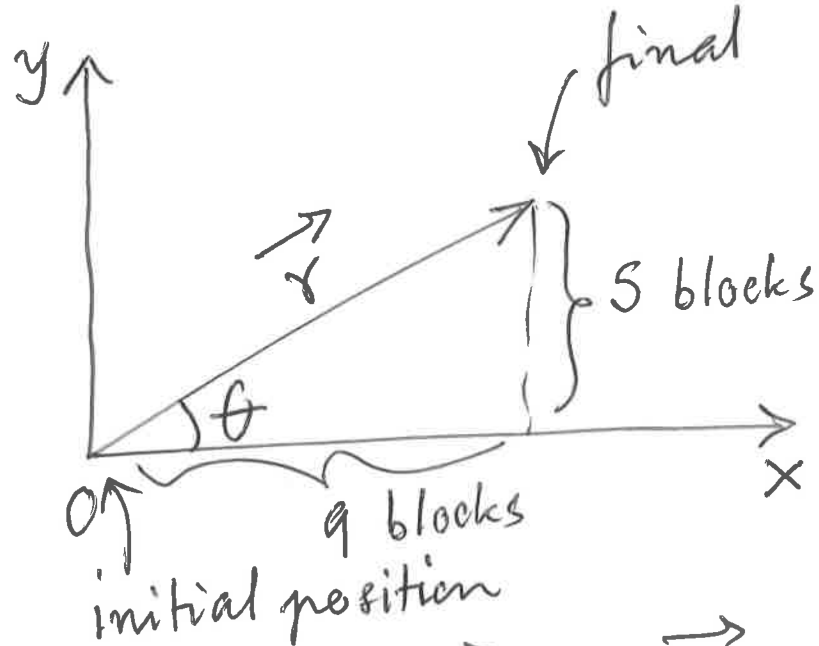
Mechanics 2

Example 1



$$\vec{r} = x\vec{i} + y\vec{j} = 9\vec{i} + 5\vec{j}$$

unit vectors

$$|\vec{r}| = \sqrt{x^2 + y^2} = \sqrt{9^2 + 5^2} = 10.3$$

magnitude of vector $\vec{r}$

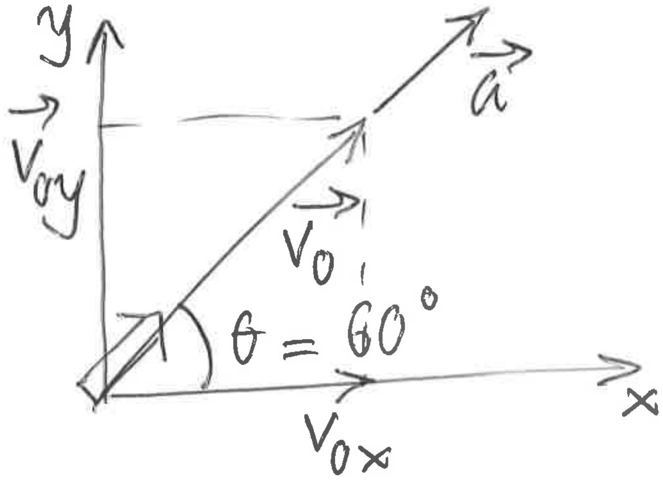
$$\tan\theta = \frac{y}{x} = \frac{5}{9} \quad \therefore \theta = \tan^{-1}\left(\frac{5}{9}\right)$$

$$\therefore \theta = 29.1^\circ$$

Final position : – 10.3 blocks away from the starting point

– 29.1° north of east

Example 2



$$\vec{v}_0 = \vec{v}_{0x} + \vec{v}_{0y}$$

$$v_{0x} = |\vec{v}_0| \cos\theta = 3500 \times \underbrace{\cos 60°}_{= 1/2}$$

$$v_{0y} = |\vec{v}_0| \sin\theta = 3500 \times \underbrace{\sin 60°}_{= \frac{\sqrt{3}}{2}}$$

$$\therefore\ v_{0x} = 1750\,\mathrm{m/s}$$

$$v_{0y} = 1750\sqrt{3}\,\mathrm{m/s}$$

$$\vec{v}_0 = 1750\,\vec{i} + 1750\sqrt{3}\,\vec{j}$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j}$$

$$= |\vec{a}| \cos 60°\, \vec{i} + |\vec{a}| \sin 60°\, \vec{j}$$

$$= 20 \times \frac{1}{2}\,\vec{i} + 20 \times \frac{\sqrt{3}}{2}\,\vec{j}$$

$$\therefore \vec{a} = 10\vec{i} + 10\sqrt{3}\,\vec{j}$$

constant acceleration

Previous lecture : $v = v_0 + at$

final velocity

initial velocity

This lecture :

$$\vec{v} = \vec{v_0} + \vec{a}t$$

$$= (1750\vec{i} + 1750\sqrt{3}\,\vec{j}) + (10\vec{i} + 10\sqrt{3}\,\vec{j}) \times 15$$

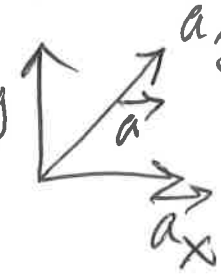
$$\therefore \vec{v} = 1900\vec{i} + 1900\sqrt{3}\,\vec{j}$$

Final velocity vector

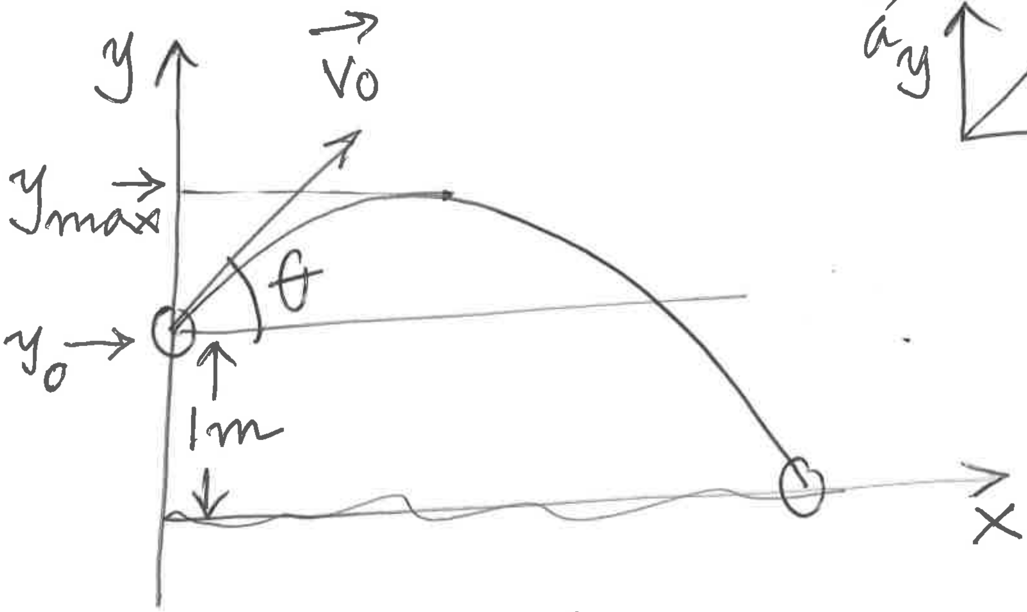
Example 3

$$|\vec{a_y}| = 9.8\,m/s^2$$

$$a_y = -g$$



$$a_x = 0$$



Along y axis

$$v_y = v_{0y} + at$$

v_{0y}: $v_0 \sin\theta$

a: $-9.8\,m/s^2$

At max height : $v_y = 0$

$$0 = v_0 \sin\theta - 9.8t$$

$$\therefore\ t = \frac{v_0 \sin\theta}{9.8} = \frac{36.5 \sin 30^\circ}{9.8} = 1.86s$$

For displacement

$$\underbrace{y_{max} - y_0}_{\text{Vertical displacement}} = v_{0y}t + \frac{1}{2}at^2$$

$$y_{max} - y_0 = v_{0y} t + \frac{1}{2} a t^2$$

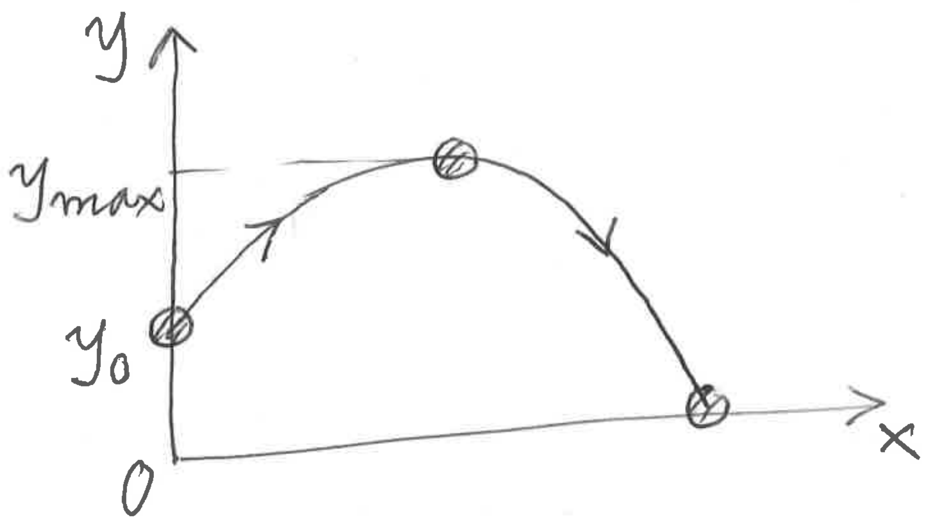
y_0: 1.0m (initial position)

v_{0y}: $v_0 \sin\theta$ ← $36.5 \sin 30^{\circ}$

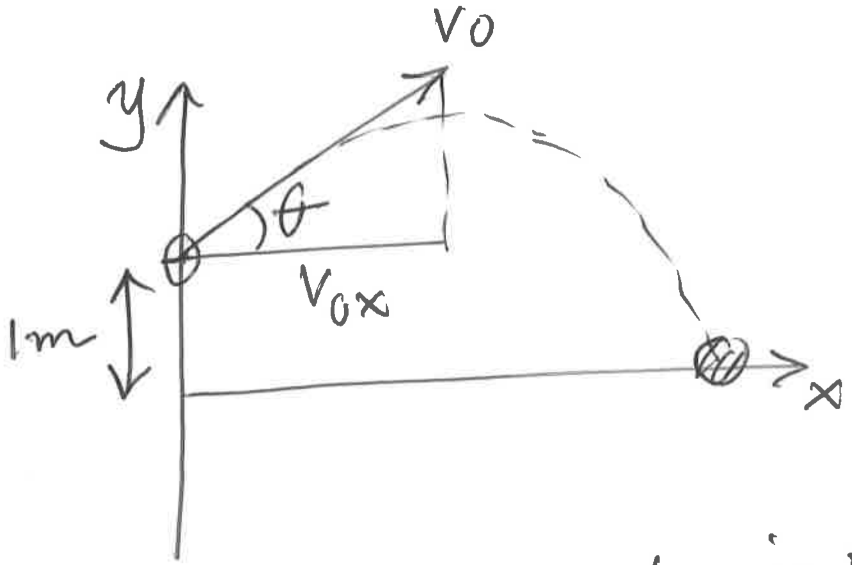
a: $-9.8 m/s^2$

t: 1.86s

$$\therefore y_{max} = 18.0m$$



Example 4



Break velocity vector into x and y components

Along x-axis

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$v_{0x} = v_0\cos\theta$, $a_x = 0$

$$\therefore\ x - x_0 = v_0\cos\theta\, t$$

Along y-axis

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$y_0 = 1.0m$ (initial position)

$v_{0y} = v_0\sin\theta$

$a_y = -9.8m/s^2$

$$y - y_0 = v_0 \sin\theta\, t - 4.9t^2$$

0m (Hits the ground) 1.0m 22m/s $\sin 30^\circ = \frac{1}{2}$

$$\therefore\ 4.9t^2 - 11t - 1 = 0$$

Solve this quadratic equation

$t = 2.3s$ (Other solution is negative)

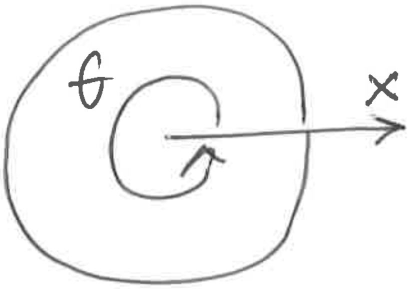
$$x = x_0 + v_0 \cos\theta\, t$$

0 22 $\cos 30^\circ$ 2.3s

$$\therefore\ x = 44m$$

Key point: treat the motion on x- and y-axis <u>separately</u>!

SI unit for angle : radian (rad)



In a full circle

$\theta = 360^\circ$ ← Not SI unit

$\theta = 2\pi$ rad

$$\therefore 1 \text{ rad} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$$

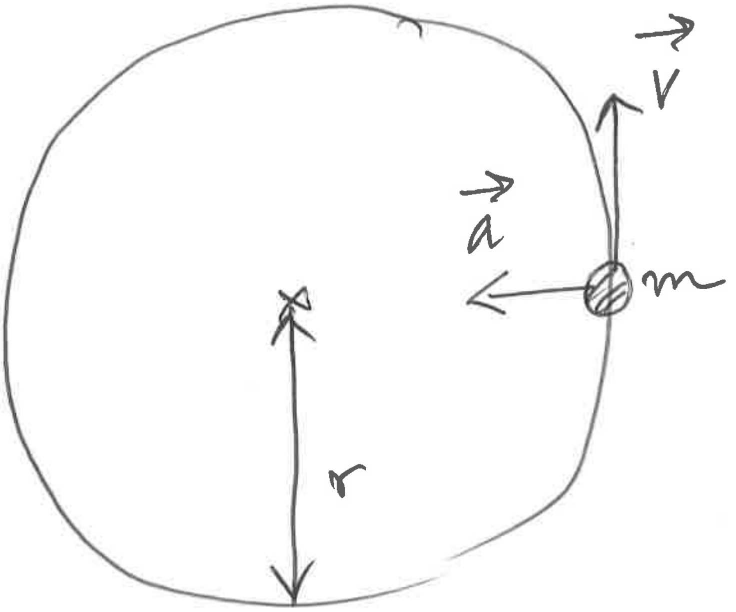
velocity

$$a = \frac{v^2}{r}$$

acceleration

radius of circle

Example 5



$F_c = 250\,F_{gravity}$

centripetal

g-force expressed in units of gravity

$F_c = 250mg$

$ma = 250mg$

$\therefore a = 250g$

$\frac{v^2}{r}$

$\therefore v = \sqrt{250g \times r}$

g: $9.8 m/s^2$, r: $0.15m$

$\therefore v = 19.2 m/s$

$$f = \frac{1}{T}$$

period (second)

frequency

Unit: $s^{-1} \equiv Hz$

rotation angle (rad)

$$w = \frac{\theta}{t}$$

time (sec)

angular velocity (rad/s)

$$v = w \cdot r$$

radius (m)

linear velocity (m/s)

angular velocity (rad/s)

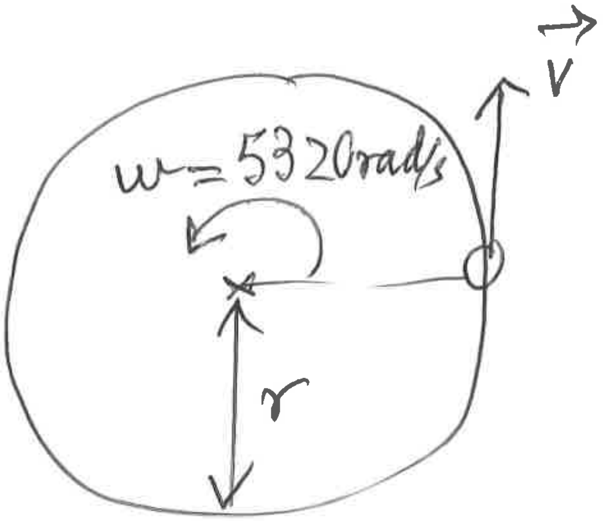
change in angular velocity

$$\alpha = \frac{\Delta w}{\Delta t}$$

time interval

angular acceleration

Example 6



$$a = \frac{v^2}{r}$$

But $v = w . r$

angular velocity

$$\therefore a = \frac{v^2}{r} = \frac{w^2 r^2}{r}$$

$$\therefore \boxed{a = w^2 r}$$

radius

angular velocity

centripetal accleration

$$a = w^2 r$$

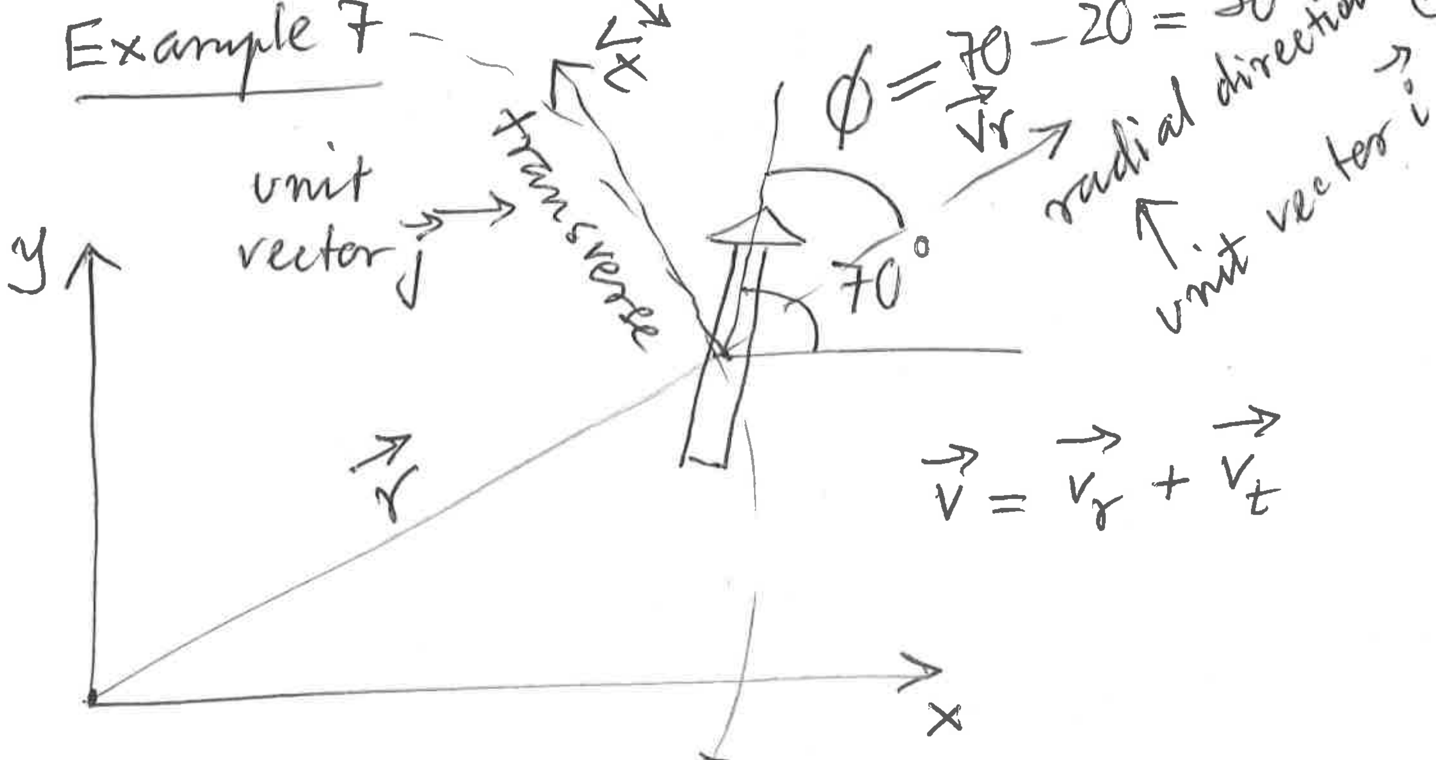
9×10^{-2} m

5320 rad/s

$$\therefore a = 2.5 \times 10^{6} \ m/s^2$$

Example 7

$\phi = 70 - 20 = 50^\circ$



$\vec{v} = \vec{v}_r + \vec{v}_t$

$$\vec{v} = v_r \vec{i} + v_t \vec{j}$$

radial dir. — transverse direction

$$v_r = v\cos\phi = 400\cos 50^\circ = 257\text{ m/s}$$

$$v_t = v\sin\phi = 400\sin 50^\circ = 306\text{ m/s}$$

$\vec{v}_t$ is perpendicular to position vector $\vec{r}$ along a circular path

$$v_t = w \cdot r$$

v_t: 306 m/s — w: angular velocity — r: 3600 m

$$\therefore\ w = 0.085\text{ rad/s} = 4.9^\circ/\text{s}$$