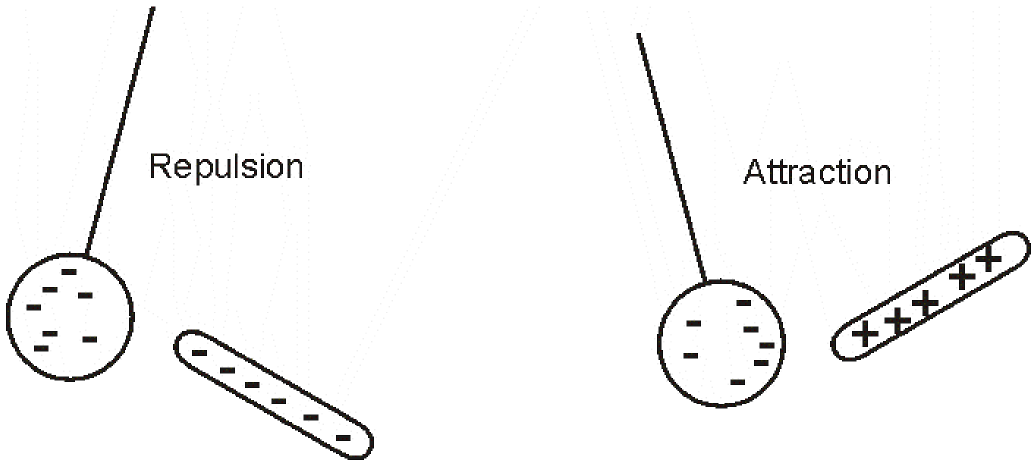
Electric Force
and
Electric Field

These lectures slides were prepared by
Dr. Danica Solina

Text: Walker *etal. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition*
John Wiley & Sons Australia (HW)

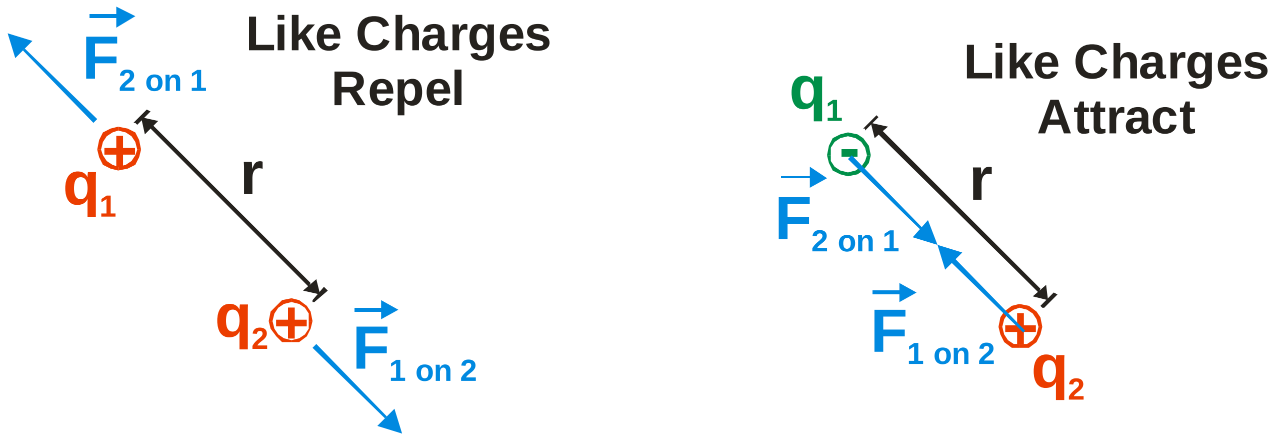
Electric Force



•Electric charge produces a force on another charge

•Like charges repel while unlike charges attract

•Between point charges the electric forces obey Newton's Third Law.



Coulomb's Law

Charles Augustin de Coulomb (1736 – 1806) studied point charges using a torsional balance (illustrated). He found:

$$\text{Electric Force} \propto \frac{1}{\text{distance square}} \text{ or } \frac{1}{r^2}$$

For two point charges q_1 and q_2

$$\text{Electric Force} \propto q_1 \times q_2$$

https://nationalmaglab.org/education/magnet-academy/history-of-electricity-magnetism/museum/torsion-balance-1785

Coulomb's Law

The magnitude of the electric force between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

$$\boxed{F = k\frac{|q_1 q_2|}{\mathrm{r}^2}}$$

$$\text{where } k = \frac{1}{4\pi\varepsilon_o} = 8.988\times10^9\,\mathrm{N.m^2/C^2} \cong 9.0\times10^9\,\mathrm{N.m^2/C^2}$$

$$\text{and } \varepsilon_o = 8.854\times10^{-12}\,\mathrm{C^2\,/\,N.m^2}$$

Important Points regarding Charge

Charge is quantised (restricted to certain values):

$$\boxed{q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots\ldots}$$

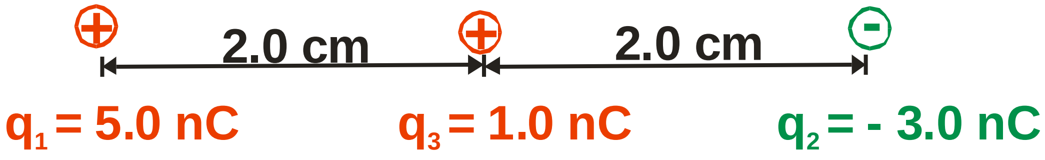
Where e is the elementary charge and $e = 1.602 \times 10^{-19}\ \mathrm{C}$

Charge is Conserved where the PRINCIPLE OF CONSERVATION OF CHARGE is :

The algebraic sum of all electric charges
in any closed system is conserved.

Example

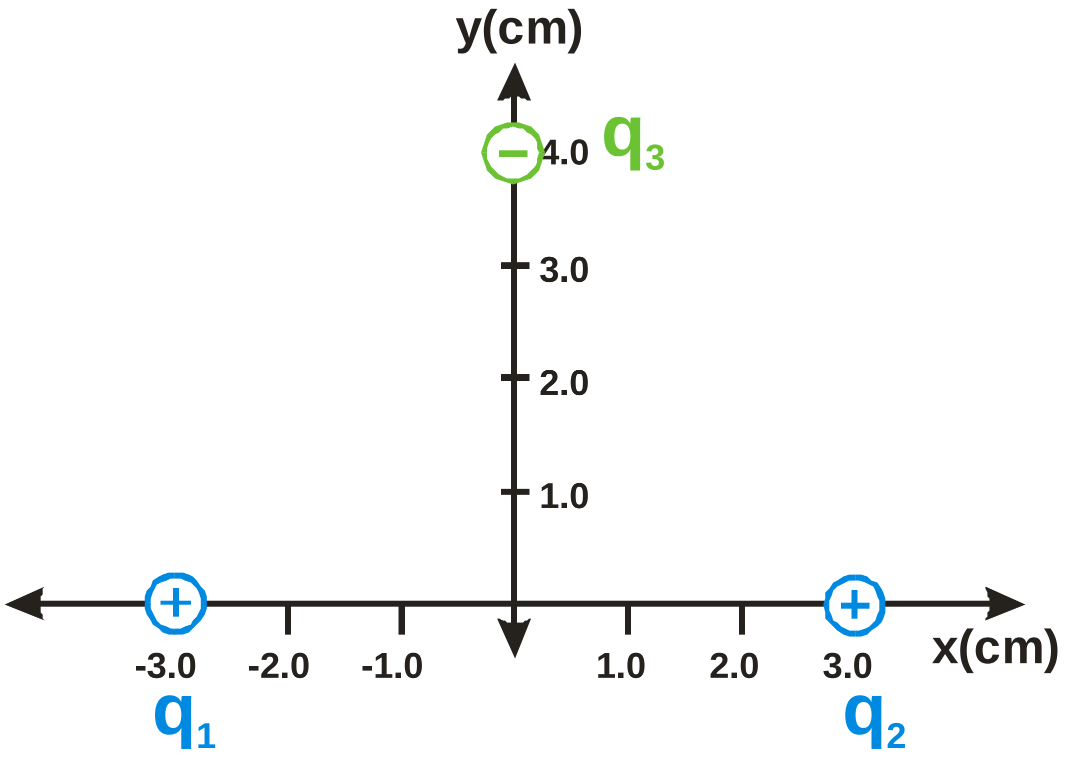
In the diagram below, what is the electric force on q_3?



SOLUTION

Example

In the diagram below, what is the electric force on q_3 given $q_1 = 2.0 \times 10^{-6}$ C, $q_2 = 2.0 \times 10^{-6}$ C and $q_3 = - 4.0 \times 10^{-6}$ C?



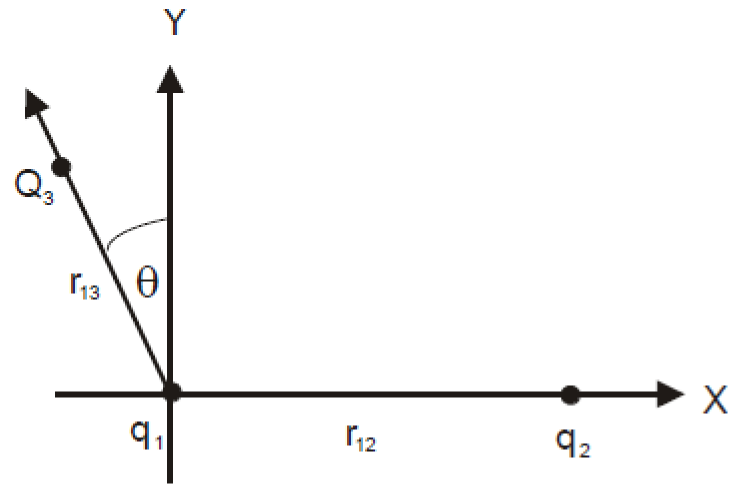
Example

Three charges q_1, q_2 and q_3 are arranged as shown where

$q_1 = -1.0\times10^{-6}$ C $q_2 = +3.0\times10^{-6}$ C $q_3 = -2.0\times10^{-6}$ C

$r_{12} = 0.15$ m $r_{13} = 0.10$ m $\theta = 30°$

What is the force on q_1 due to the other 2 charges?



Example cont….

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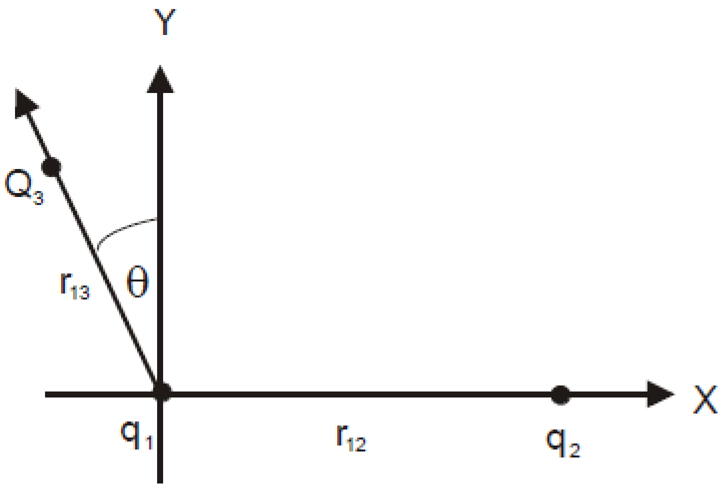


Electric Field and Electric Forces

How does a charge know the other is there?

>>> Electric Field

Imagine that somehow the charge that a body carries modifies the space around it. Then another body as a result of its charge senses this change. The second body responds by experiencing the force F_0 to this modification.

The modification is the Electric Field (E).

"The electric force on a charged body is exerted by the electric field created by other charged bodies."

Electric field E is defined as the electric force F_0 experienced by a test charge q_0 given by:

$$\boxed{\vec{E} = \frac{\vec{F}_0}{q_0}}$$

Unit: N/C

Rearranging:

$$\vec{F}_0 = \vec{E} q_0$$

If q_0 is positive, the force and field are in the same direction.
If q_0 is negative, the force and field are in opposite directions.
The electric field is directed from positive to negative.

This applies to point charges!!!

Now

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{|qq_0|}{\mathrm{r}^2} \quad \Rightarrow \quad E = \frac{F_0}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{\mathrm{r}^2}$$

As a vector:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\mathrm{r}^2}\hat{r}$$

Example

Determine the size of the electric field 2.0 m from a point charge of 3.0 μC.

Example

Determine the electric field at (1.5 m, -2.0 m) from a point charge of $q = -5.0\ \mu C$ at the origin.

As with Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

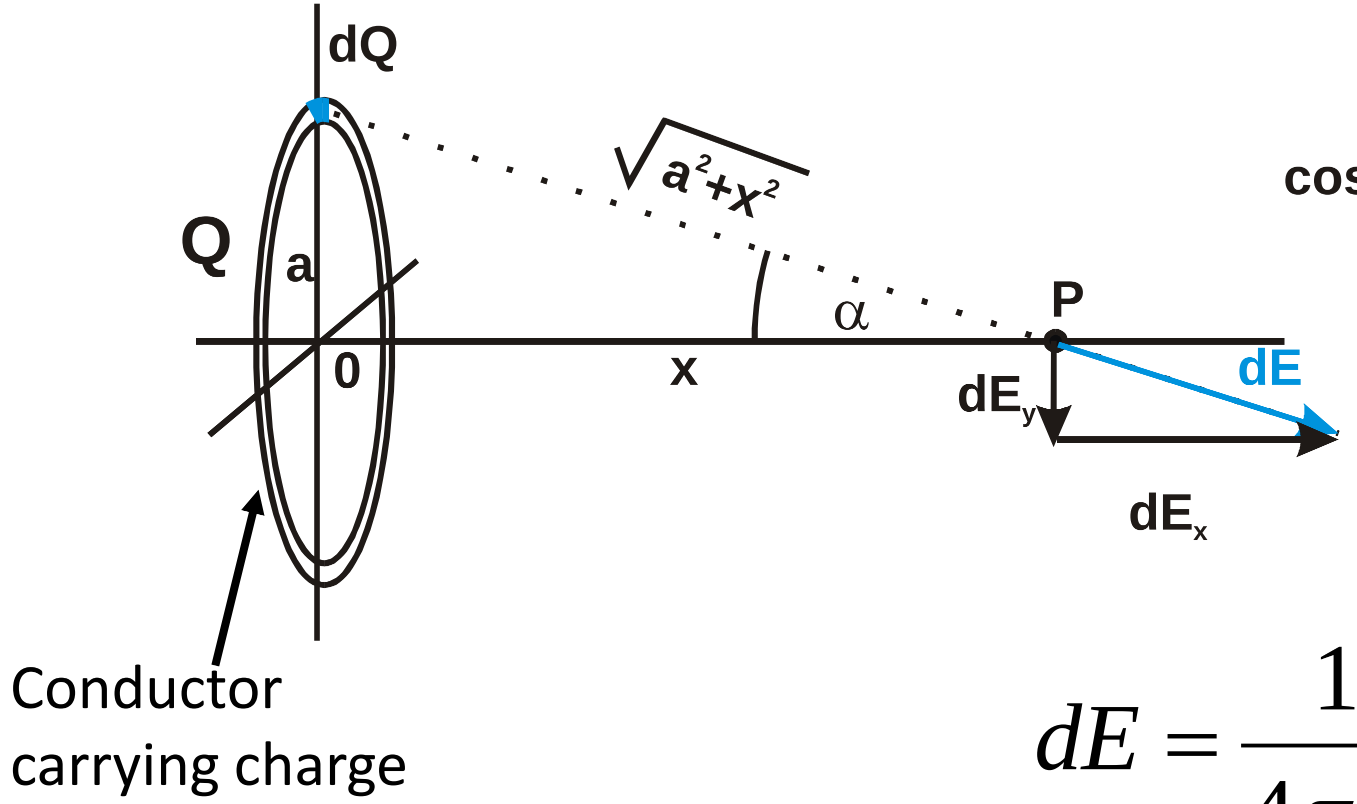
$$\vec{F}_0 = \sum \vec{F} = \sum q_0 \vec{E} = q_0 \sum \vec{E}$$

You will see this as 'Principle of Superposition'

What this means is that the electric field owing to a number of charges is additive at a point.

Field of a Ring of Charge

What is the field at point P?



$$\cos\alpha = \frac{x}{\sqrt{a^2+x^2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{a^2+x^2}$$

Field of a Ring of Charge

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{a^2+x^2}\bullet\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\varepsilon_0}\frac{dQx}{\left(a^2+x^2\right)^{3/2}}$$

$$E_x = \int\frac{1}{4\pi\varepsilon_0}\frac{dQx}{\left(a^2+x^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(a^2+x^2\right)^{3/2}}$$

Giving

$$\boxed{\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(a^2+x^2\right)^{3/2}}\hat{i}}$$

Electric Field Lines

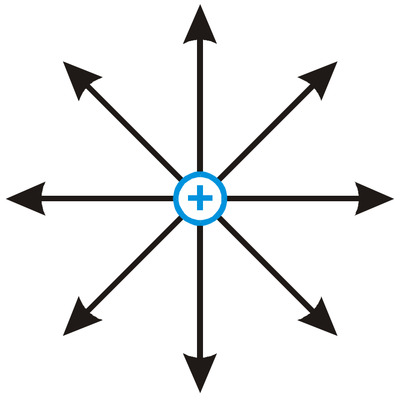
Electric field lines are imaginary lines giving the direction of the electric-field vector at that point.

Their spacing gives an indication of the strength of the electric field at that point.

The direction is away from the positive charge and toward the negative charge.

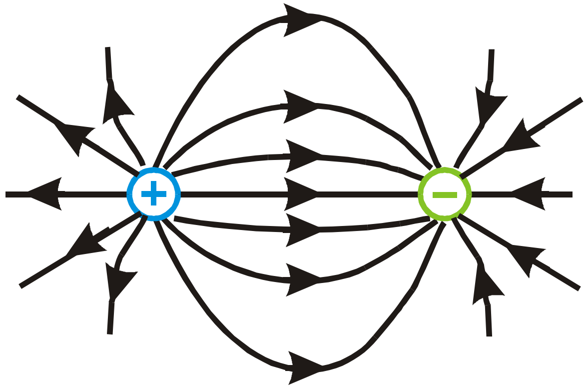
Electric Field Lines

A)



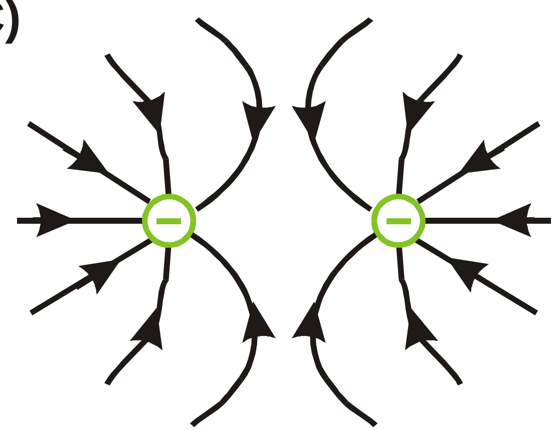
Positive Charge

B)



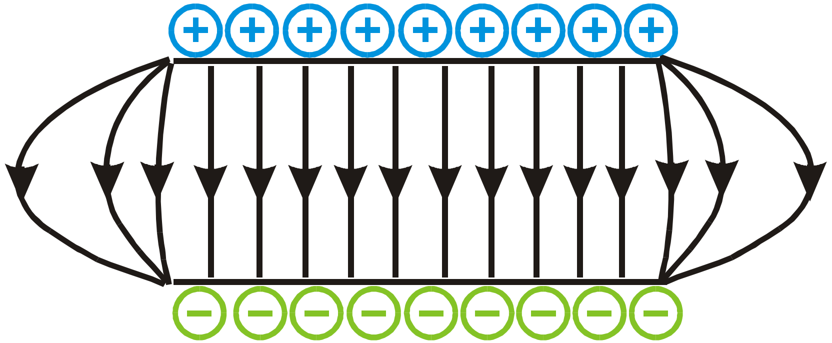
Dipole

C)



Two negative charges

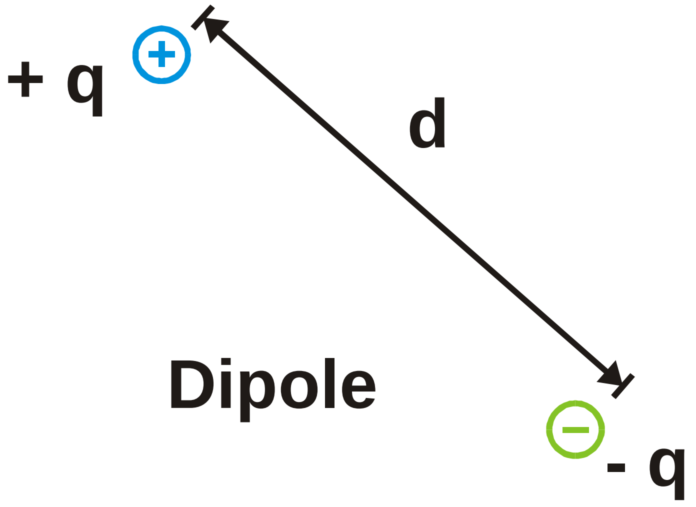
D)



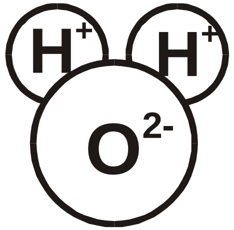
Between two charged plates

Electric Dipoles

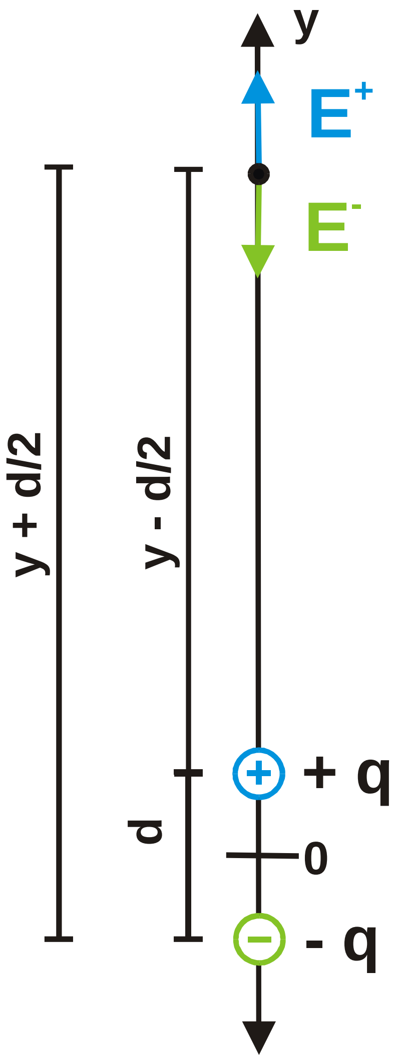
Electric dipole is a combination of a positive and negative charge with equal magnitude separated by a distance, d.



e.g. a water molecule



Field of an Electric Dipole



Derive an expression for E at a distance y when y>>d.

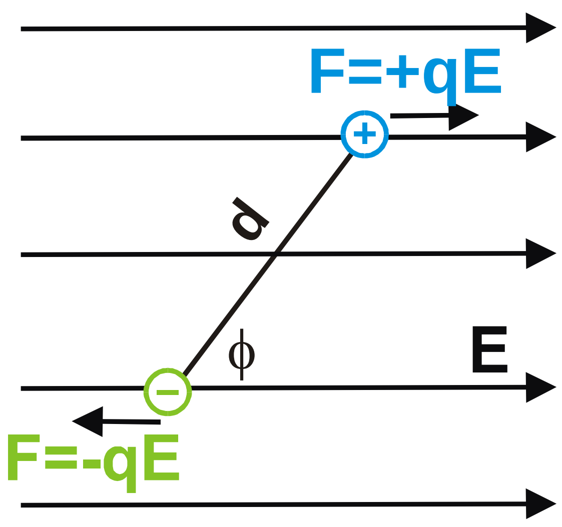
$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(y-\frac{d}{2}\right)^2} - \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(y+\frac{d}{2}\right)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2yd}{\left(y^2-\frac{d^2}{4}\right)^2} \qquad when\, y >> d$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2yd}{y^4}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{qd}{y^3} = 2k\frac{qd}{y^3} \qquad when\, k = \frac{1}{4\pi\varepsilon_0}$$

Force and Torque on an Electric Dipole



$$\vec{\Gamma} = \vec{r} \times \vec{F}$$

$$\Gamma = qEd\sin\phi$$

The product *qd* is the magnitude of the electric dipole moment.

$$\boxed{p = qd}$$

(Units C.m)

This means for a dipole:

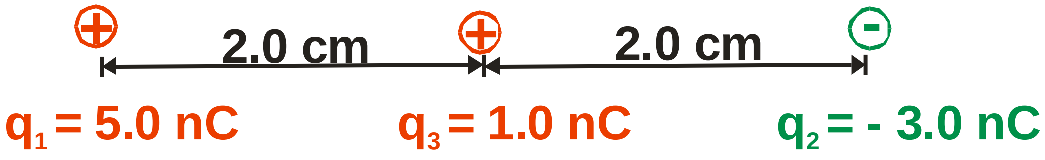
$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{y^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{y^3}$$

and

$$\boxed{\vec{\Gamma} = \vec{p} \times \vec{F}}$$

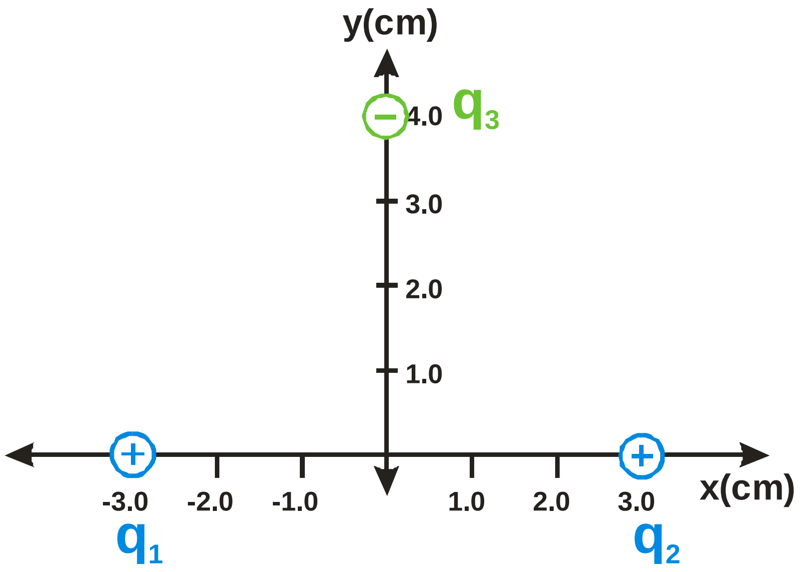
Example

What is the electric field at the position of q_3?



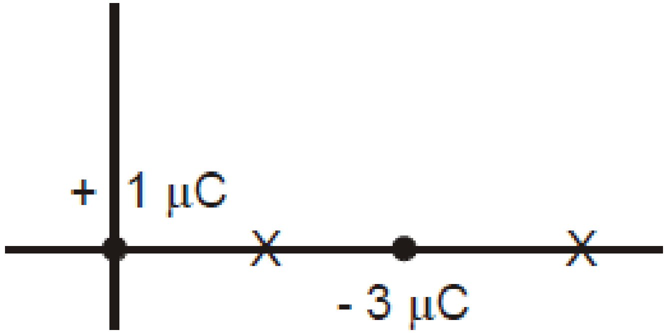
Example

In the diagram below, what is the electric field at the position of q_3 given $q_1 = 2.0 \times 10^{-6}$ C, $q_2 = 2.0 \times 10^{-6}$ C and $q_3 = -4.0 \times 10^{-6}$ C?



Example

Two charges of +1.00 μC and -3.00 μC are located on the *x*-axis. The positive charge is at the origin, while the negative charge is at *x* = 1.00 m. What is the magnitude and direction of the electric field due to these charges at a. *x*=0.50m and b. *x*=1.50m?



Electric Potential

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Review

First

$$W_{a \to b} = \int_a^b \vec{F} \bullet d\vec{l} = \int_a^b F \cos\phi dl$$

This is work done by a force.

Second

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

For work done by a conservative force.

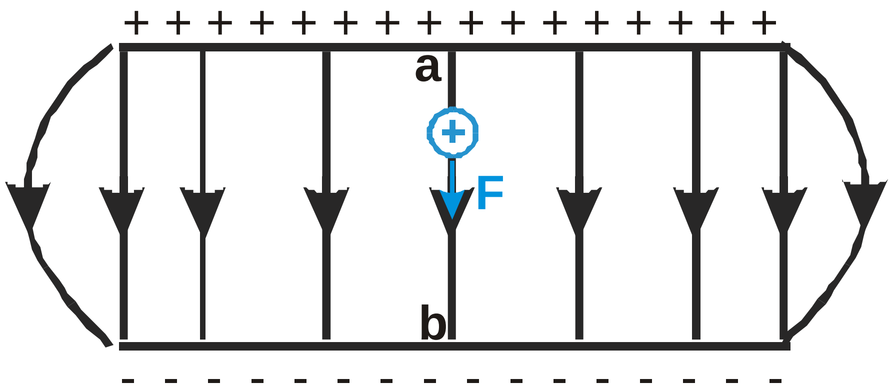
Third

$$K_a + U_a = K_b + U_b$$

Work-energy theorem when work is done by a conservative force i.e. total energy is conserved.

Electric Potential Energy in a Uniform Field

$$W_{a\to b} = -\Delta U$$
$$= Fd$$
$$= q_0 E d$$

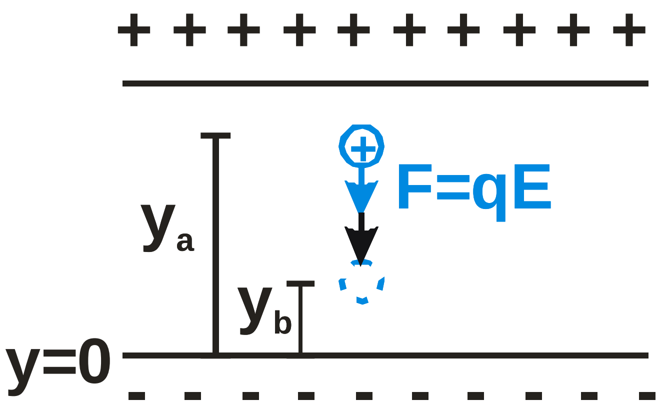


N.B. Potential energy for gravity, F_y=-mg was U=mgy, thus potential energy for the electric force, $F_y=q_0E$ is $U=q_0E\ y$.

$$W_{a\to b} = -\Delta U = -(U_b - U_a)$$
$$= -(q_0 E y_b - q_0 E y_a) = q_0 E(y_a - y_b)$$

POSITIVE CHARGE

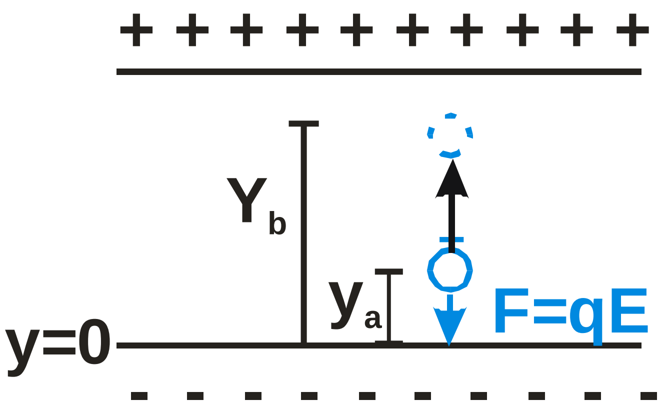
NOTE: Same behaviour as GPE



Positive charge moves in direction of E

Field does positive work

U decreases



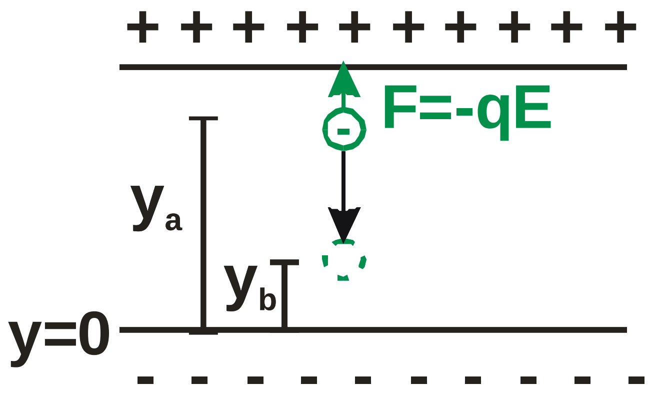
Positive charge moves opposite to E direction

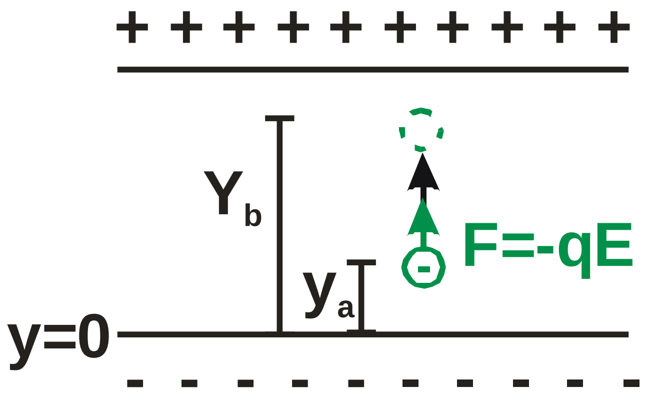
Field does negative work

U increases

NEGATIVE CHARGE

General rule: U increases if test charge q_0 moves in the direction that is opposite the electric force.





Negative charge moves
in direction of E, opposite F

Field does negative work

U increases

Negative charge moves
opposite to E in direction F

Field does positive work

U decreases

The presence of q and q_0!

Remember a field exists in the presence of a charge and the effect on another charge is an electric force $F_0=q_0E$.

The potential energy, U, when the test charge, q_0 is any distance r from q is then:

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}}$$

Units: Joules (J)

Electric Potential (V)

Potential is potential energy per unit charge.

$$\boxed{V = \frac{U}{q_0} \quad or \quad U = q_0 V}$$

Unit: Volt, V
or J/C

Potential energy and charge are scalars, therefore potential is a scalar quantity.

N.B. Whereas potential is a property of the field, potential energy is the energy of a charged object in the field.

Work done

Now work done by an electric force from a to b is $-\Delta U=-(U_b-U_a)$, then the work per unit charge is:

$$\frac{W_{a\to b}}{q_0}=\frac{-\Delta U}{q_0}=-\left(\frac{U_b}{q_0}-\frac{U_a}{q_0}\right)=-(V_b-V_a)$$

$$\frac{W_{a\to b}}{q_0}=V_a-V_b$$

This gives the potential of a w.r.t b.

Equals the work done to move a UNIT charge slowly from b to a against the electric force.

Calculating electric potential

Potential due to a point charge

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad or \quad V = \frac{F \times d}{q_0} = E \times d$$

Due to a collection of point charges

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

HW : Sec. 24.7

Some useful notes

Unit of electric field:

1 N/C = 1 V/m

Electron Volts (eV)

Uses magnitude of electron charge to define a useful unit of energy. If V_{ab}=1 V then:

$$U_a - U_b = qV_{ab} = 1.602 \times 10^{-19} \times 1 = 1.602 \times 10^{-19}\, J$$

$$\Rightarrow \quad 1.602 \times 10^{-19}\, J = 1eV$$

THIS IS A UNIT OF ENERGY NOT POTENTIAL!!!!

Example

Calculate the electric potential of a charge of 5.0 μC experiencing a force of 3.0 N over 2.0m.

Example

Find the work done in assembling the charges as illustrated

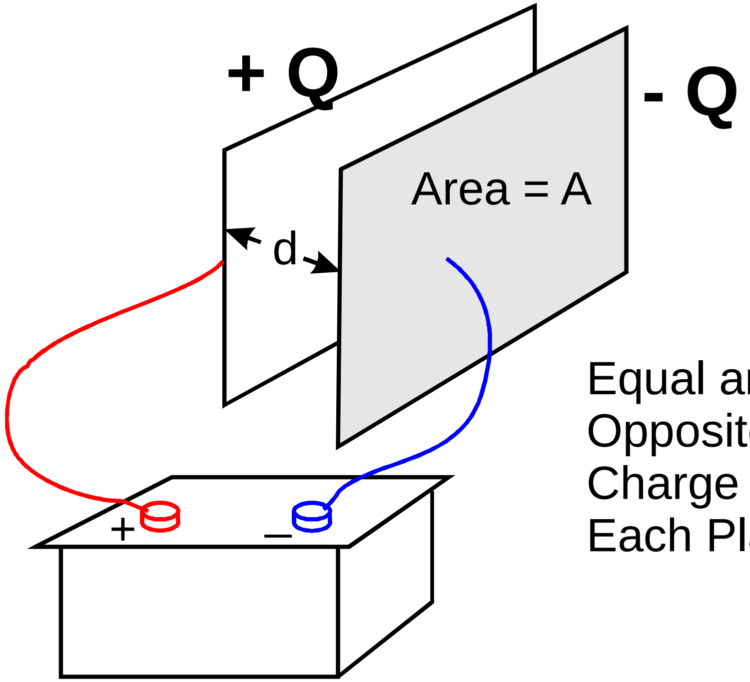
Capacitor

Used in a variety of electric circuits:

- to tune the frequency of the radio receivers,
- eliminate sparking in automobile,
- store short term energy for rapid release in electronic flash units.

Configuration:

Two parallel plates of area, A, are separated by a distance ‘d’. In an electric circuit, plates are connected to the terminals of a battery. Electrons are pulled off one plate transferred through the battery and deposited on the other leaving it with charge – Q. Transfer stops when the potential difference is the same as that from the battery.

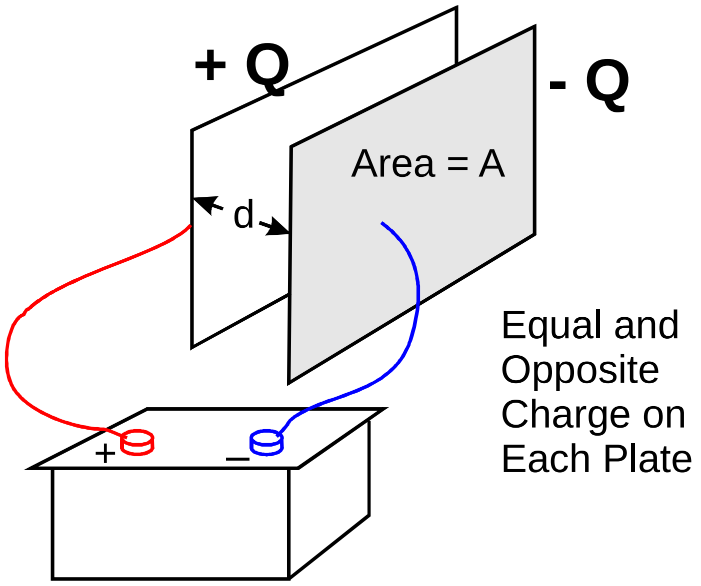


Equal and Opposite Charge on Each Plate

Capacitance, C, of a capacitor

Ratio of the magnitude of the charge on either conductor to the potential difference between the conductors:

$$C_0 = \frac{Q_0}{\Delta V_0} = \varepsilon_0 \frac{A}{d}$$



C = capacitance (farad (F)
= colomb per volt (C/V).

Q = charge (colombs, C)

ΔV = Potential Difference (volts, V)

ε_0 =permittivity of free space = 8.85 × 10^{-12} $C^2/N{\cdot}m^2$

A = area (m^2)

d = distance between the slides (m)

Dielectric Material and the Dielectric Constant, *κ*

A dielectric is an insulating material. When it is inserted between the plates the capacitance increases and we have:

$$C = \kappa \varepsilon_0 \frac{A}{d} = \kappa \frac{Q_0}{\Delta V_0} = \kappa C_0$$

κ = relative permittivity of dielectric material between the plate

Material	Dielectric constant, κ	Material	Dielectric constant, κ
Air	1.00059	Pyrex® glass	5.6
Bakelite®	4.9	Silicone Oil	2.5
Fused Quartz	3.78	Strontium titanate	233
Neoprene rubber	6.7	Teflon®	2.1
Nylon	3.4	Vacuum	1.00000
Paper	3.7	Water	80
Polystyrene	2.56		

Example

Some cell walls in the human body have a double layer of surface charge, with a layer of negative charge inside the wall and a layer of positive charge of equal magnitude on the outside. Assume that the surface area of the cell is 25 μm^2, the cell wall is 5.0×10^{-9} m thick with a dielectric constant of 6.
If the potential across the cell is 55 mV, calculate,

a) the magnitude of the electric field in the wall between the two charge layers

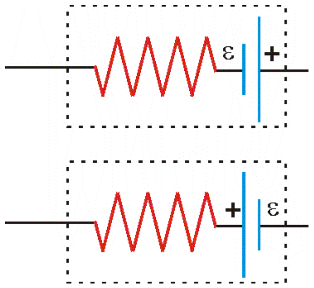
b) the capacitance of the cell wall

c) the charge on the cell wall

Symbols for Circuit Diagrams

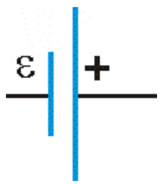
Conductor with negligible resistance

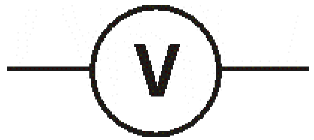
Resistor



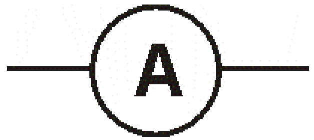
Source of emf with internal resistance

Source of emf





Voltmeter – used to measure potential difference between terminals. A voltmeters has large resistance and is connected in parallel.



Ammeter – measures current through section. Has very low resistance and is connected in series.

Capacitors in Parallel

A parallel combination of two capacitors is shown. In this configuration, the potential difference, V, across each capacitor is the same value as the battery. The total charge stored on the two capacitors is :

since $$Q = Q_1 + Q_2$$

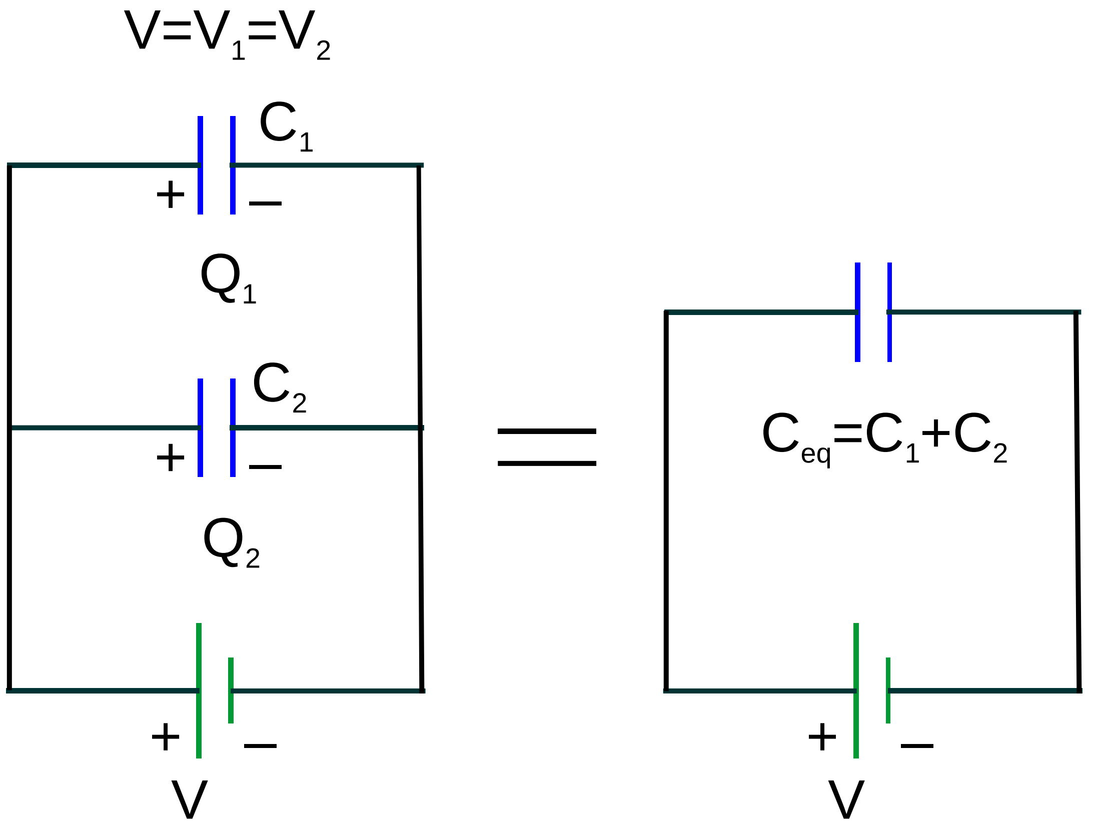
$$Q_1 = VC_1 \text{ and } Q_2 = VC_2$$

so

$$Q = VC_1 + VC_2 = VC_{eq}$$

the equivalent capacitance is:

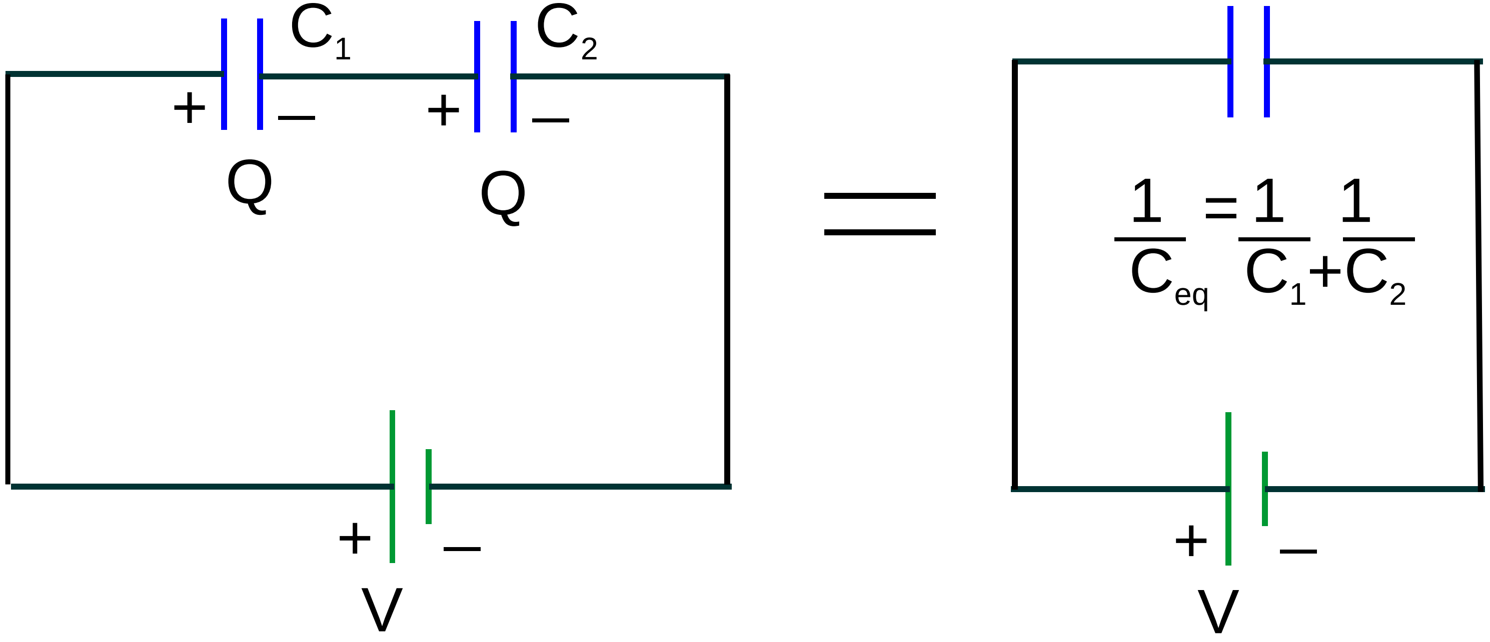
$$C_{eq} = C_1 + C_2$$



Capacitors in Series

A series combination of two capacitors is shown. In this configuration the charge on the two ‘inner’ plates must be equal and opposite so the charge on the outer plates must also be equal and opposite. The potential difference on the outer plates must be the same as that of the battery.

$$V=V_1+V_2$$



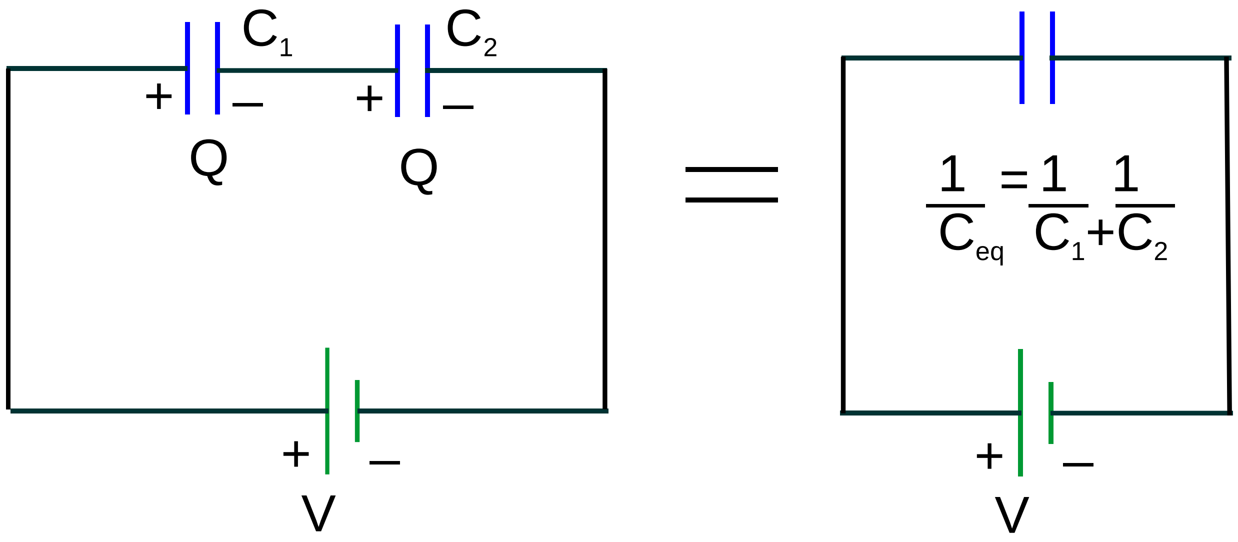
Capacitors in Series

Since

$$V_1 = \frac{Q}{C_1} \text{ and } V_2 = \frac{Q}{C_2}$$

then

V=V$_1$+V$_2$



$$V = V_1 + V_2 \text{ then } \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

the equivalent capacitance, C_{eq}, can be found from:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Summary Equivalent Capacitance:

The equivalent capacitance for capacitors in parallel is the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + ...$$

The equivalent capacitance for capacitors in parallel is always greater than any individual capacitance.

For capacitors in series, the INVERSE of the equivalent capacitance is the sum of the inverses of the individual capacitances:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

The equivalent capacitance for capacitors in series is always smaller than any individual capacitance.

Example

Two capacitors of value 12.5 μF and 8.8 μF are connected a) in parallel and then b) in series. Calculate the effective capacitance of each arrangement, the charge and voltage drop on each capacitor when connected to a 16.0 V source.

Example cont…..

Two capacitors of value 12.5 μF and 8.8 μF are connected a) in parallel and then b) in series. Calculate the effective capacitance of each arrangement, the charge and voltage drop on each capacitor when connected to a 16.0 V source.

Example

A potential difference of 300 V is applied to a 2.0 mF and an 8.0 mF capacitor in series.

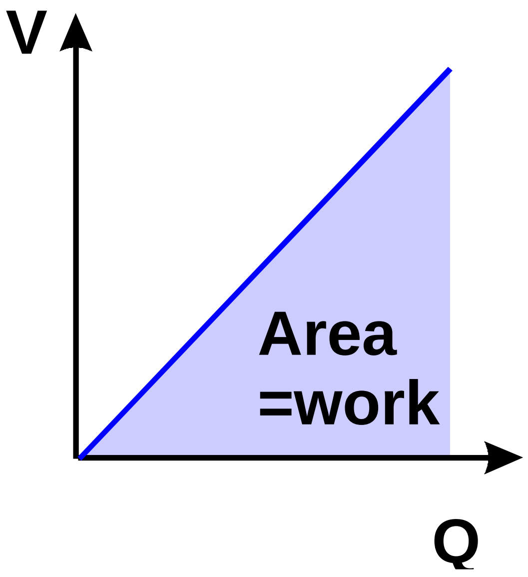
Find the charge and the potential difference for each capacitor.

Stored Energy on a Capacitor

We know:

$$V = \frac{W}{Q} \quad \Rightarrow \quad W = VQ$$

Plotting V versus Q gives a straight line. The area under the line is the work done to fill the capacitor with charge or energy stored on the capacitor. It is a triangle.



Work:

$$\text{Area } \Delta = \frac{1}{2}\text{Base} \times \text{Height}$$

$$\text{W} = \frac{1}{2}\text{QV}$$

Stored Energy on a Capacitor

From

$$W = \frac{1}{2} QV \qquad \text{and} \qquad V = \frac{Q}{C}$$

Energy stored:

$$\textit{Energy Stored} = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

Example

Three capacitors having capacitances of 10.0 μF, 10.0 μF, and 6.0 μF are connected in series across a 20-volt line.

a. Calculate the charge on the 6.0 μF capacitor.

b. Calculate the total energy of all three capacitors.

Example cont….

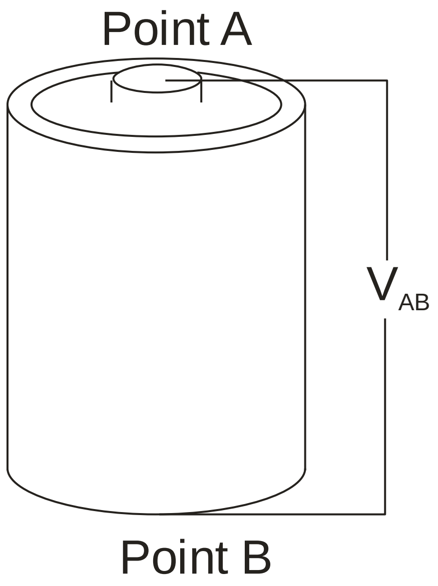
The capacitors are disconnected from the line and reconnected in parallel with the positively charged plates connected together.

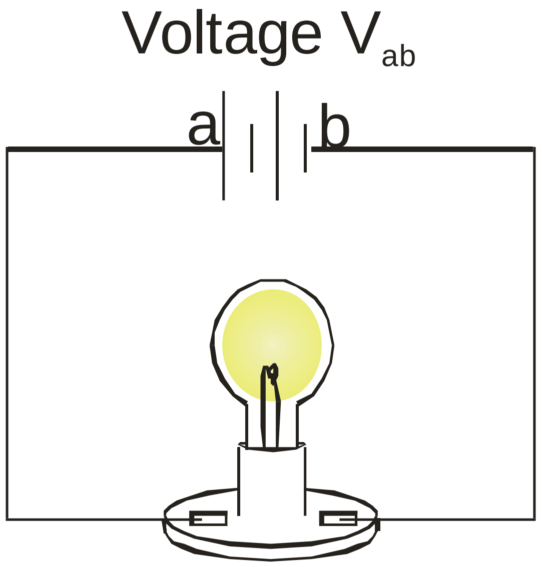
c. Calculate the voltage of the new combination.
d. Calculate the energy stored in the combination.

What is the importance of all this?

Batteries

Power Supplies





Have the potential to move charge!!!!! Have the ability to store charge for later use.