega t) + \sin(kx_2 - \omega t)\right]$$

$$kx_2 = kx_1 + k\Delta x = kx_1 + \phi$$

$$y = 2A\cos\left(\frac{k\Delta x}{2}\right)\sin\left(kx_1 - \omega t + \frac{k\Delta x}{2}\right)$$

Exercise



- Two speakers emitt sound waves.
- Listener originally at point P_1 hears a maximum of sound intensity
- Listener slowly moves to point P_2 and hears the first minimum.
- Find the frequency of the sound wave

Standing Waves

- Superposition of two equal waves travelling in opposite directions = standing wave

- $$y_1 = A\sin(kx - \omega t)$$
  $$y_2 = A\sin(kx + \omega t)$$

$$y = y_1 + y_2 = (2A\sin kx)\cos\omega t$$

Exercise

- Why is this a 'standing wave'?

Exercise

Standing Waves Trig.

Need:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$y = y_1 + y_2$$

$$= A\sin(kx-\omega t) + A\sin(kx+\omega t)$$

$$= A[\sin kx \cos\omega t - \cos kx \sin\omega t] +$$

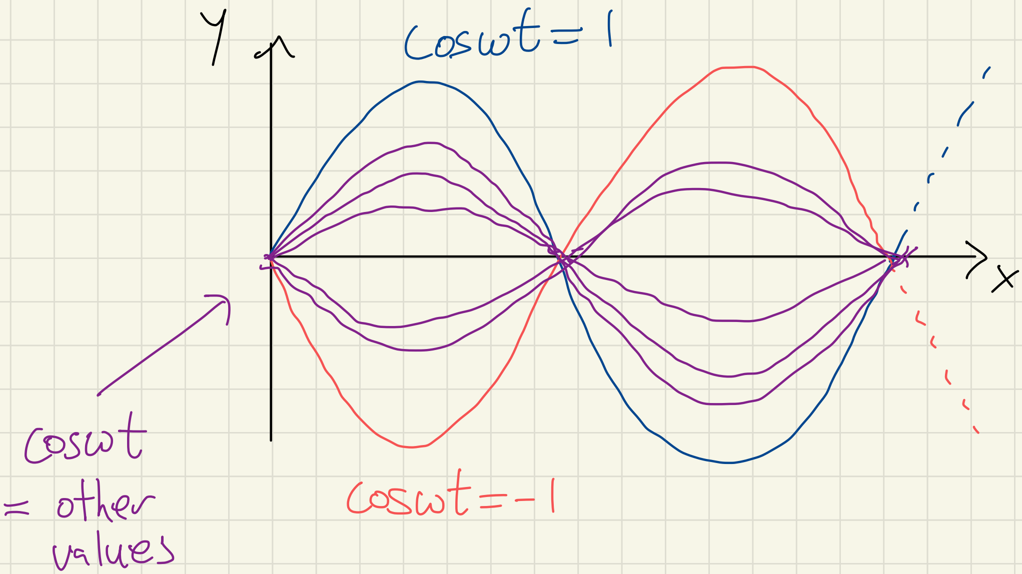
$$+ A[\sin kx \cos\omega t + \cos kx \sin\omega t]$$

$$= 2A\sin kx \cos\omega t$$

Standing Waves

- Why $y = (2A\sin kx)\cos\omega t$ doesn't travel?

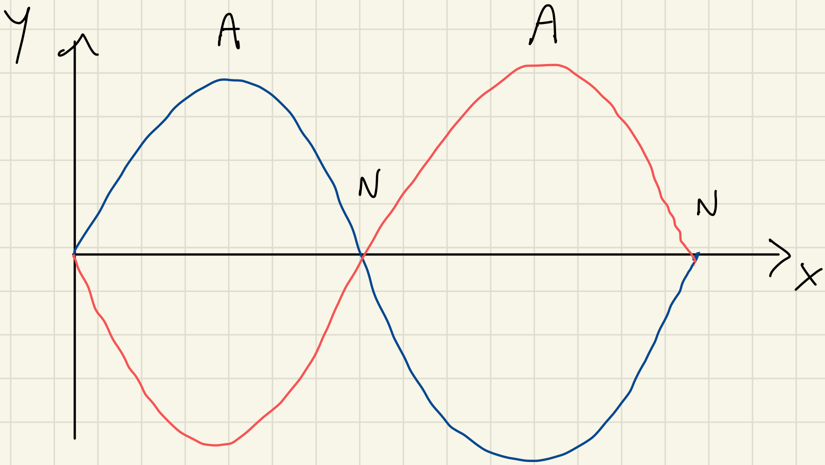
- Plot:



- Oscillates in time, but...

  Fixed spatial locations of (anti) nodes

Standing Waves



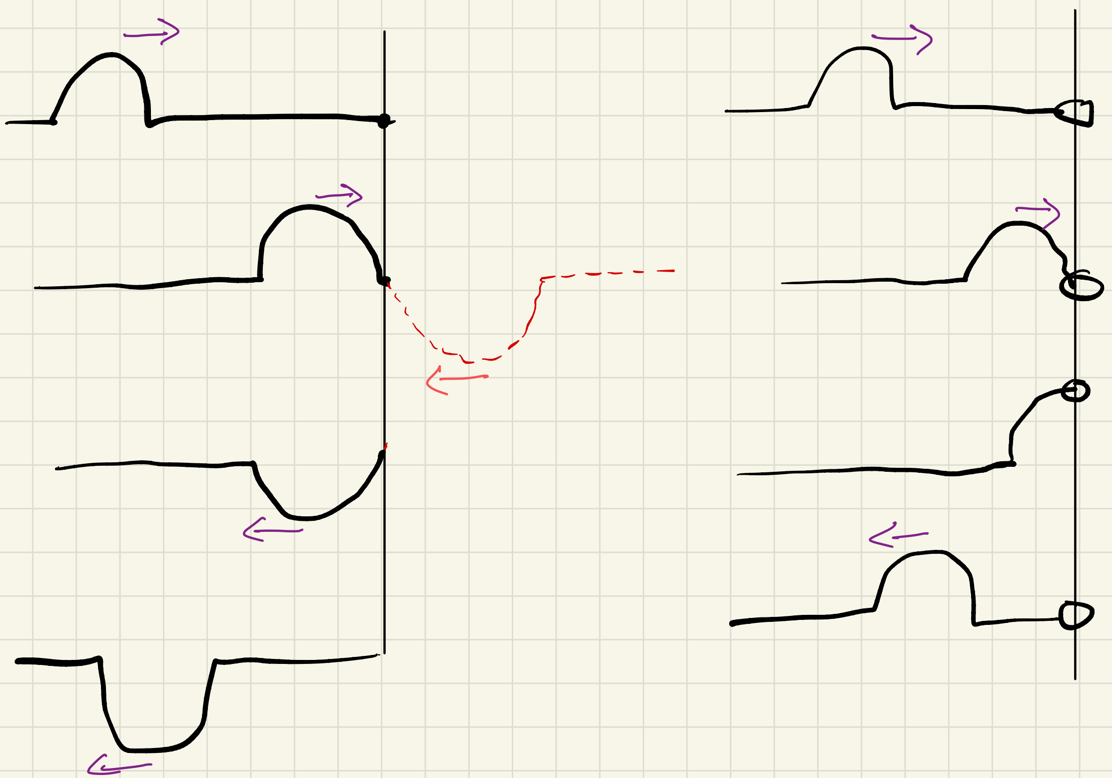
A = antinode = destructive

N = node = constructive

Boundary Conditions

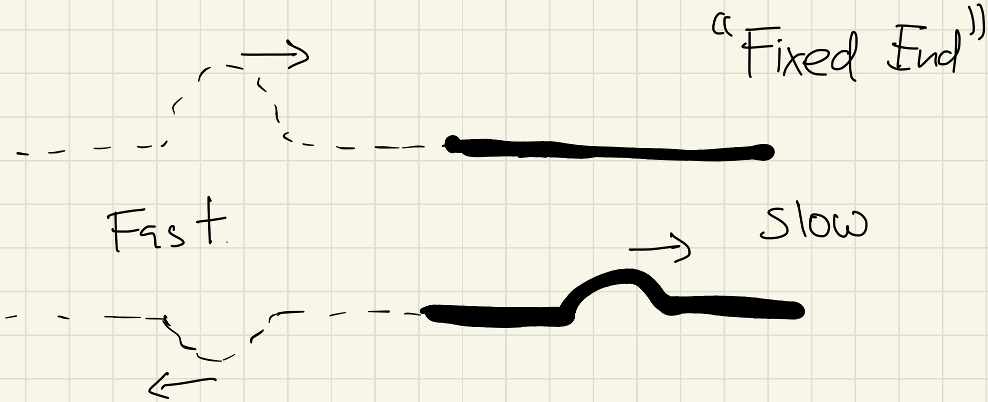
(i) Fixed End

(ii) Free End

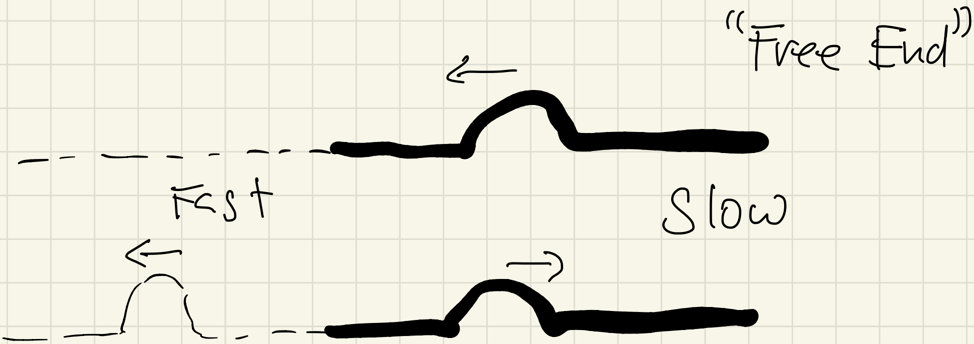


Boundary Conditions

(iii)

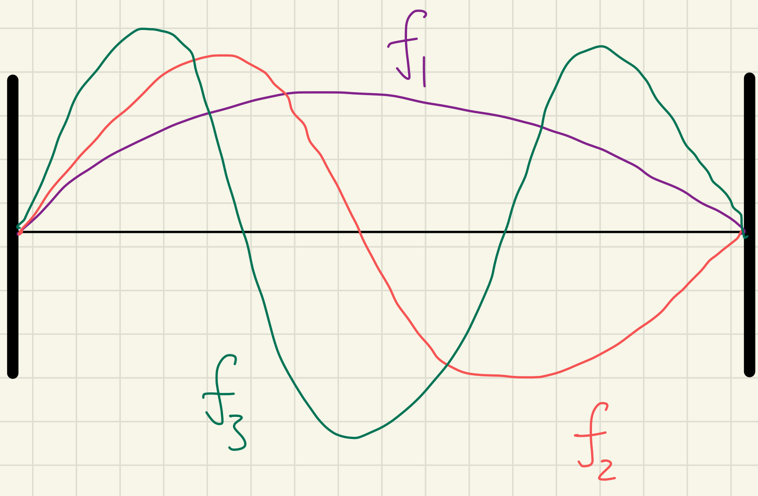


(iv)

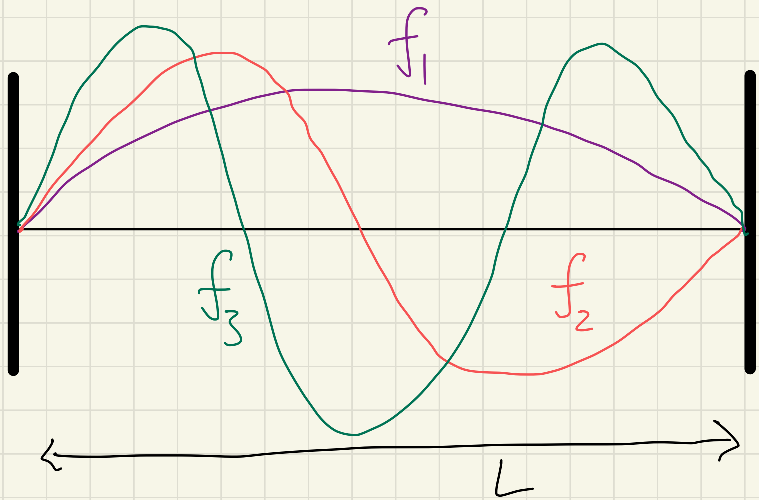


Quantized Waves

- Waves subject to fixed boundary conditions are restricted to certain 'Quantized' frequencies



Quantized Waves



Wavelengths

$\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2}{3}L$

frequencies

$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$

$f_3 = v/\lambda_3 = \frac{3v}{2L} = 3f_1$

$\vdots$

$f_n = nf_1$

Aside

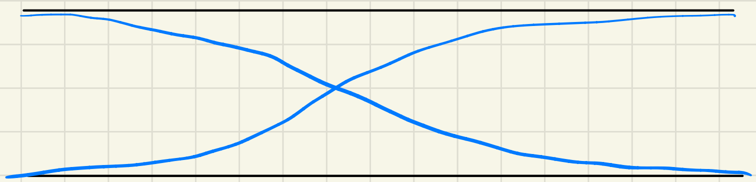
Recall

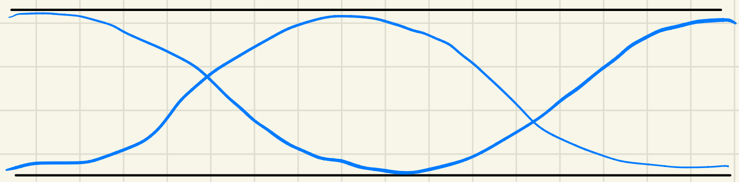
$$v = \sqrt{\frac{T}{\mu}} = \text{fixed}$$

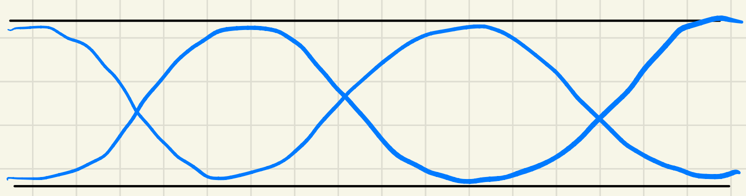
$$\therefore \quad f\lambda = v = \text{fixed}$$

More Quantized Waves

(ii) Two open ends



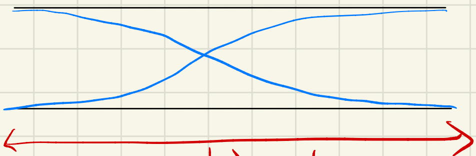




More Quantized Waves

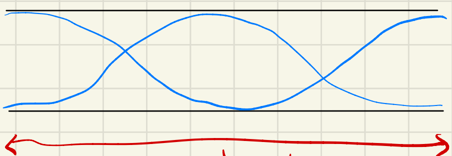
(ii) Two open ends

$n=1$



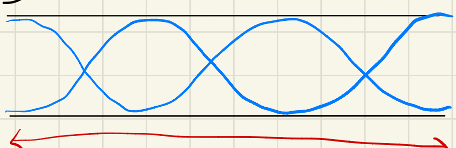
$\frac{1}{2}\lambda = L$

$n=2$



$\lambda = L$

$n=3$



$\frac{3}{2}\lambda = L$

$\vdots$

$$\frac{n}{2}\lambda_n = L$$

$$v = f_n \lambda_n = \text{constant}$$

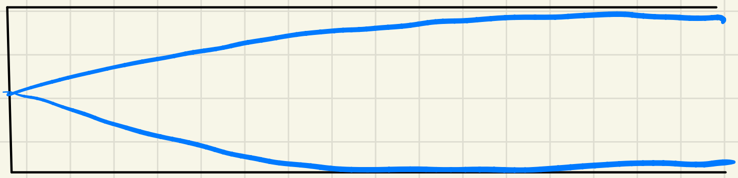
$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n}$$

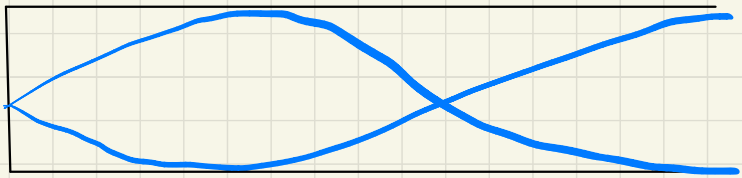
$$= n\frac{v}{2L} = n f_1$$

More Quantized Waves

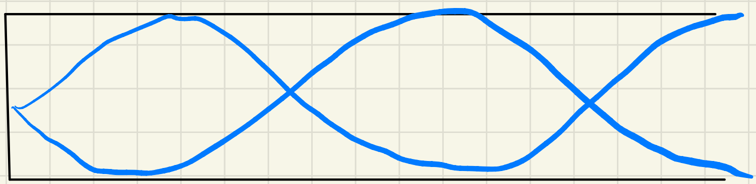
(ii) closed + open ends



$\frac{1}{4}\lambda = L$



$\frac{3}{4}\lambda = L$

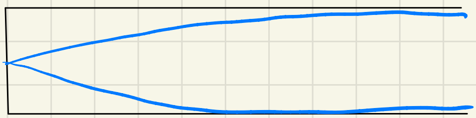


$\frac{5}{4}\lambda = L$

More Quantized Waves

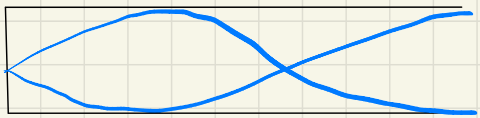
(i) closed + open ends

$m=1$



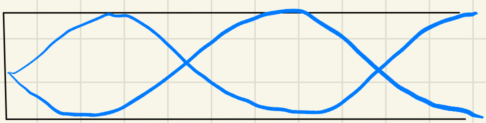
$\frac{1}{4}\lambda = L$

$m=2$



$\frac{3}{4}\lambda = L$

$m=3$



$\frac{5}{4}\lambda = L$

$\vdots$

$$\frac{n}{4}\lambda_n = L$$

$v = f_n \lambda_n$

$\lambda_n = \frac{4L}{n}$

$f_n = n\frac{v}{4L} = nf_1$

$n = 1, 3, 5$

$m = 1, 2, 3, \ldots$

$n = 1, 3, 5, \ldots$

$[n = 2m-1]$

Loose Ends

Superposition: Beats

- Add two waves with slightly different frequencies

$$y_1 = A\cos(\omega_1 t) = A\cos(2\pi f_1 t)$$
$$y_2 = A\cos(\omega_2 t) = A\cos(2\pi f_2 t)$$

$$y = y_1 + y_2 = \dots$$
$$= 2A\cos\left(2\pi \frac{f_1 - f_2}{2} t\right)\cdot \cos\left(2\pi \frac{f_1+f_2}{2} t\right)$$

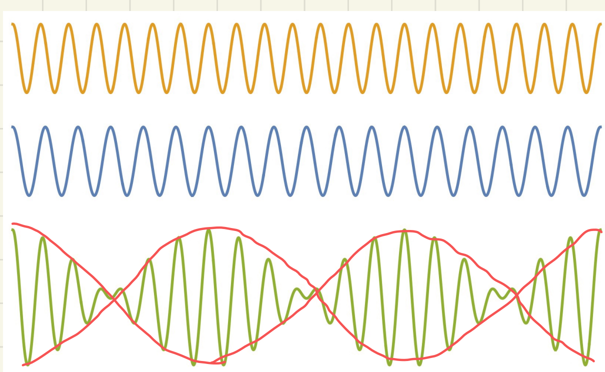
f_{beat}

slow 'envelope' wave

f_1

f_2

$f_{beat} = f_1 - f_2$



Example — Beat frequency

- Two identical piano strings of length 0.75 m are each tuned to 440 Hz. The tension in one of the strings is then increased by 1%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

Superposition — Validity

Properties

- $y = y_1 + y_2$
- Waves don't interact

Hold true only for SHM

- $m\ddot{y} = -ky$
- $\frac{1}{2}m\dot{y}^2 + \frac{1}{2}ky^2 = \text{constant}$

Examples from Nature:

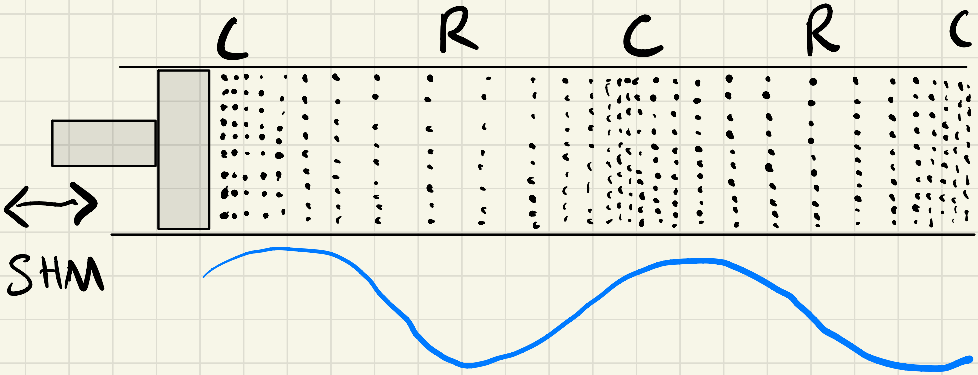
- Electromagnetic waves
- sound waves

Counter-examples from Nature:

- Gravitational waves
- "Gluons" (strong nuclear force)

Sound Waves

(i)



Sound in fluids = pressure fluctuations

Sound Waves

(ii)

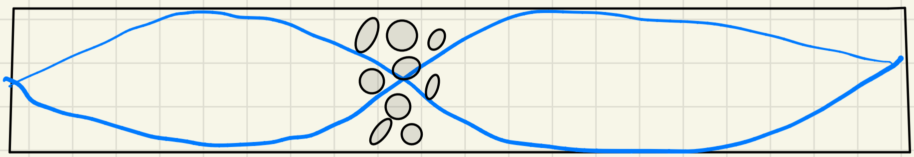
$$y(x,t) = A\cos(kx - \omega t + \phi)$$

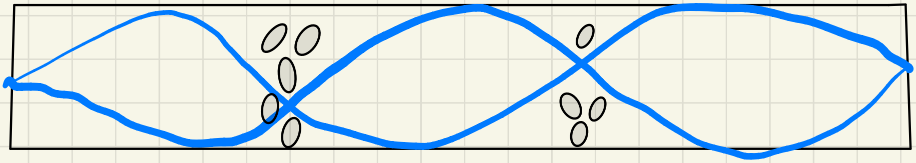
$\longrightarrow$

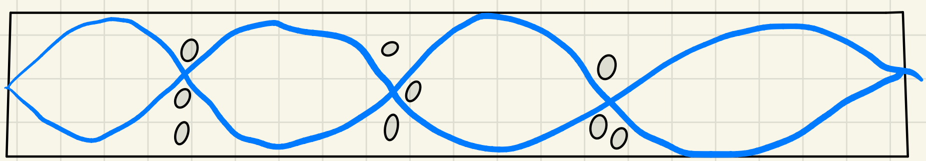
$y(x,t)$ compresses along x

$\boxed{\text{Sound waves are longitudonal}}$

Demonstration: Kundt's Tube

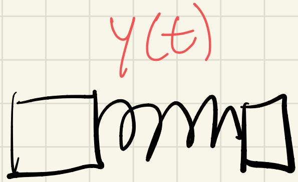




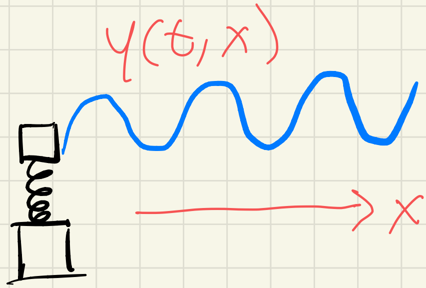


Multidimensional Waves

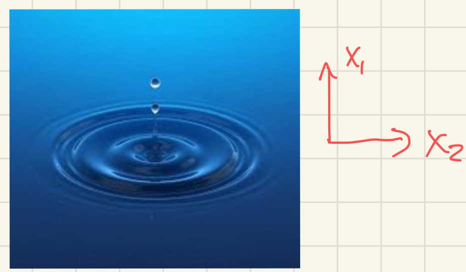
0D



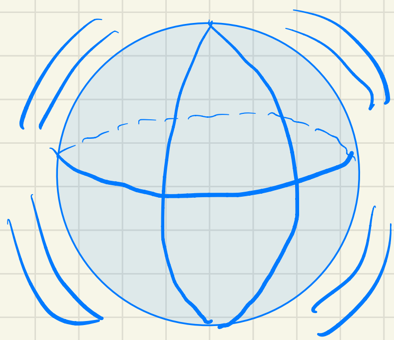
1D



2D



3D



Intensity of Multidimensional Waves

- $y(x,t) = A \sin(kx - \omega t + \phi)$
- $\langle \text{Power} \rangle \sim \int_0^T dt \, [y(x,t)]^2 \frac{1}{T}$

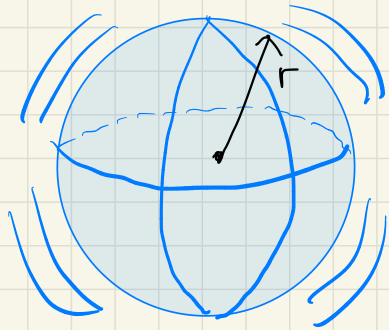
Not in test/exam

- $\text{Intensity} = \frac{\text{Power}}{\text{Area}}$

$$I = \frac{P}{A}$$

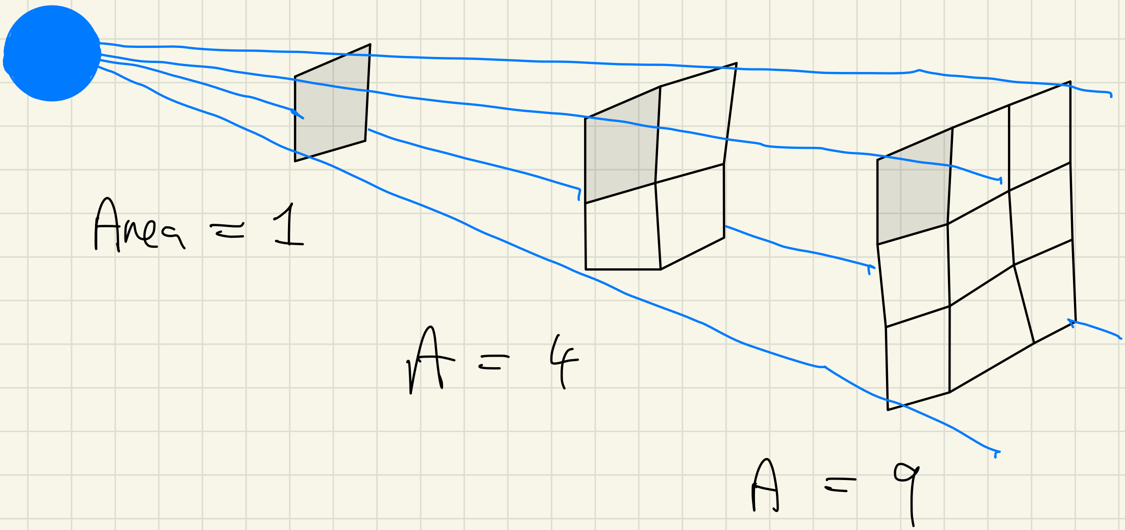
Inverse-Square Law

3D



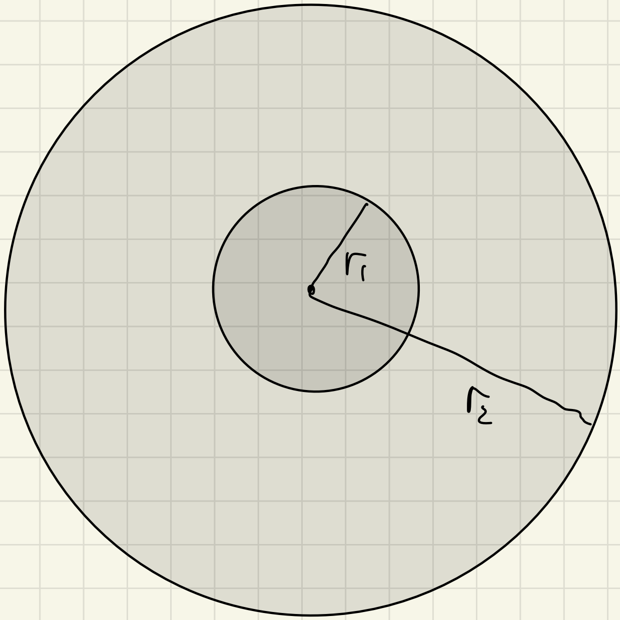
$$I_{3D} = \frac{P}{4\pi r^2}$$

$$I_{2D} = \frac{P}{2\pi r}$$



Inverse-Square Law

$P = \text{fixed}$



$$I_1 = \frac{P}{4\pi r_1^2}$$

$$I_2 = \frac{P}{4\pi r_2^2}$$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

Example

- Source of sound radiates uniformly in all (3D) directions at

  * $P = 1.5\ W$ (Energy/time)

(a) Find intensity at a point $r_0 = 25m$ from the source

(b) If we halve the power and distance, by what ratio does the intensity change relative to part (a)?