Current, Resistance and Electromotive Force

These lectures slides were prepared by

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CONDUCTOR

In ordinary metal e.g. copper, some electrons free to move, and do so randomly ($v > 10^6\ ms^{-1}$) having collisions with atoms.

They are attracted to the positive ions of the material therefore do not escape meaning there is no ‘net flow’ of electrons.

APPLY A STEADY ELECTRIC FIELD INSIDE THE CONDUCTOR

Charged particles are then subject to a steady force

$$\vec{F} = q\vec{E}$$

There are still collisions with slight random direction changes but together with the field there is a slow net motion in the direction of the field.

Drift Velocity, $\vec{v}_d$

Term for the net motion of the particles

Random average motion ~ 10^6 ms^{-1}

Drift velocity ~ 10^{-4} ms^{-1}

This field is set up in the wire with the speed of light .

The electron move ~ at the same time even though slow therefore instantaneously turned ‘on’ regardless’ of the speed.

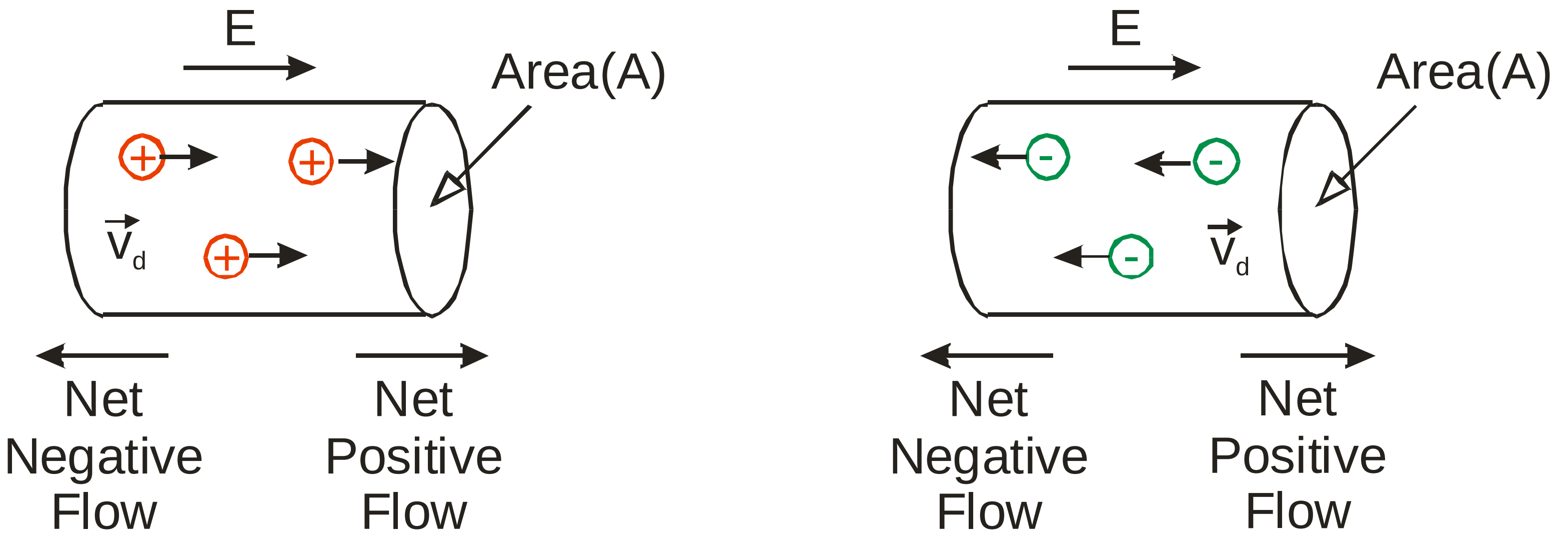
Work and Energy : Field and Current

- $\vec{E}$ does work on the moving charges.
- Moving charges have kinetic energy.
- Kinetic energy transferred to conductor materials by means of collisions with ions.
- Ions vibrate about their equilibrium positions.
- Energy transfer results in an increase in temperature
- Most of the work done by the $\vec{E}$ goes into heating the conductor, not into moving the charges faster.

Direction Current Flow

Moving particles	$\longrightarrow$	positive or negative
Negative	$\longrightarrow$	metals
Both	$\longrightarrow$	ionized gas or ionic solution
Holes and Electrons	$\longrightarrow$	semiconductors

Direction of Charge Flow



Conventional Current

Current (I) is defined to be in the direction of flow of positive charge called conventional current.

This is historical but the sign of the moving charges is of little importance in electric circuit analysis.

Current

Current is defined as the net charge (dQ) through the area per unit time:

$$\boxed{I = \frac{dQ}{dt}}$$

(A- amperes)

Current, Drift Velocity and Current Density

- Assume free charge is positive, then $\vec{v}_d$ is in the direction of $\vec{E}$.

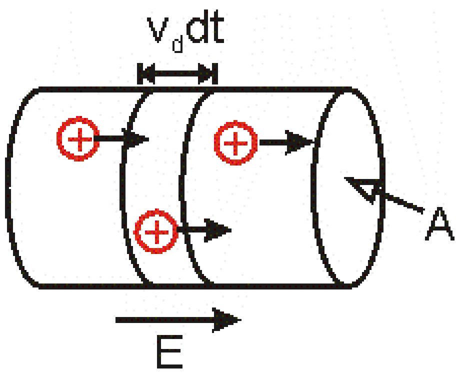
- Let n = moving charged particles per unit volume (m^{-3}).

- Assume all move $v_d \Rightarrow s_d = v_d dt$ in the interval dt.

- Volume $V = Av_d dt$ $\quad$ $\text{Number Particles} = nAv_d dt$

Now:

$$dQ = nAv_d dtq \Rightarrow I = \frac{dQ}{dt} = nAv_d q$$



Current Density (J)

- Current per unit cross-sectional area.

$$J = \frac{I}{A} = nv_d \left| q \right|$$

- This is independent of the charge sign.

Current Density Vector ($\vec{J}$)

- Current per unit cross-sectional area.

$$\vec{J} = nq\vec{v}_d$$

- This is dependent on the sign of the charge but when q is positive, $\vec{v}_d$ is in the direction of E.

- When q is negative, $\vec{v}_d$ is in opposite direction of E.

- Both cases $\vec{J}$ is positive and in the direction of E.

Example

Copper wire of nominal diameter 1.00 mm carries a constant current of 1.50 A. The free electron density is 8.5×10^{28} electrons per m^3.

Find the magnitude of (a) current density (b) drift velocity.

Resistivity (ρ)

At a given temperature

$$\vec{J} \; \alpha \; \vec{E}$$

Ratio

$$\text{constant} = \frac{\vec{E}}{\vec{J}} = \rho$$ (units: Ω.m)

The greater the resistivity the larger field needed to produce a given J.

Metals: small $\rho \sim 10^{-14}$ Insulators: > by × 10^{22}

N.B. $$\sigma = \frac{1}{\rho} = \frac{\vec{J}}{\vec{E}}$$ This is conductivity (units: $\Omega^{-1}.m^{-1}$)

Conductors

- Good electrical conductors are usually good heat conductors e.g. silver, copper, tungsten

- Poor electrical conductors are usually poor heat conductors e.g. ceramic, plastic

Resistivity and Temperature

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

where α is the temperature coefficient of resistivity

- This is valid over a temperature range of ~ 100 °C.

Resistance (R)

$$\vec{E} = \rho \vec{J}$$

Now $$E = \frac{V}{L}$$ and $$J = \frac{I}{A}$$

Substitute:

$$\frac{V}{L} = \rho \frac{I}{A} \quad \Rightarrow \quad V = \boxed{\rho \frac{L}{A}} I$$

when ρ = constant then V α I. Let resistance (R) be

$$R = \frac{V}{I} = \rho \frac{L}{A}$$

Units: Ω - Ohms

Temperature Coefficients of Resistivity, α ($(°C)^{-1}$) (near room temperature)

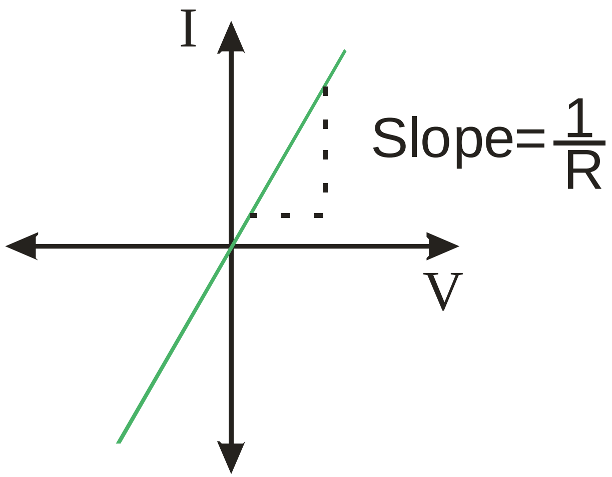
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon(graphite)	-0.0005	Mercury	0.0088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

HW : Sec. 26.3

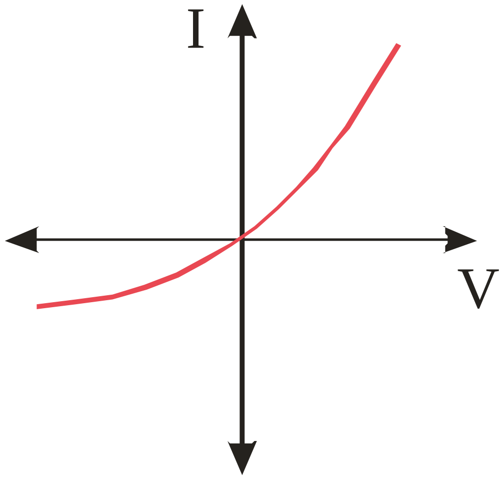
Ohm's Law (as we know it)

$$V = IR$$

Ohmic resistor



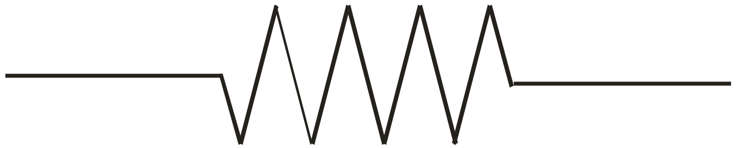
Semiconductor



Also because ρ α T and $R = \rho \frac{L}{A}$ then $R(T) = R_0[1 + \alpha(T - T_0)]$

Resistor

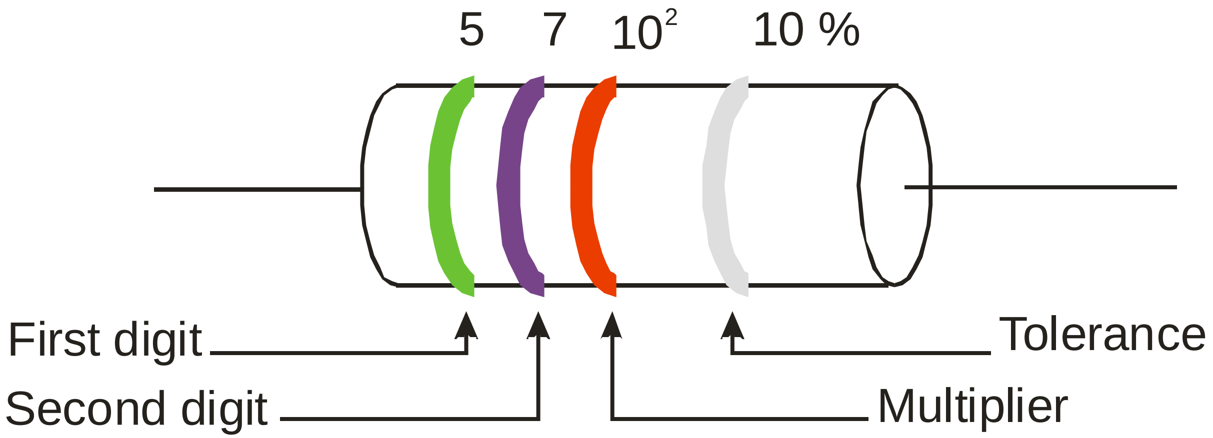
Circuit device with specific resistance value.

Symbol: 

$57 \times 10^2\,\Omega \pm 10$

%

$5.7\ k\Omega \pm 10\ \%$



Resistor Colour Table

Colour	Digit Value	Multiplier Value
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Tolerance:	no band	± 20 %
	silver band	± 10 %
	gold band	± 5 %

Example

For the previous example with copper wire ($d = 1.00$ mm, $I = 1.50$A, $n = 8.5 \times 10^{28}$ m^{-3}) find

a. The magnitude of E within the wire,

b. The potential difference if $L = 50.0$ m

Example cont……

c. The resistance of the wire given $\rho = 1.72 \times 10^{-8}\ \Omega$m for copper at 20 °C

d. Given the temperature coefficient of resistivity, α, is 0.00393 $°C^{-1}$, what is the resistance at 0 °C and 100 °C?

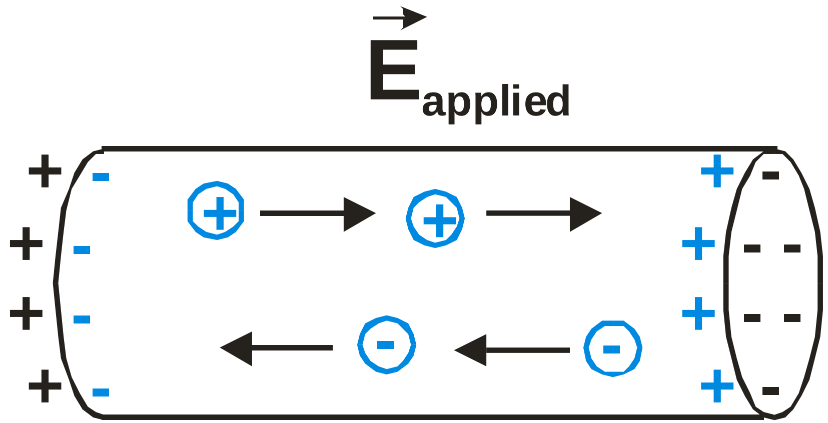
Points of Importance

- Resistance is a property of the device/object.

- Resistivity is a property of the material from which the device/object is made.

Electromotive Force and Circuits

- Steady current flow requires a closed circuit.
- Why? Let's look at a piece of wire



$\vec{E}_2$ **forms in opposite direction owing to accumulated charge**

$$\Rightarrow \quad \vec{E} = \vec{E}_{app} + \vec{E}_2 = 0 \quad \Rightarrow \quad \vec{J} = 0$$

- So current stops thus no steady motion occurs in an incomplete (open) circuit

HW : Sec. 27. 1

How is a steady current maintained in a closed circuit?

Think fountain – water falls into the basin

- PE >> KE : High PE >> Low PE
- Water is then **pumped** back up
- KE >> PE : Low PE >> High PE

Electrostatic force – push charge from high PE to low PE but Electromotive Force, aka EMF, is the pump that pushes the charge from low PE to high PE.

N.B. EMF is not a real force

but energy per charge quantity

http://www.openclipart.org/detail/382/fountain-by-johnny_automatic

Electromotive Force (EMF), ε

Units: V or J/C

Sources : batteries, electric generators, solar cells, fuel cells which convert energy of some form to electrical energy.

Ideal Source: maintains constant potential difference between terminals

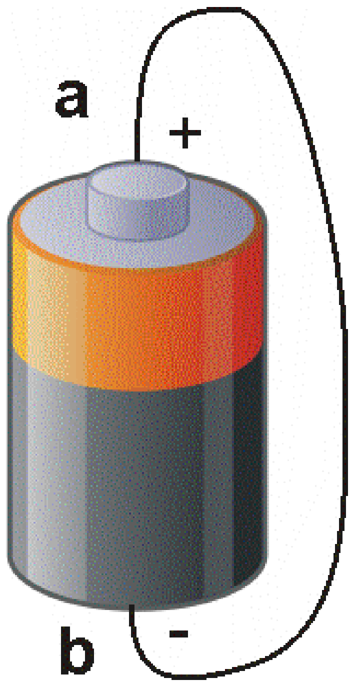
$$\boxed{V_{ab} = \varepsilon = IR}$$

Internal resistance, r

A real source has internal resistance i.e. the charge encounters resistance. If ohmic then r is constant.

As current travels through r, it experiences a drop in potential of Ir therefore as charge goes from negative terminal b to positive terminal a the potential difference between them is V_{ab}

$$V_{ab} = \varepsilon - Ir = IR$$



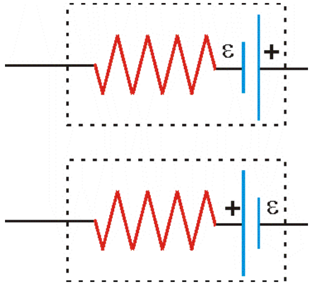
Where R is the external resistance in the circuit.
The current in the circuit is then given by

$$I = \frac{\varepsilon}{R + r}$$

Revision Symbols for Circuit Diagrams

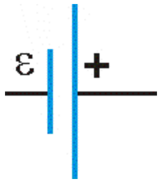
Conductor with negligible resistance

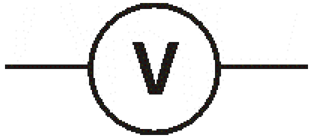
Resistor



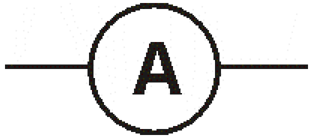
Source of emf with internal resistance

Source of emf





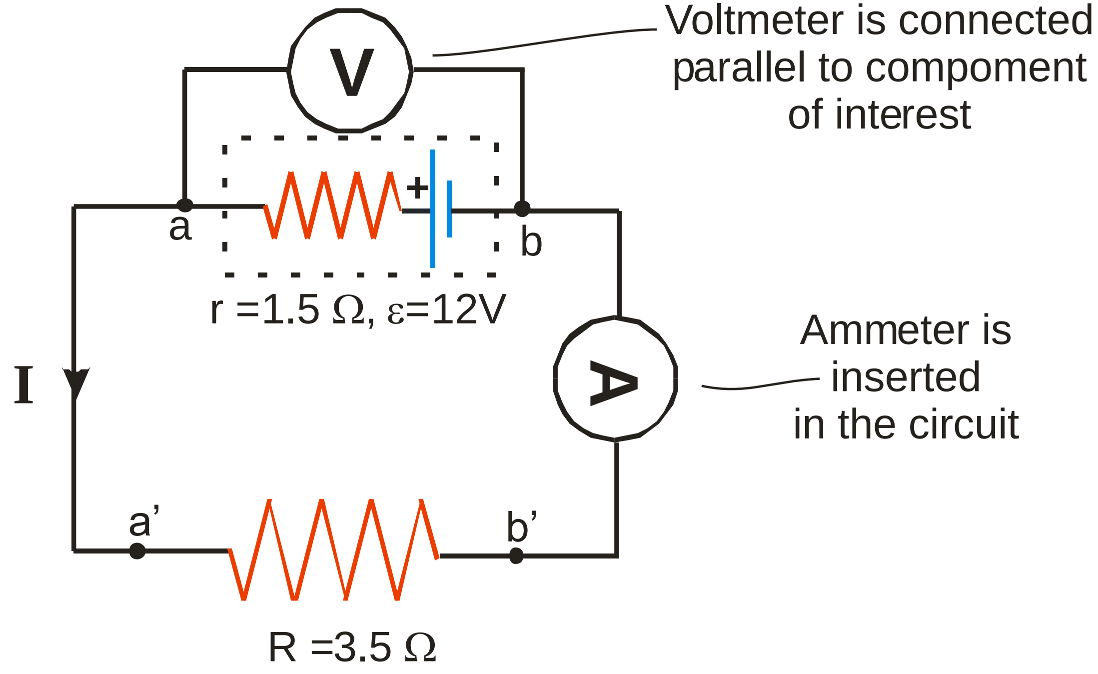
Voltmeter – used to measure potential difference between terminals. A voltmeters has large resistance and is connected in parallel.



Ammeter – measures current through section. Has very low resistance and is connected in series.

Example

What are the voltmeter and ammeter readings?



Potential Change around a Circuit

This is for ohmic resistors only.

$$V_{ab} = \varepsilon - Ir = IR$$

$$\Rightarrow \varepsilon - Ir - IR = 0$$

$$or$$

$$\varepsilon = Ir + IR$$

Energy and Power in Electric Circuits

There is a steady state flow therefore no increase in kinetic energy occurs but there is transfer of energy into the circuit elements e.g. heat for toaster element, or out of circuit e.g. battery.

$$dW = dQV_{ab} \quad \text{but} \quad dQ = Idt$$

$$P = \frac{dW}{dt} = \frac{dQ}{dt}V_{ab} = V_{ab}I$$

$$\boxed{P = VI}$$

Rate energy delivered to/extracted from element.

Power (P)

$$P = V_{ab} I \qquad \text{in J/s or W}$$

But $V_{ab} = IR$ for a pure resistor.

$$\Rightarrow \quad P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$$

Power <u>Out</u> of a Source:

$$P = V_{ab} I = I\varepsilon - I^2 r$$

Example

For the previous example, calculate the power dissipated in the 3.5 Ω resistor and in the internal resistance.

Example

A storage battery has an emf of 25 V and an internal resistance of 0.20 Ω. Find its terminal voltage when

a. it is delivering 8.0 A

b. it is being charged with 8.0 A.

DIRECT CURRENT CIRCUITS

Direction of current does not change with time

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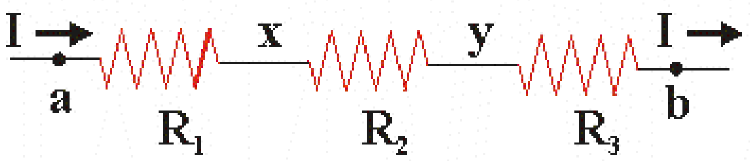
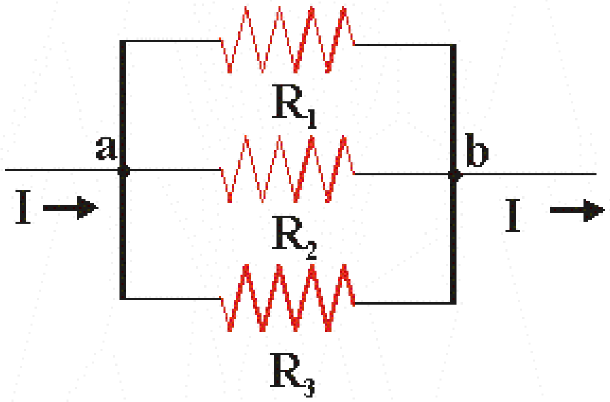
Resistor Circuits

Potential V_{ab} between points a and b

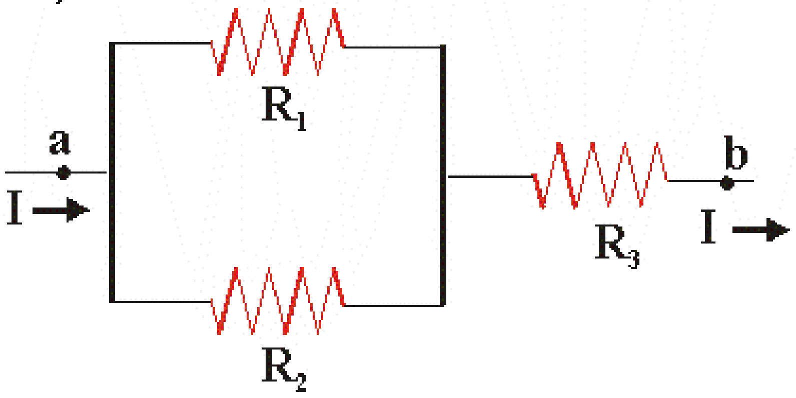
A) **In series**



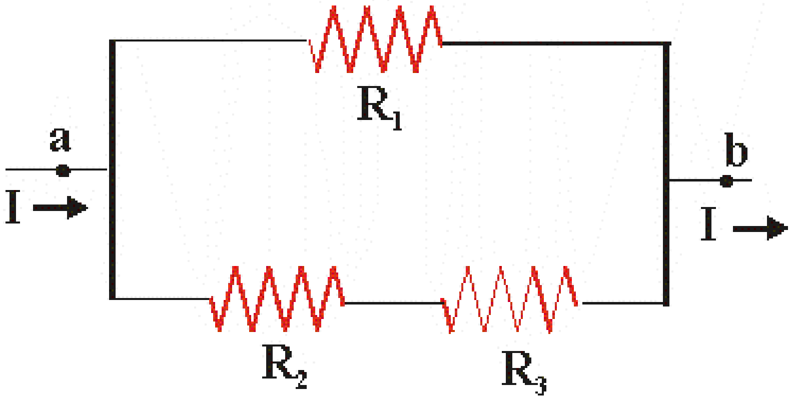
B) **In parallel**



C) **Combination**



D) **Combination**



Resistor Circuits

- In circuit A the potential drop across R_1, R_2, and R_3, adds up to V_{ab}.

- The current has nowhere else to go but through R_1, R_2, and R_3.

- This is a SERIES connection of resistors.

- In circuit B the potential drop across **each** resistor R_1, R_2, and R_3 is the same. Each resistor provides an **alternate** path for current.

- This is a PARALLEL connection of conductors.

- C and D are combinations of parallel and series.

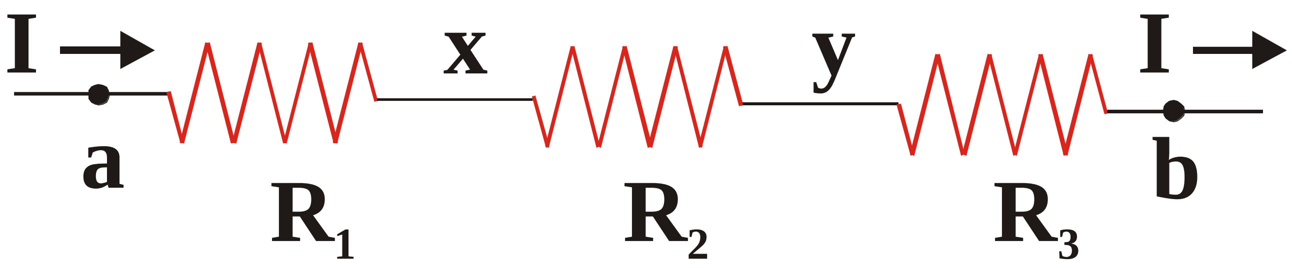
Resistor Circuits

- Between points a and b the circuit could be replaced with a **single resistor** to give the same total current and potential difference. This is called an equivalent resistance.

$$V_{ab} = IR_{eq} \quad \text{or} \quad R_{eq} = \frac{V_{ab}}{I}$$

•Finding R_{eq} - Assume V_{ab} across actual network.
- Compute corresponding I (or measure)
- Apply ohm's law and take ratio V_{ab}/I

Resistors in Series



Voltage Drops:

$$V_{ax} = IR_1 \quad V_{xy} = IR_2 \quad V_{yb} = IR_3$$

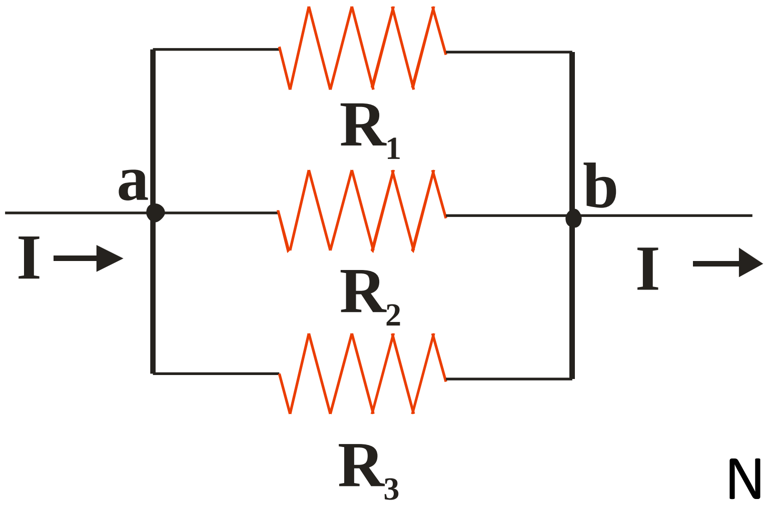
Now:

$$\begin{aligned} V_{ab} &= V_{ax} + V_{xy} + V_{yb} \\ &= IR_1 + IR_2 + IR_3 \\ &= I\left(R_1 + R_2 + R_3\right) \end{aligned}$$

$$\boxed{R_{eq} = \frac{V_{ab}}{I} = R_1 + R_2 + R_3}$$

Resistors in Parallel

Voltage Drops:



$$V_{ab} = V_1 = V_2 = V_3$$

$$I_1 = \frac{V_{ab}}{R_1}, \quad I_2 = \frac{V_{ab}}{R_2}, \quad I_3 = \frac{V_{ab}}{R_3}$$

Now:

$$I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Summary

Series

$$R_{eq} = \frac{V_{ab}}{I} = R_1 + R_2 + R_3 + \ldots\ldots$$

$$R_{eq} = \sum_{k=1}^{n} R_k$$

R_{eq} is greater than any individual resistance.

Parallel

$$\frac{1}{R_{eq}} = \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots$$

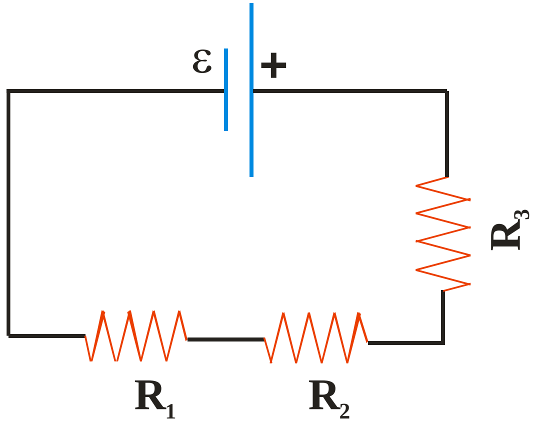
$$\frac{1}{R_{eq}} = \sum_{k=1}^{n} \frac{1}{R_k}$$

R_{eq} is smaller than any individual resistance.

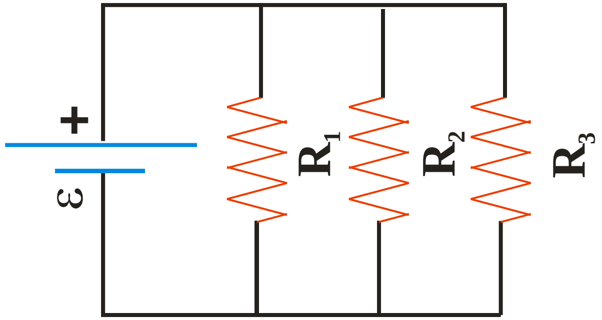
Example

For the circuits below, determine R_{eq}, I and the power dissipated in the circuit when V_{ab}=12 V, R_1=3.0 Ω, R_2=4.0 Ω, and R_3=5.0 Ω.

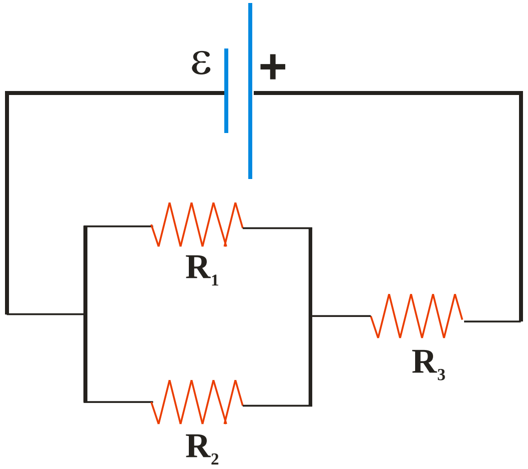
A)



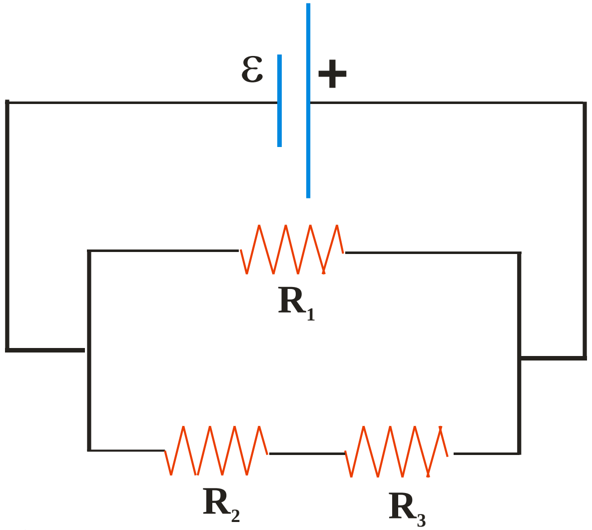
B)



C)

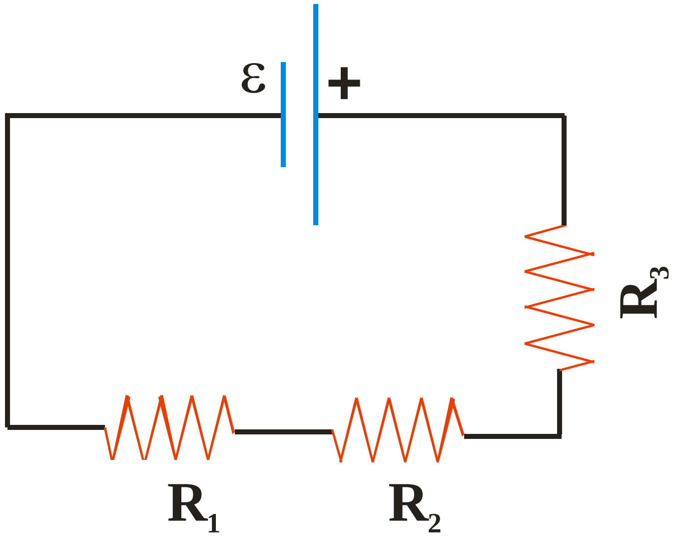


D)



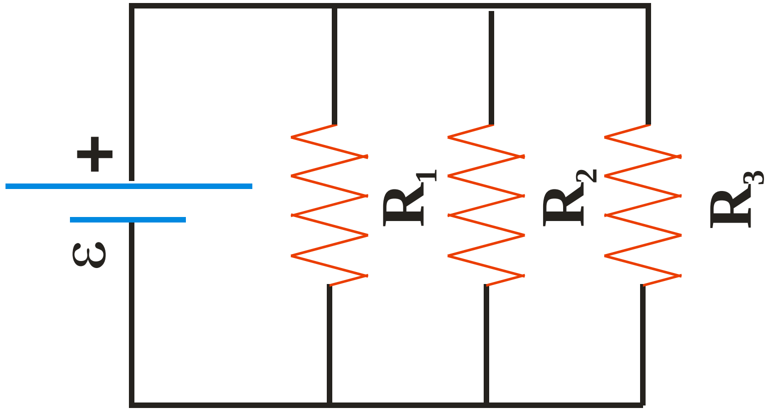
Example part A

A)



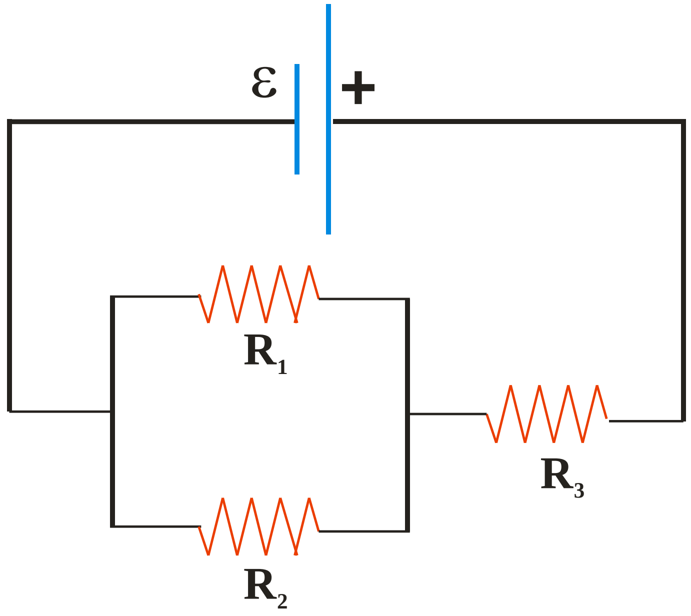
Example part B

B)



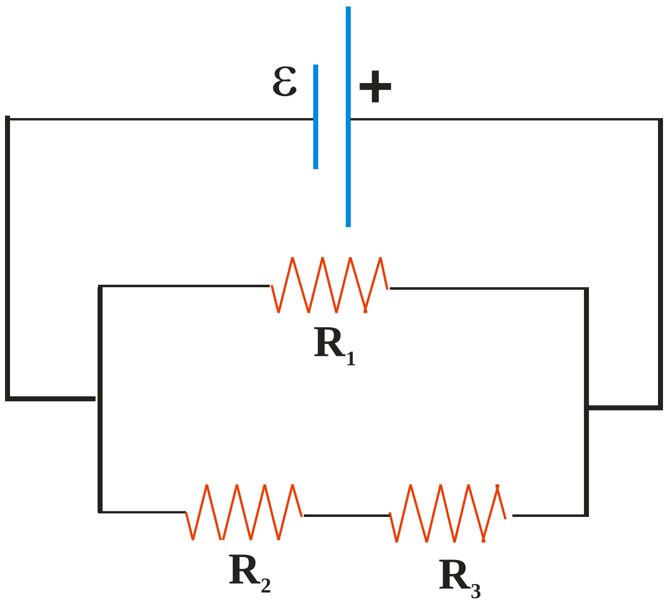
Example part C

C)



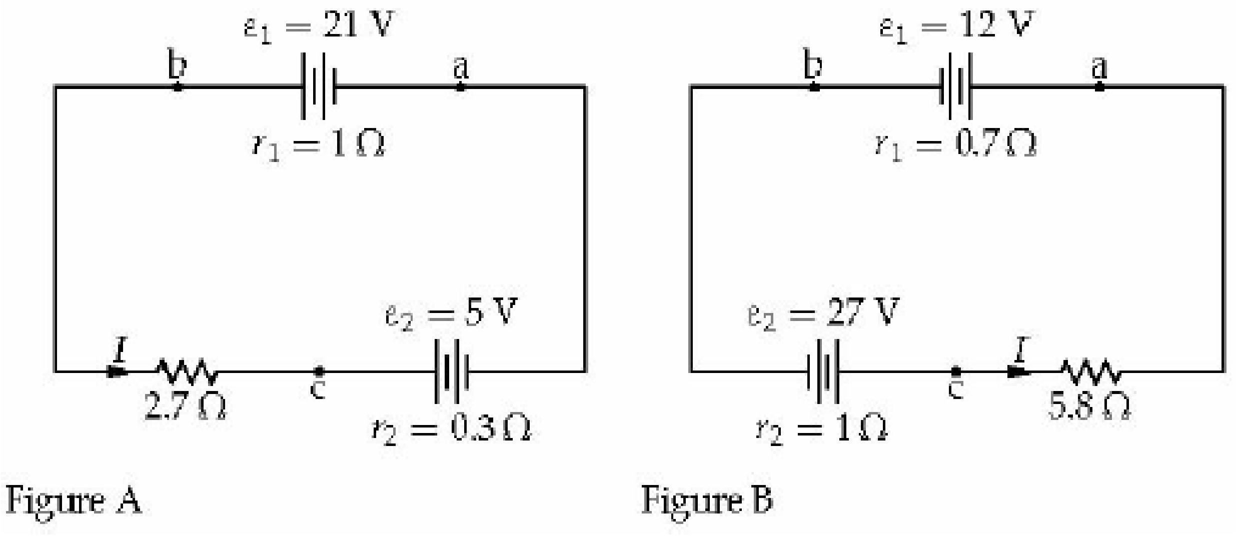
Example part D

D)



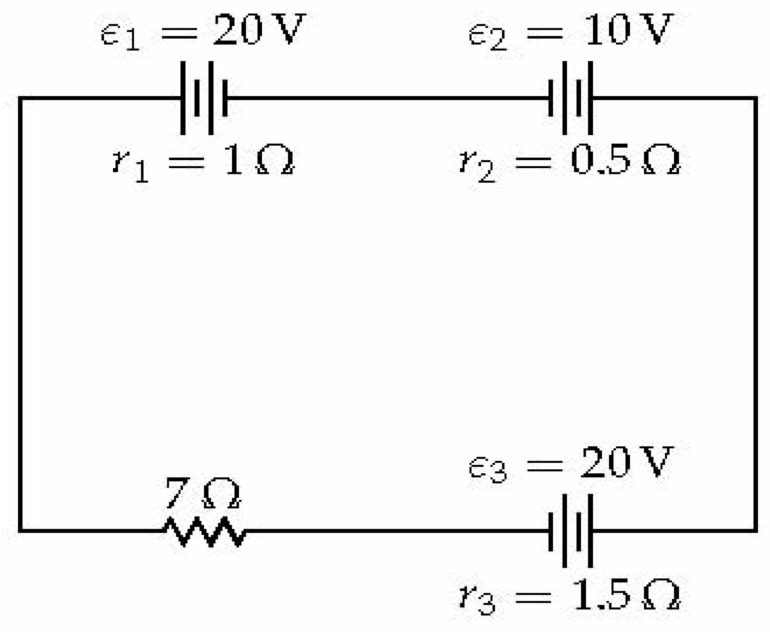
Example

Referring to Figures A and B below, for each circuit determine the current, *I*, and the potential differences V_{ab}, V_{bc} and V_{ca}.



Example

Referring to the circuit diagram, find the current in the circuit and the terminal voltage of each battery.



Example

Two identical cells are connected first in parallel, and then in series, to a resistance of 15 Ω. If the readings of a voltmeter connected across the 15 Ω are 2.52 V and 3.2 V respectively, find

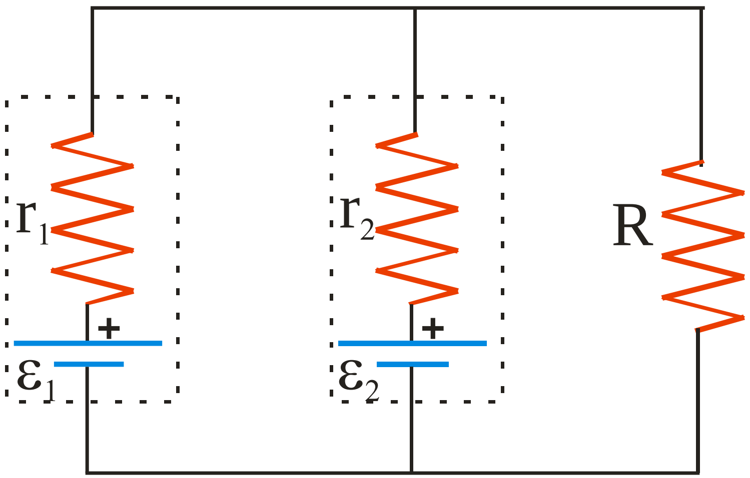
a. the internal resistance

b. the emf of the cells. Assume that the voltmeter resistance is infinite.

Example cont….

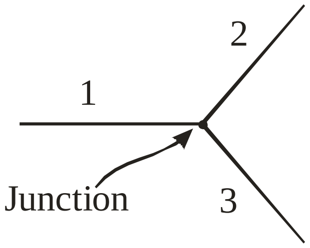
What happens when the circuit is more complex and cannot be reduced easily???

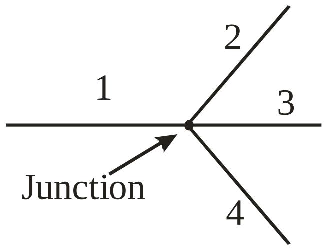
Two sources of EMF
with internal resistances



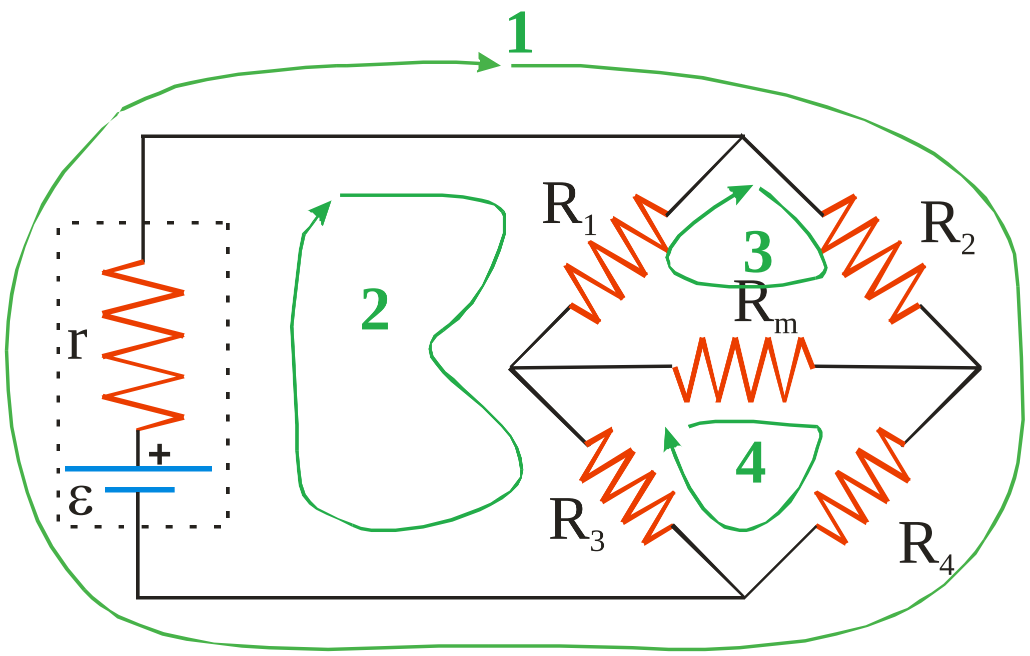
First Some Definitions

Junction: a point in the circuit where three or more conductors meet





Loop: any closed conducting path.



Kirchoff’s Rules

Junction Rule: the algebraic sum of the currents into any junction is zero (conservation of charge)

$$\sum I = 0$$

Loop Rule: the algebraic sum of the potential differences in any loop must equal zero (electrostatic force is conservative)

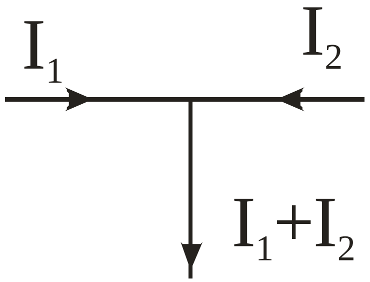
$$\sum V = 0$$

HW : Sec. 27.2

Kirchoff’s Rules

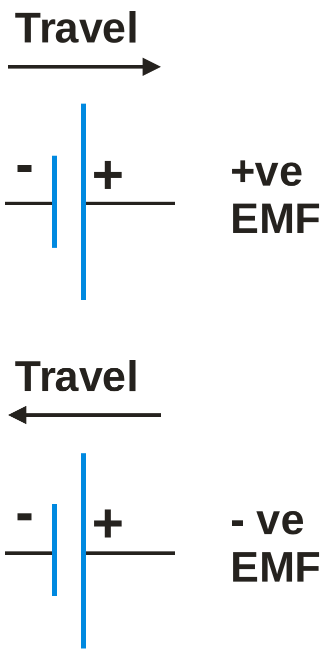
Junction Rule:

what goes in must come out

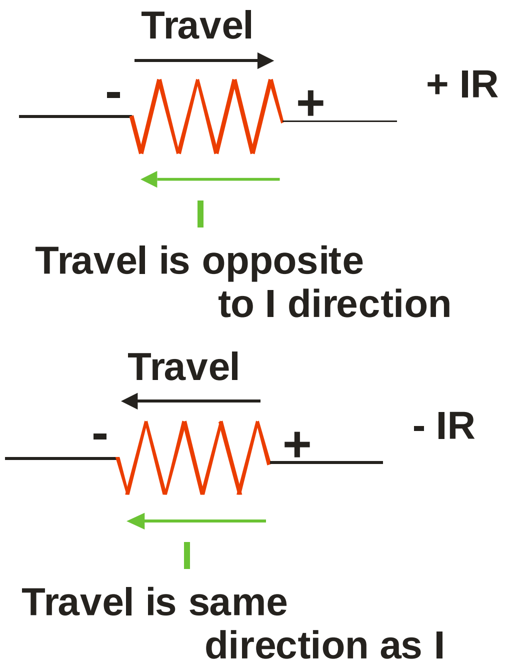


Sign conventions for the loop rule: first assume an I direction

For EMF

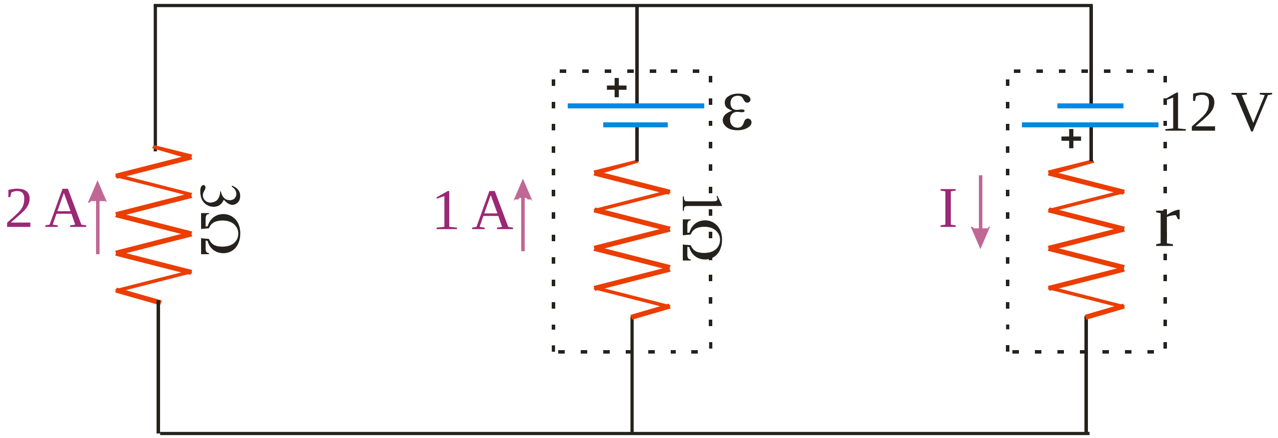


For Resistors



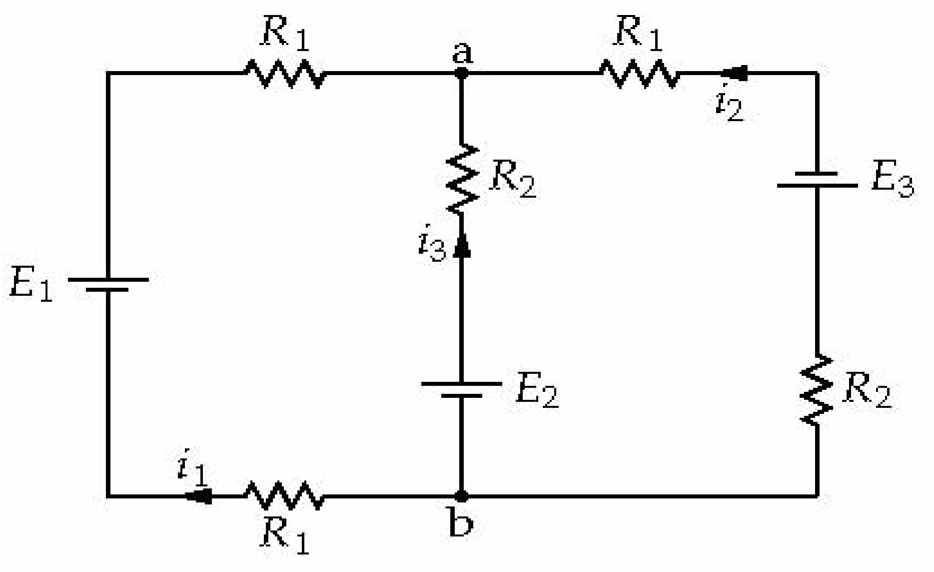
Example

Find ε, I and r



Example

In the circuit shown below, R_1 = 1.0 Ω, R_2 = 2.0 Ω, E_1 = 2.0 V, E_2 = 4.0 V and E_3 = 8.0 V.



a. Find the current in each branch of the circuit, i.e. find i_1, i_2 and i_3.

b. Find the potential of *b* with respect to *a*, V_{ab}.

Example – cont…