Fluid Mechanics

Fluid = liquids and gases

characteristics : ρ for density (rho)

P for pressure

Fluid Statics

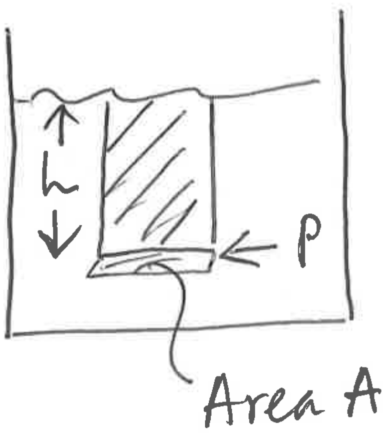
Density

$$\underset{(m^3)}{\frac{\overset{(kg)}{m}}{V}} = \underset{(kg/m^3)}{\rho}$$

$\rho(H_2O) = 1000\ kg/m^3$

$\rho(\text{air}) = 1.3\ kg/m^3$

Pressure



P is pressure at depth h

$$P = \frac{\overbrace{\text{Force } F}^{\text{weight}}}{\text{Area } A} = \frac{m \cdot g}{A}$$

$$\therefore\ P = \frac{V \cdot \rho \cdot g}{A} \quad \text{but } \frac{V}{A} = h$$

$$\boxed{P = \rho \cdot g \cdot h}$$

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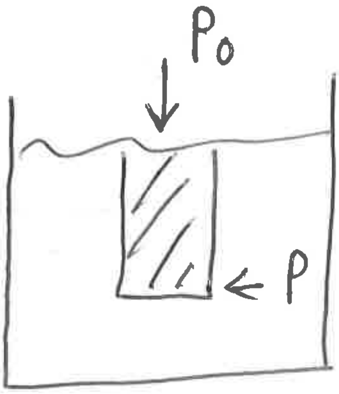
depth (m)

Pressure (Pa) $\quad$ kg/m^3 $\quad$ $9.8\,m/s^2$

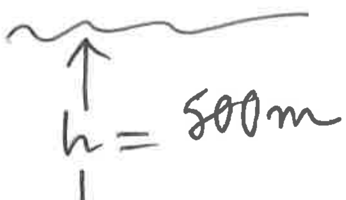
This is gauge pressure of the fluid



P_0 is pressure at surface e.g. atmospheric P_{atm}

Total $\boxed{P = P_0 + \rho g h}$

Example 1



$$P = P_0 + \rho(\text{water}) \times g \times h$$

1.01×10^5 Pa $\quad$ $1000\,kg/m^3$ $\quad$ $9.8\,m/s^2$ $\quad$ 500m

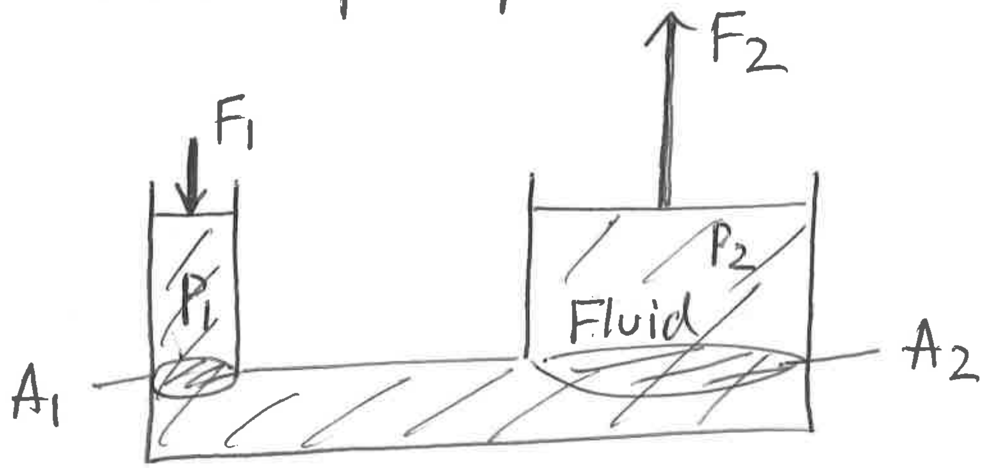
$\therefore\ P = 5.0 \times 10^6$ Pa

$$F = P \times \text{Area (window)}$$

5.0×10^6 Pa $\quad$ $\pi R^2 = \pi (17 \times 10^{-2} m)^2$

$\therefore\ F = 4.5 \times 10^5$ N

Pascal's principle



Hydraulic system

Applied pressure $P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore \boxed{F_2 = \left(\frac{A_2}{A_1}\right) \cdot F_1}$$

Because $A_2 > A_1$, $F_2 > F_1$

Force is amplified.

Example 2

(a) $F_2 = \left(\frac{A_2}{A_1}\right) \cdot F_1$

(b) $F_2 = \text{weight} = mg$

$$F_1 = \left(\frac{A_1}{A_2}\right) \cdot F_2 = \frac{\pi r_1^2}{\pi r_2^2} \cdot F_2 = \frac{r_1^2}{r_2^2} \cdot F_2$$

0.05^2 0.2^2 1000×9.8

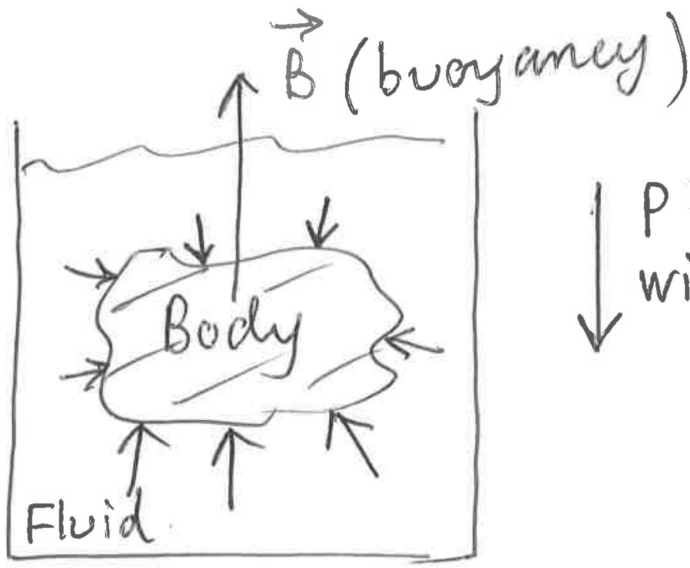
$\therefore F_1 = 610N$

mmHg : pressure unit

Blood presure 80 mmHg / 120 mmHg

Atmospheric $P = 760$ mmHg

Buoyancy



P increases with depth

Upward force = buoyancy B

$$B = \rho_{fluid} \times g \times V_{object}$$

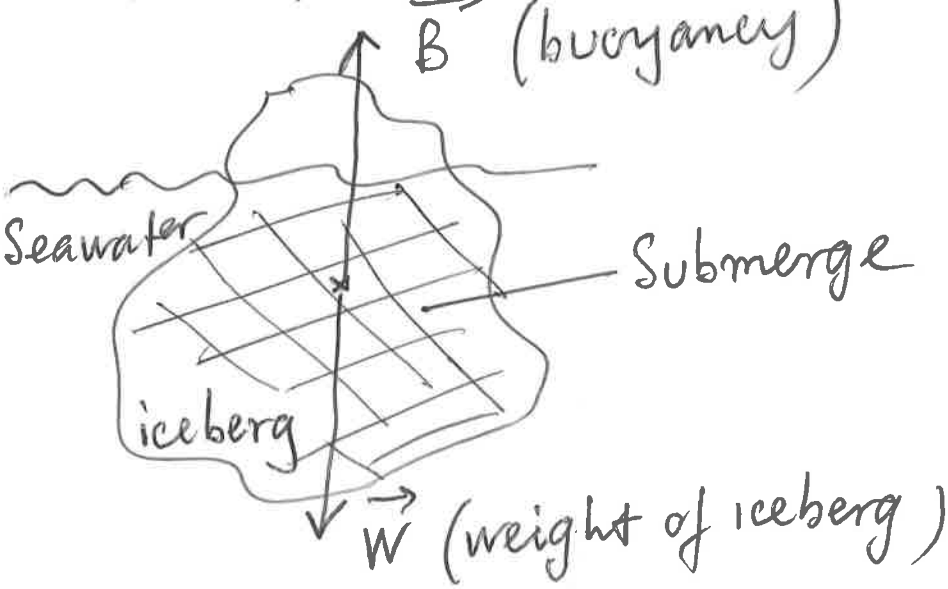
B: force (N)

ρ_{fluid}: density of fluid (kg/m^3)

g: 9.8 m/s^2

V_{object}: volume (m^3)

Example 3



$$\vec{B} + \vec{W} = 0$$

Magnitude $\quad B = W$

$$\rho_{water} \times \cancel{g} \times V_{submerge} = \rho_{iceberg} \times \cancel{g} \times V_{iceberg}$$

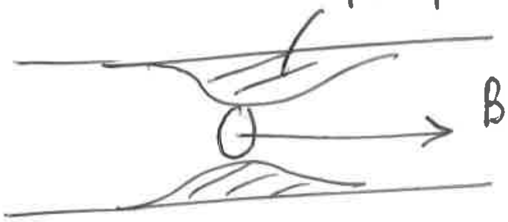
$$\frac{V_{submerge}}{V_{iceberg}} = \frac{\rho_{iceberg}}{\rho_{water}}$$

$\rho_{iceberg}$ — $920 kg/m^3$

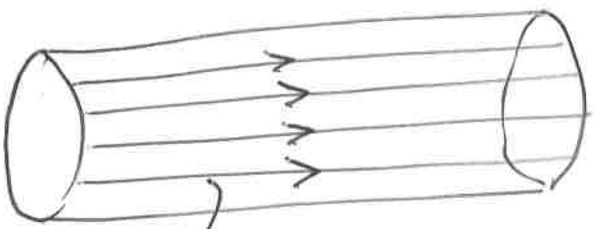
ρ_{water} — $1025 kg/m^3$

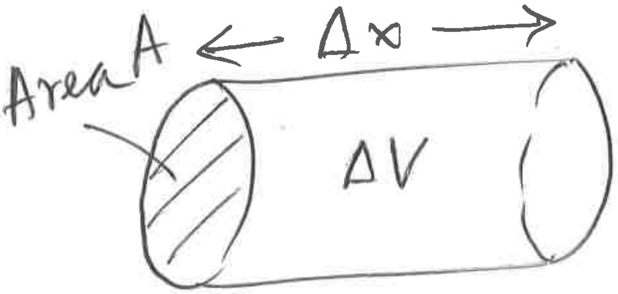
$$\therefore \frac{V_{submerge}}{V_{iceberg}} = 0.9 \text{ or } 90\%$$

Fluid dynamics



Laminar flow : flow lines are straight and parallel





Over time interval Δt

Δx : fluid displacement

volume of fluid (m^3)

Flow rate $Q = \dfrac{\Delta V}{\Delta t}$ (second)

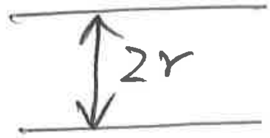
m^3/s

$$\therefore\ Q = \frac{A \cdot \Delta x}{\Delta t} \Rightarrow \boxed{Q = A \cdot v}$$

cross-sectional area (m^2)

velocity of fluid (m/s)

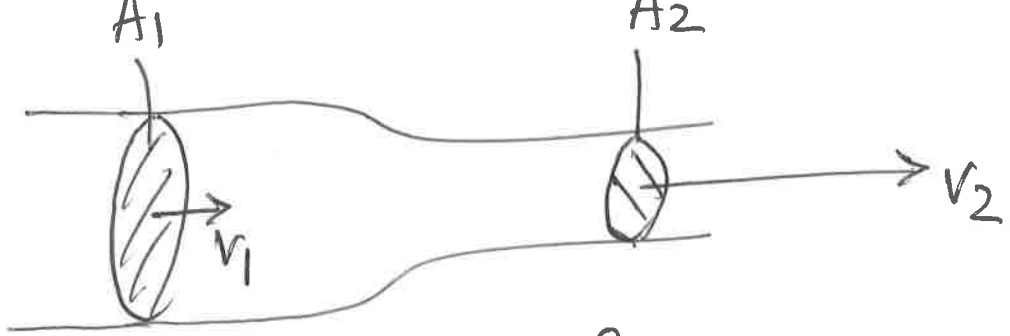
Example 4



$$Q = A \cdot v = \pi r^2 \cdot v$$

$(1.5 \times 10^{-2}\,m)$ $1.0\,m/s$

$$\therefore \quad Q = 7.1 \times 10^{-4}\,m^3/s$$

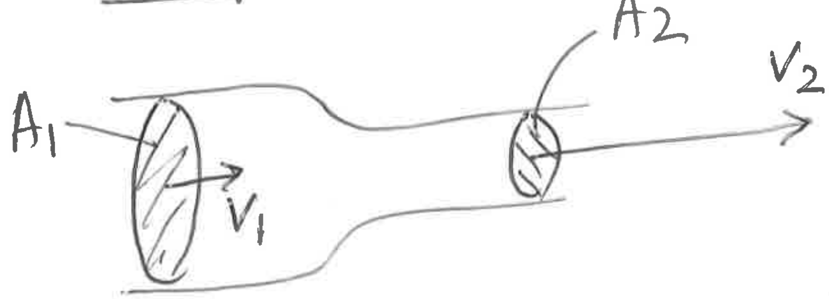


If no leak, $Q_1 = Q_2$

$A_1 \cdot v_1 = A_2 \cdot v_2$ ← continuity equation

Since $A_1 > A_2$, $v_1 < v_2$

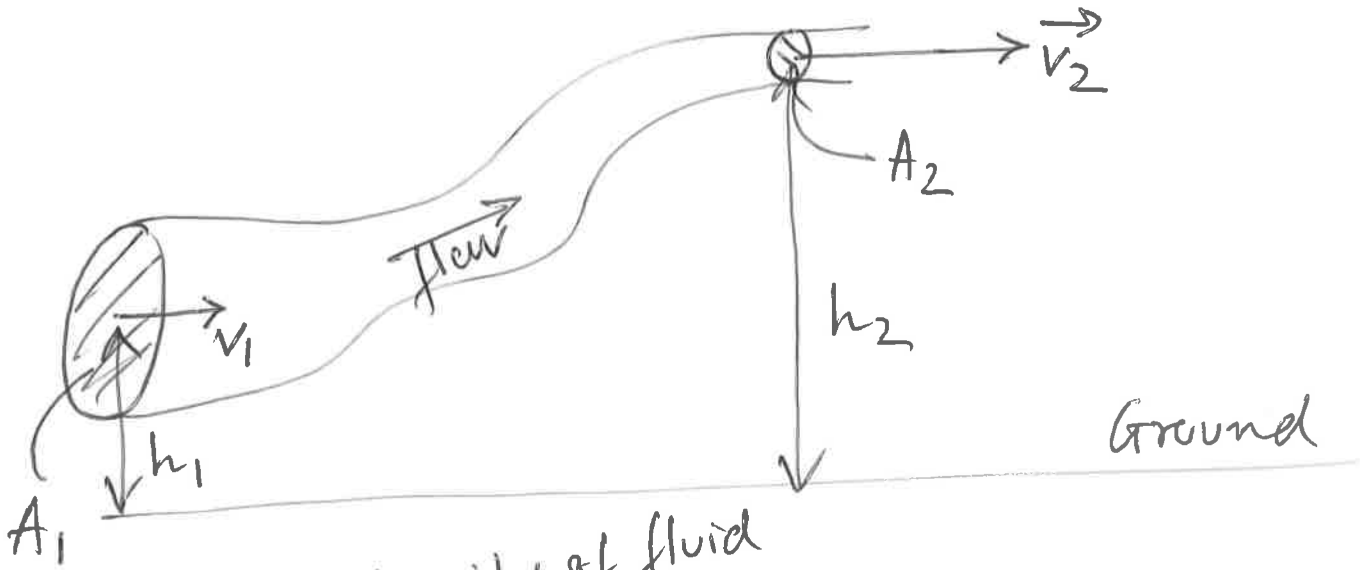
Example 5



$$A_1 \cdot v_1 = A_2 \cdot v_2 \quad \therefore \quad v_2 = \frac{A_1}{A_2} \cdot v_1 = \frac{0.11}{0.07} \times 1.5$$

$$\therefore \quad v_2 = 2.4\,m/s$$

Bernoulli's principle : total energy of fluid system is unchanged



density of fluid

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

fluid pressure (Pa)

kinetic energy per unit volume ($J/m^3 = Pa$)

potential energy per unit volume (Pa)

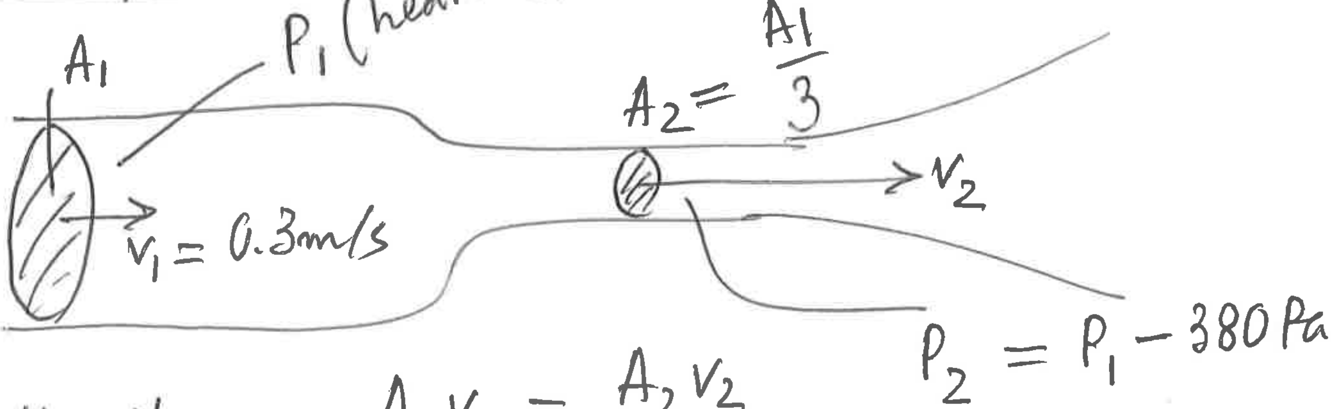
$\frac{1}{2}mv^2 = KE$
↑
(J)

$\frac{1}{2}\rho v^2$ ← unit Pa

Energy density

$1\ J/m^3 = 1\ Pa$

Example 6



Continuity equation : $A_1 v_1 = A_2 v_2$

$$v_2 = \underbrace{\left(\frac{A_1}{A_2}\right)}_{3} \times \underbrace{v_1}_{0.3m/s}$$

$\therefore \; v_2 = 0.9 m/s$

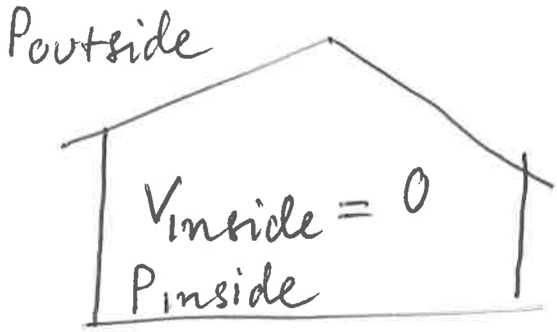
$$P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g h_1} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2}$$

Assume $h_1 = h_2$ i.e. flow is horizontal

$$P_1 - P_2 = \frac{1}{2}\underbrace{\rho}_{1050 kg/m^3} (\underbrace{v_2^2}_{0.9m/s} - \underbrace{v_1^2}_{0.3m/s})$$

$\therefore \; P_1 - P_2 = 380 Pa$

Example 7



$$v_{outside} = 150\,km/h$$

$$KE_{inside} < KE_{outside}$$

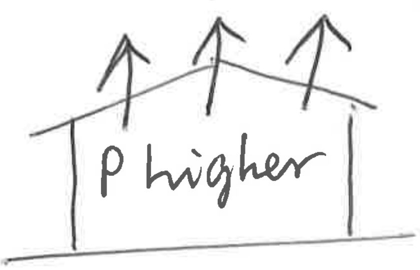
$$P_{inside} > P_{outside}$$

$$P_{inside} - P_{outside} = \frac{1}{2}\underbrace{\rho_{air}}_{1.3\,kg/m^3}\Big(\underbrace{v_{outside}^2}_{150\,km/h} - \underbrace{v_{inside}^2}_{0}\Big)$$

$$150\,km/h = \frac{150 \times 1000\,m}{3600\,s}$$

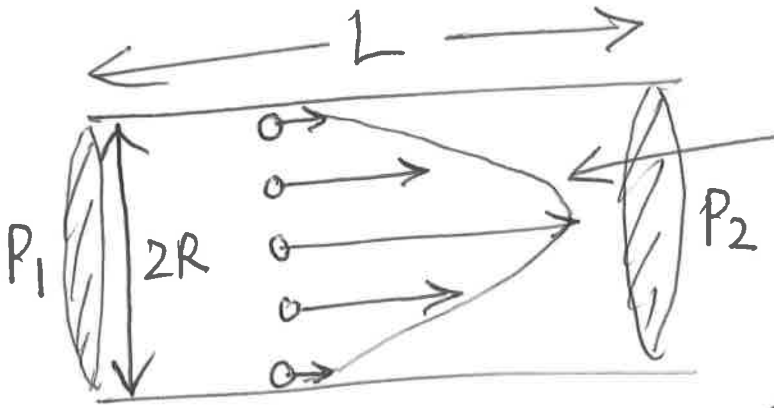
$$\therefore \underbrace{P_{inside} - P_{outside}}_{\text{Dynamic pressure due to fluid motion}} = 1130\,Pa$$

$$Force = \underbrace{P_{dynamic}}_{1130\,Pa} \times \underbrace{A}_{200\,m^2}$$



Viscosity (next week's lab)

Friction in fluid



Velocity profile of viscous fluid

Poiseuille's law

$$Q = \underset{\text{pi}}{\frac{\pi}{8}} \cdot \frac{\overset{\text{radius (m)}}{R^4}\,\overbrace{(P_1 - P_2)}}{\eta \cdot L}$$

$P_1 - P_2$: is pressure difference

Flow rate (m^3/s)

length (m)

Viscosity coefficient (Pa.s)

Example 8

$$Q = \frac{\pi}{8} \cdot \frac{R^4 (P_1 - P_2)}{\eta\ L}$$

$R = 2.5 \times 10^{-3}$ m

$P_1 - P_2 = 420$ Pa

$L = 8.2 \times 10^{-2}$ m

η of blood $= 2.7 \times 10^{-3}$ Pa.s

$\therefore\ Q = 2.9 \times 10^{-5}\ m^3/s$

Key points:

$Q \propto R^4$

$Q \propto \frac{1}{L}$