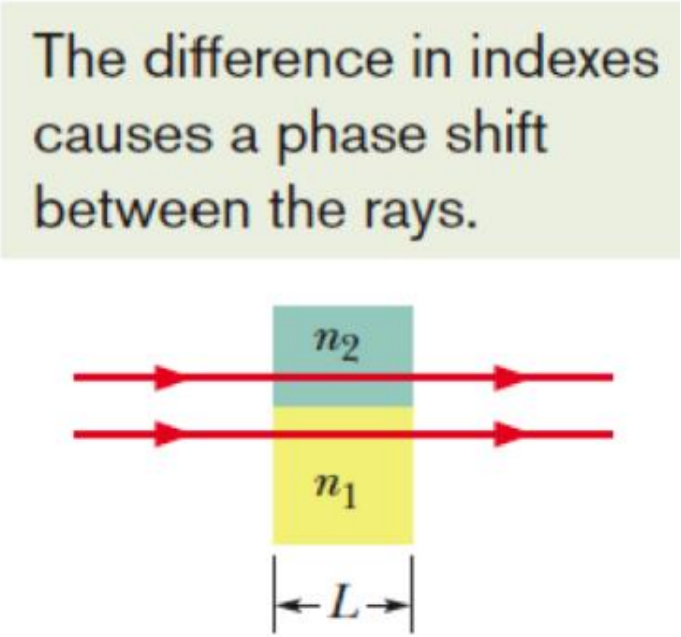
Phase Shifts and Diffraction

Text: Walker *etal. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition*
John Wiley & Sons Australia (HW)

Phase shift and refractive index

Recall $n = \dfrac{c}{v}$, as frequency does not change it follows that: $n = \dfrac{\lambda}{\lambda_n}$

When identical waves travel through material with the same thickness but different refractive index, a phase difference is introduced owed to the difference in effective path lengths.



To find this new phase difference consider the length of the medium.

In medium 1 the number of waves is $N_1 = \dfrac{L}{\lambda_1}$ while in 2 the number of waves are $N_2 = \dfrac{L}{\lambda_2}$

Phase shift and refractive index

Now $\dfrac{n_0}{n_1} = \dfrac{\lambda_1}{\lambda_0}$ and $\dfrac{n_0}{n_2} = \dfrac{\lambda_2}{\lambda_0}$

where n_0 is the refractive index of the incident medium and λ_0 is the wavelength in the incident medium.

Rearranging $\lambda_1 = \dfrac{n_0 \lambda_0}{n_1}$ and $\lambda_2 = \dfrac{n_0 \lambda_0}{n_2}$

substituting $N_1 = \dfrac{L n_1}{n_0 \lambda_0}$ and $N_2 = \dfrac{L n_2}{n_0 \lambda_0}$

Wavelength difference:

$$N_2 - N_1 = \frac{L n_2}{n_0 \lambda_0} - \frac{L n_1}{n_0 \lambda_0} = \frac{L}{n_0 \lambda_0}\left(n_2 - n_1\right)$$

Phase shift and refractive index

Phase difference:

$$\phi = 2\pi\left(N_2 - N_1\right)$$

Note for full wavelengths the phase difference is 2π so the effective difference is only the decimal of $N_2 - N_1$

Example 1

What is the phase difference when 633 nm light incident from air onto a junction of two glasses (n = 1.40 and n = 1.52) of thickness 1.00 μm?

Interference and Diffraction

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Light…..colour……wavelength…

Nature is divine………….

What we see is a result of optical interference and changes as your viewing perspective changes.

Examples of interference of light waves

- Interference occurs when waves are ‘added’ together.

Examples:

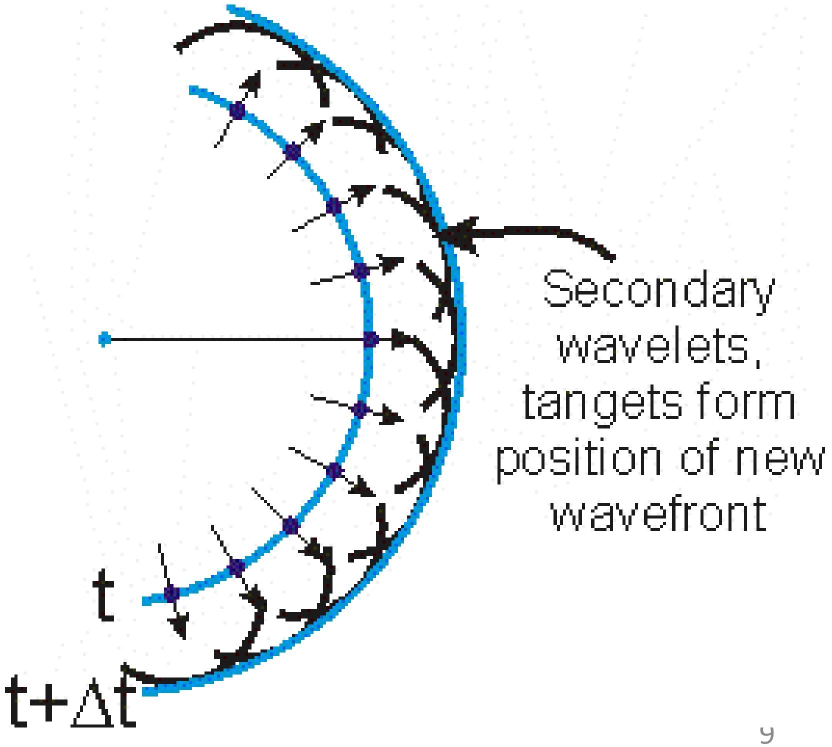
Soap bubbles, oil slicks, holograms, concert hall acoustics etc.

- The intensity of the new wave is not the simple sum of the constituent waves.
- The phase difference between the waves has to be taken into account.

Huygen’s Principle

Geometrical method for finding the shape of a wavefront some time later when we know the position of a wavefront at time, t.

Huygen’s assumption is that every point of a wave front may be considered the source of secondary a wavelet. These spread out in all directions with a speed equal to the speed of propagation of the wave.



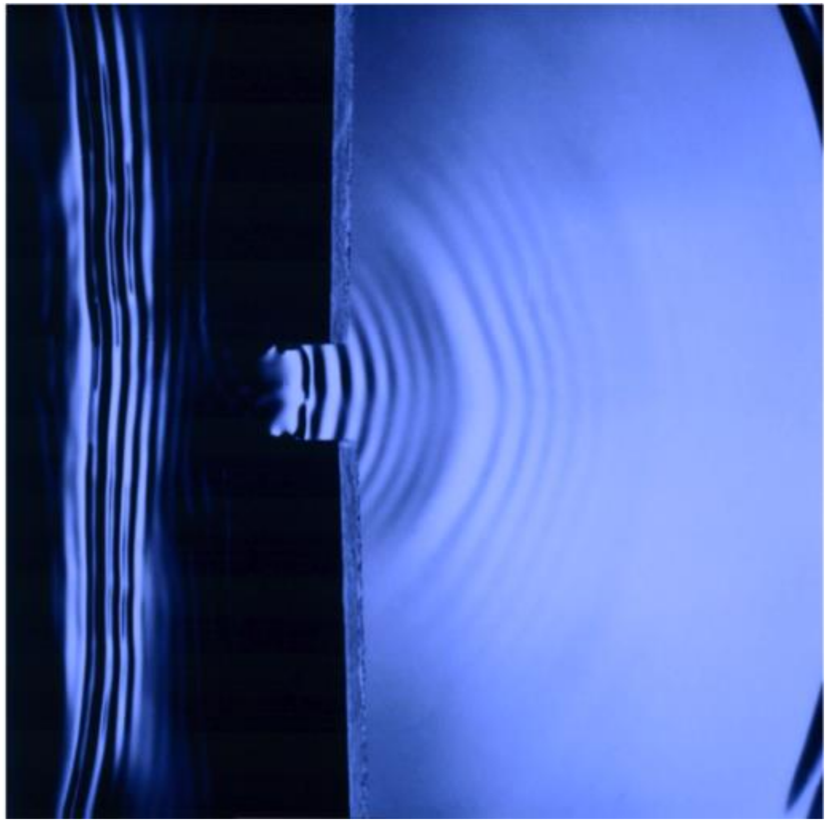
The new wavefront can be determined by constructing a tangent to the wavelets.

We will use this when examining Young’s experiments.

http://www.mincel.com/featured/how-to-photograph-water-drops/ The photo is from Mincel’s blog …. Divine.

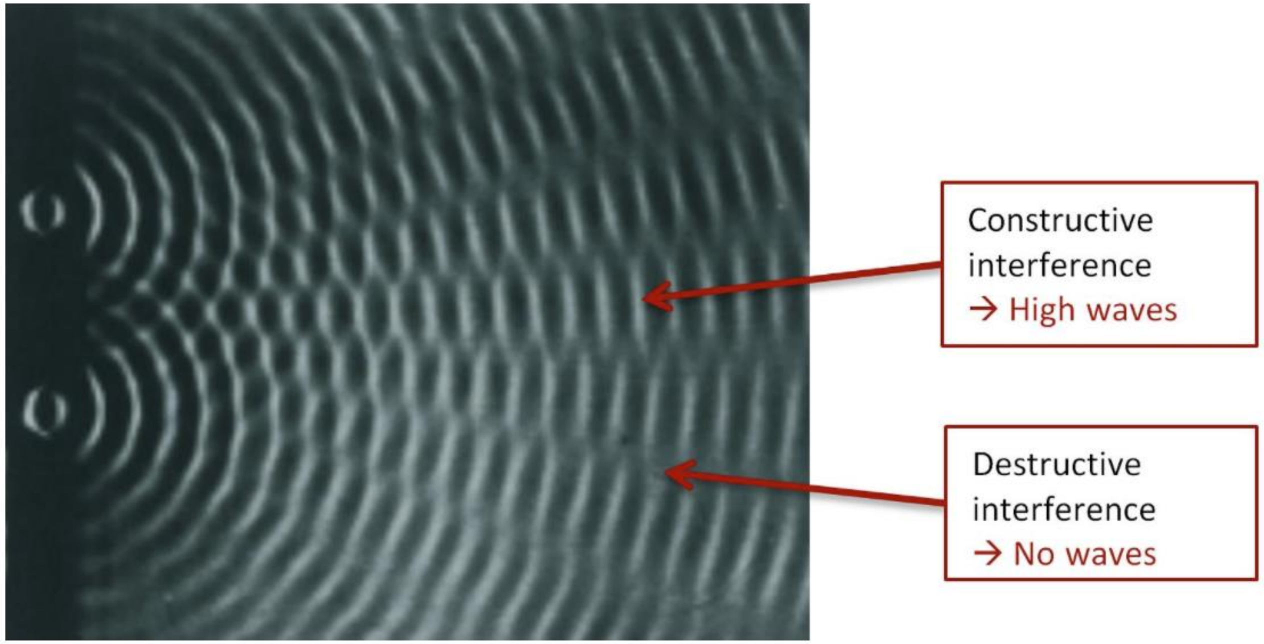
Diffraction of water waves

If a wave encounters a barrier that has an opening of similar dimension to the wavelength, the part of the wave that passes through the opening will flare out, or 'diffract' into the region beyond the barrier. This is consistent with the spreading of wavelets according to Huygen's principle. Diffraction occurs for waves of all types.



Examples of diffraction: hearing 'around corners', holding thumbs close together and looking at light source, diffraction grating etc.

Interference and diffraction of waves



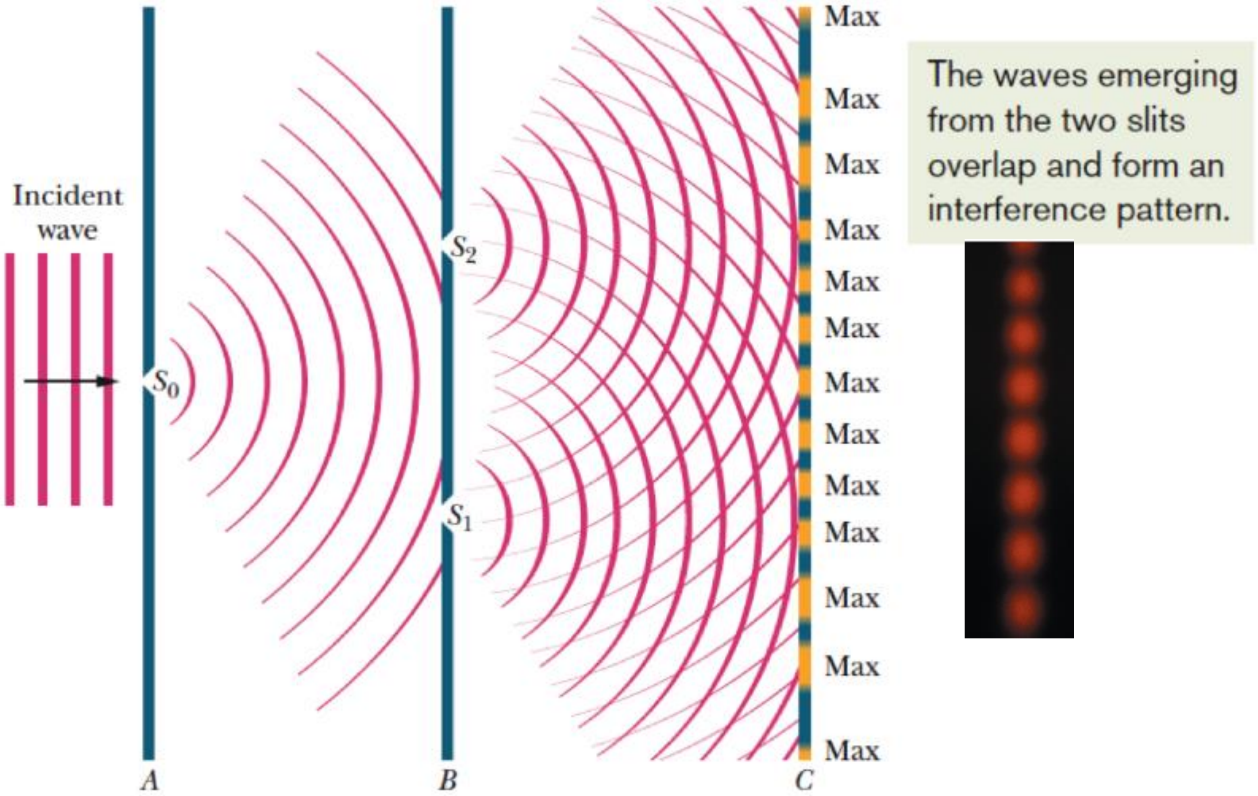
The figure on the left shows how water waves spread out in circles from two points and how they interfere.

Whenever a wave crest coincides another wave crest this results in constructive interference (high waves), whereas a wave crest which meets a wave trough results in destructive interference.

Picture https://wiki.anton-paar.com/en/double-slit-experiment/

Young's Interference Experiment – light – double slit diffraction

Key experiment (1801) showing that light travels as waves. The pattern observed with maxima and minima was similar to that observed for water waves passing through two slits.

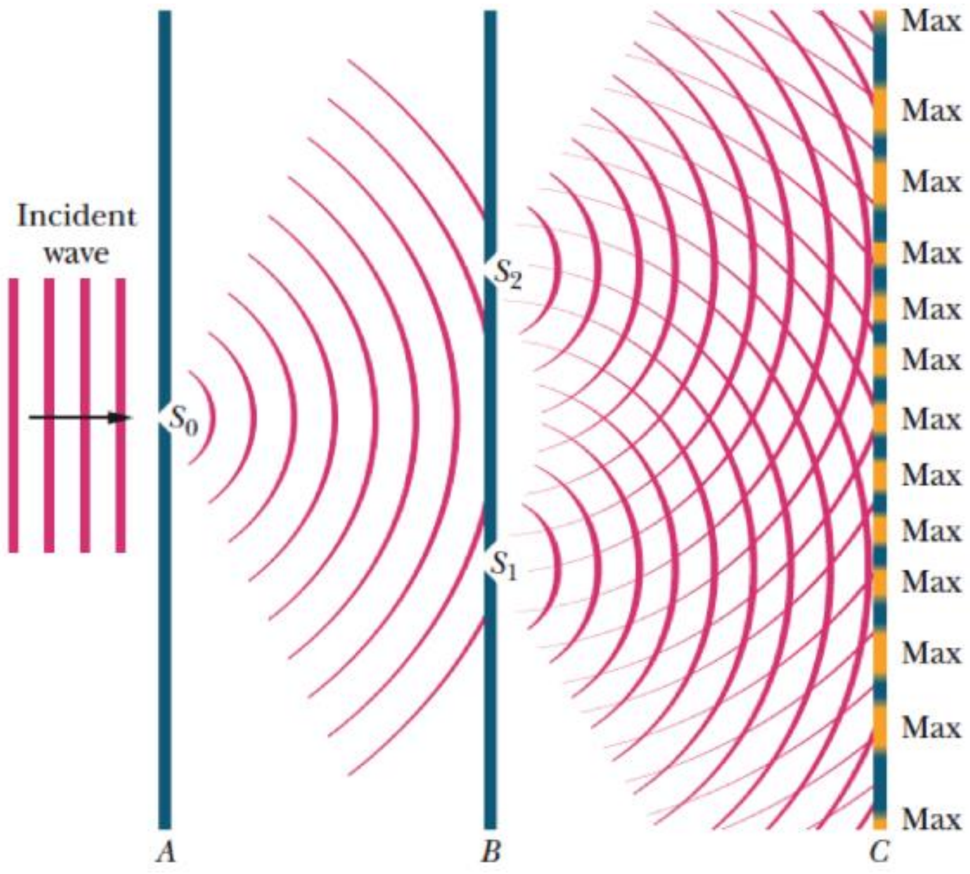


Here monochromatic light is incident on S_0 and diffracted, acting as a point source of light. This is incident on S_1 and S_2 which are a distance 'd' apart acting as two point sources, these then interfere on a screen a perpendicular distance 'D' from the slits.

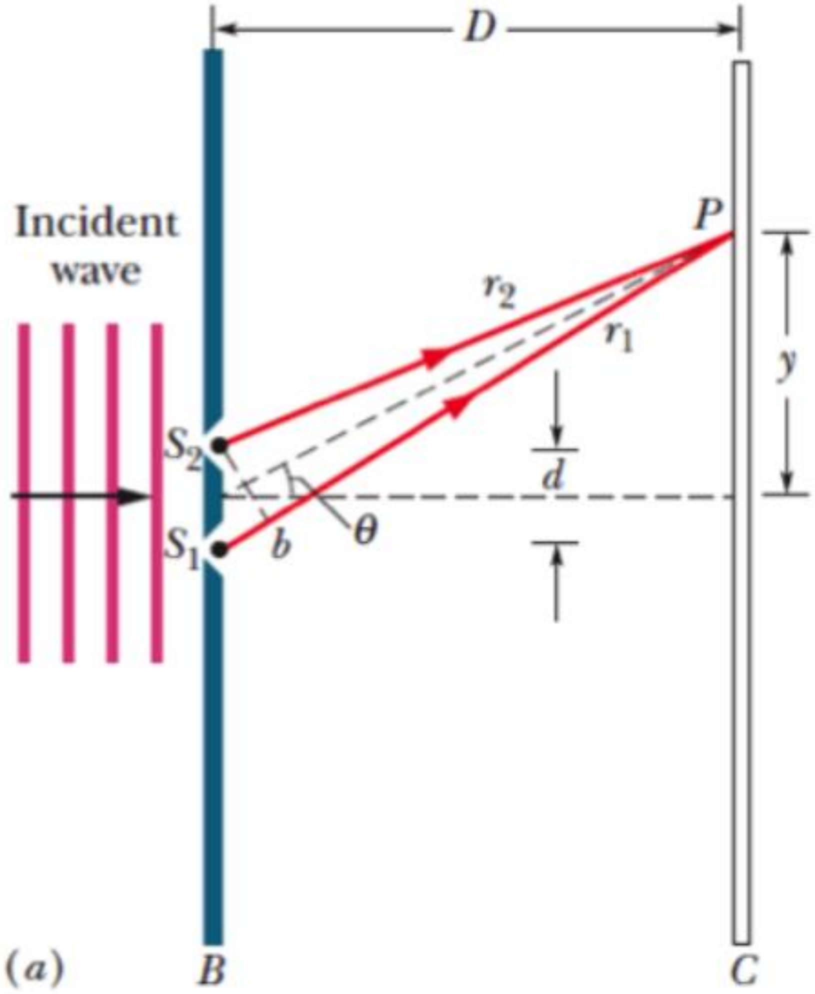
Coherence

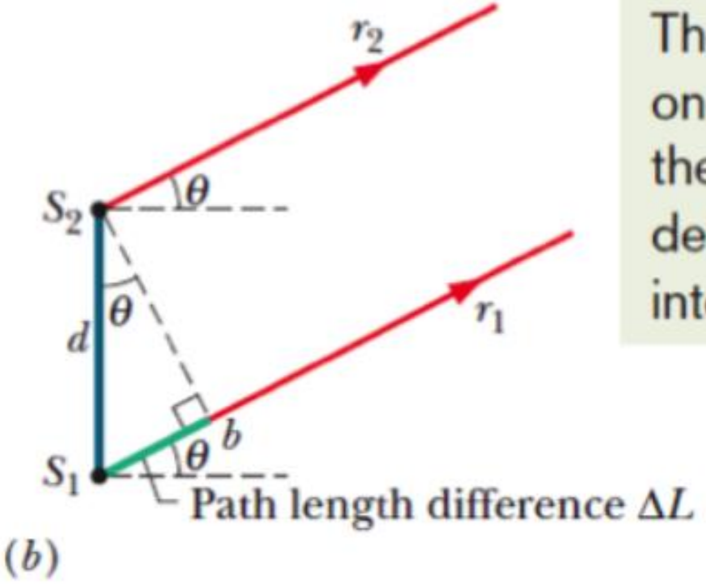
To obtain an interference pattern, the light waves reaching any point P on the screen must have a phase difference that does not vary with time. When the phase difference remains constant, the light from the slits S_1 and S_2 is said to be completely coherent.

If the phase difference varies with time, then the light is said to be incoherent.



Calculation of the Fringe Positions





The ΔL shifts one wave from the other, which determines the interference.

Calculation of the Fringe Positions cont…..

For a bright fringe (maxima), ΔL must be either zero or an integer number of wavelengths:

$$\Delta L = d\sin\theta = n\lambda \quad (n = 0,1,2\ldots\ldots)$$

For a dark fringe (minima), ΔL must be the odd multiple of half a wavelength:

$$\Delta L = d\sin\theta = \left(\frac{2n-1}{2}\right)\lambda \quad (n = 0,1,2\ldots\ldots)$$

Example 2

A Young's experiment is carried out using 546 nm light and a slit separation of 0.12 mm. If the screen is 55 cm from the slits, what is the distance on the screen between the central maximum and the next minimum?

Interference and Diffraction Continued

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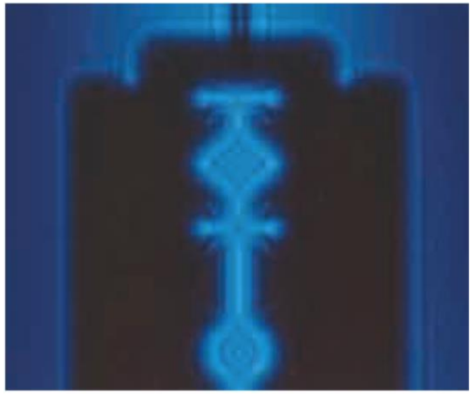
Single Slit Diffraction

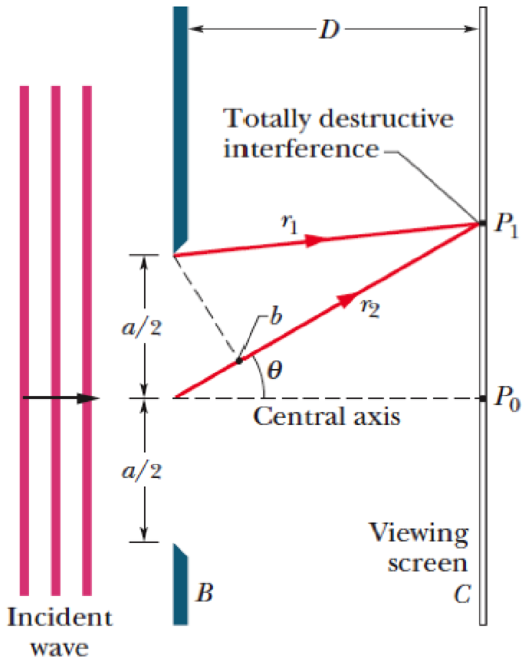
Young's slit showed that light bends as it emerges from a narrow slit. This is not all. An interference pattern also occurs from this single slit called a **diffraction pattern** something like the left image.

The pattern includes a broad intense central maximum with a number of narrowed, less intense maxima (known as **secondary** or **side** maxima). Between the maxima are minima.

Diffraction also occurs when light passes a sharp edge like a razor blade:



Single Slit Diffraction



When the diffracted light reaches the screen, waves from different points of slit undergo interference resulting in a diffraction pattern of bright and dark fringes.

Central fringe, P_0, all points travel about same distance thus in phase – bright.

For minimum – let the pair of rays from the top of the two halves cancel each other at P_1, similar for other such pairings.

For a dark fringe, r_1 and r_2 must arrive out of phase by $\frac{\lambda}{2}$.

Single Slit Diffraction Cont…..

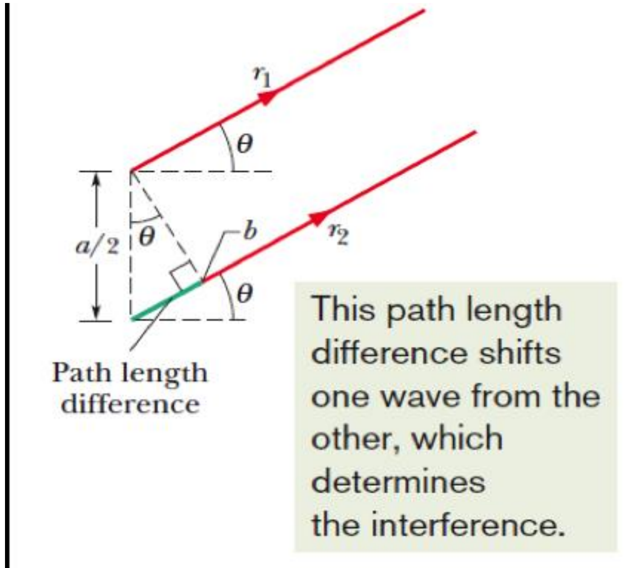


Fig. 36-5 For $D \gg a$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

The phase difference is owed to the path length difference.

If $D >> a$ then can consider the rays as parallel.

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad \Rightarrow \quad a\sin\theta = \lambda$$

Generally for minimum (dark fringes)

$$a\sin\theta = m\lambda \quad m = 1, 2, 3, ...$$

In a single slit experiment, dark fringes are produced when the path length differences ($a\sin\,\theta$) between the **top and bottom** rays are equal to λ, 2λ, 3λ,…

Phase Difference ….

Imagine that we divide our slit into N zones of equal width Δx which are small enough so that the assumption that each zone acts as a Huygen's wavelength is valid.

The phase difference between wavelets from adjacent zones is given by:

$$\begin{pmatrix} phase \\ difference \end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix} path\ length \\ difference \end{pmatrix}$$

For point P at angle θ, the path length difference is $\Delta x \sin\theta$ so the phase difference in radians between wavelets from adjacent zones is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x \sin\theta$$

Example 3

A diffraction pattern is created by illuminating a narrow slit with blue light. What happens to the pattern if we

a. Switch to yellow light (longer wavelength)?
b. Decrease the slit width?

Diffraction by a Circular Aperture

The image shows the effect of laser light passing through an small circular aperture of diameter, d. (Fresnel Bright Spot)

The image is overexposed to bring out the weak secondary maxima.

The first minimum for the diffraction pattern for a circular aperture of diameter d is located:

$$\sin\theta = 1.22\frac{\lambda}{d}$$

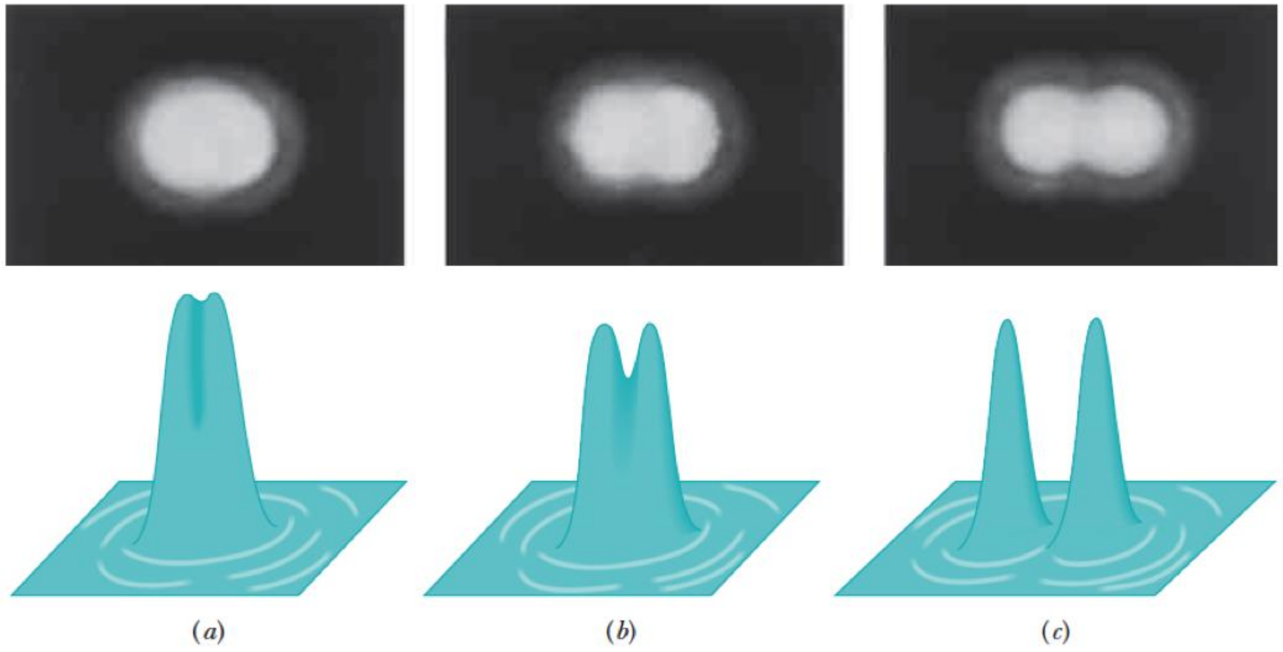
Compare to a rectangular slit of width, a:

$$\sin\theta = \frac{\lambda}{a}$$

Main difference is the factor 1.22 owed to the shape of the aperture.

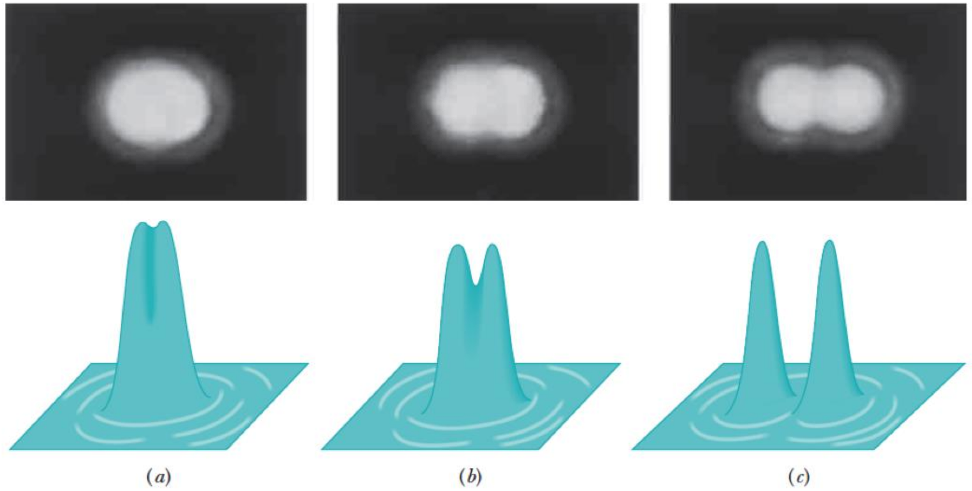
Rayleigh's Criteria and Resolvability

Imagine the image of two stars formed by a converging lens.



Whereas in (a) the angular separation is too small for the stars to be resolved, i.e. their diffraction patterns overlap in (c) they are clearly distinguishable.

Rayleigh's Criteria and Resolvability



In (b) they are marginally distinguishable. In (b) the central maximum of one source is centred on the first minimum of the other. This condition is called **Rayleigh's Criterion** for resolvability.

From the equation for a circular aperture this criterion must have an angular separation θ_R of:

$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{d}\right) \quad \text{or} \quad \theta_R = 1.22\frac{\lambda}{d}$$

in radians for small angle where $\theta \sim sin\theta$.

Example 4

Suppose you can barely resolve two headlights of a distance car, due to the diffraction by the pupil of your eye.

If the illumination is increased so the pupil decreases in diameter, does your resolvability improve of get worse?

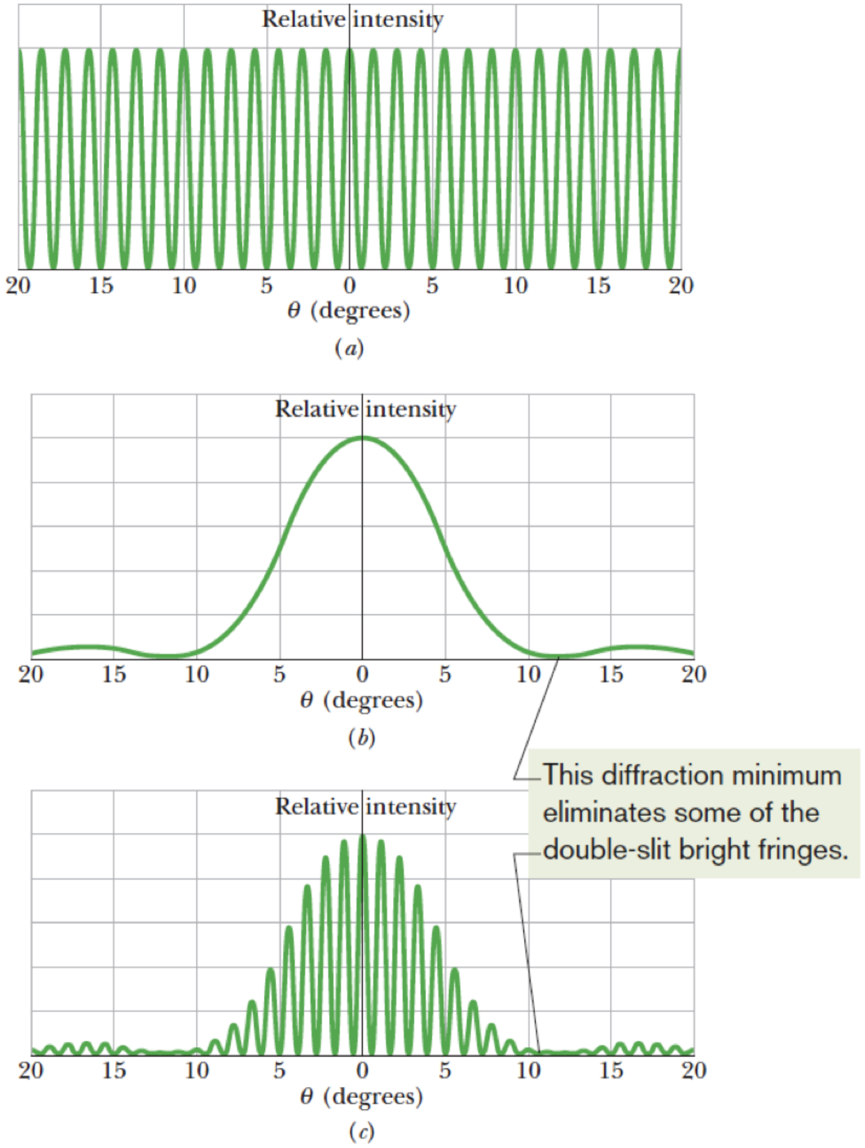
Diffraction in Young's Experiment

If $a << \lambda$ for the slits in a double slit experiment then the pattern in (a) is observed. In practice the slits are rarely so small.

(b) Diffraction pattern for a slit of width, a.

Curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a) giving

(c) for the double-slit experiment.



Diffraction in Young's Experiment cont…

When diffraction effects are taken into account, the intensity of a double-slit interference pattern is:

$$I(\theta) = I_m \left(\cos^2 \beta\right)\left(\frac{\sin \alpha}{\alpha}\right)^2$$

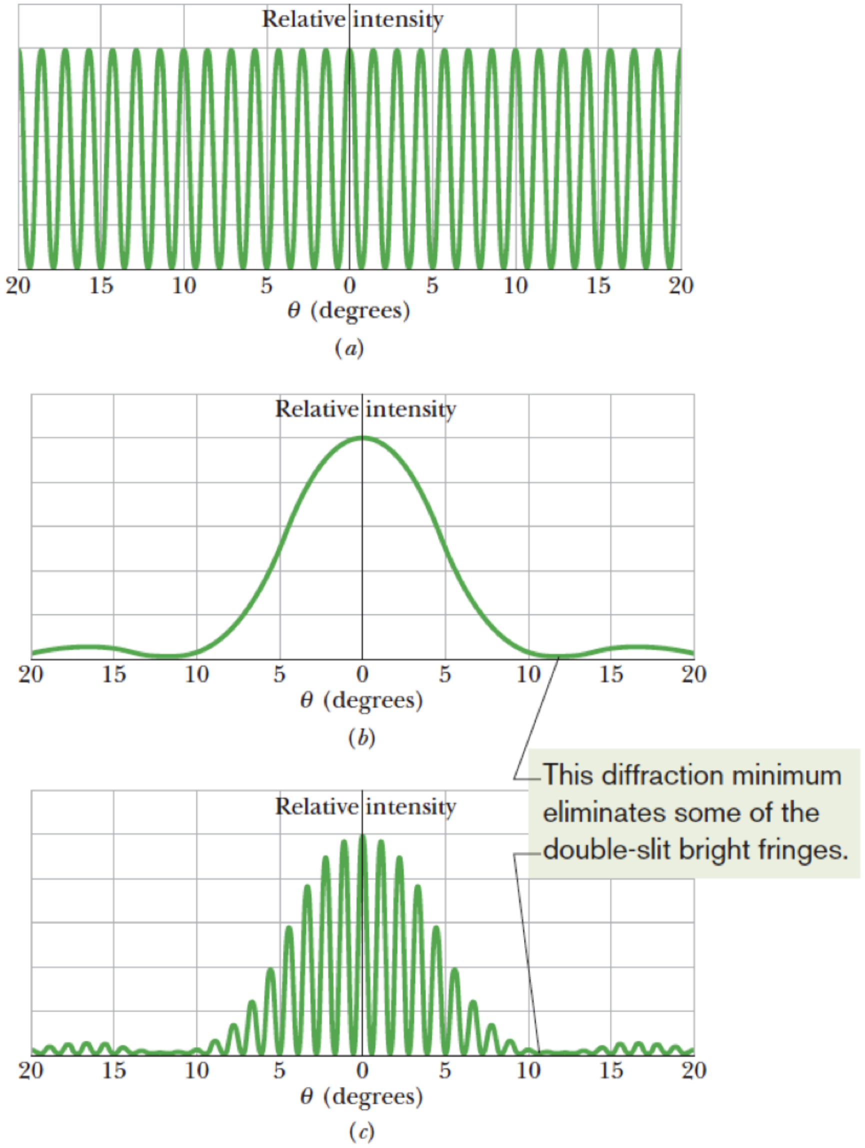
where

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

d is the distance between the slits and a is the slit width.



Diffraction Grating…

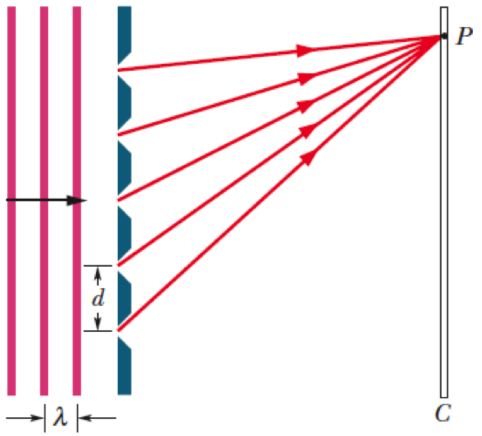
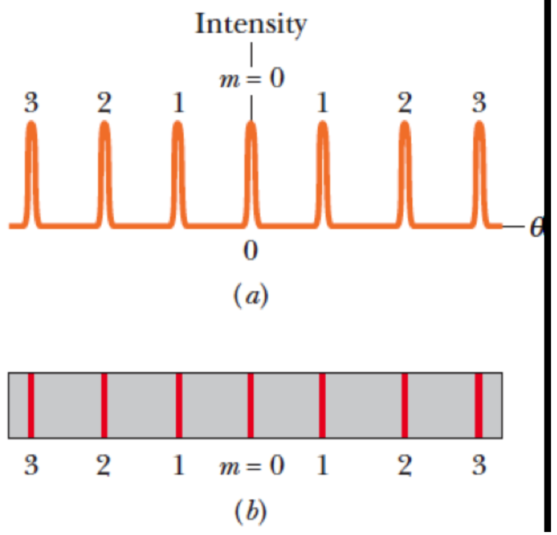


Fig. 36-18 An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen C.

The diffraction grating is like the double slit with a lot more slits, N (often called rulings).

When monochromatic light passes through the grating, narrow interference fringes are formed. The analysis of these fringes allows the determination of the wavelength of the light.



The pattern for a great many rulings consists of narrow peaks.

For maxima,

$$d \sin\theta = m\lambda$$

with $m = 0,\ 1,\ 2,1\ \ldots$

Gratings: Dispersion and Resolving Power

Dispersion (D) : spreading of diffraction lines owed to wavelength

Defined as the derivative of the diffracted angle with respect to wavelength.

Differentiating

$$d\cos\theta d\theta = m d\lambda$$

and rearranging gives: $$D = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$$

Also: $$D = \frac{\Delta\theta}{\Delta\lambda}$$

where $\Delta\theta$ is the angular separation of the two lines whose wavelengths differ by $\Delta\lambda$.

Gratings: Dispersion and Resolving Power

Resolving power

The ability to distinguish different wavelengths with a grating depend on the width of the maxima.

Resolving power is defined as $R = \frac{\lambda_{avg}}{\Delta\lambda}$, where $\Delta\lambda = \lambda_1 - \lambda_2$ with λ_1 and λ_2 being two just – resolvable wavelengths; λ_{avg} is the average of these wavelengths.

For a grating, $R = Nm,$ with N being the number of slits illuminated.

Example 5

A diffraction grating has 500 lines/mm. It is illuminated at normal incidence by a mercury vapour lamp having a yellow doublet at 577.0 nm and 579.1 nm.

At what angle does the first order maximum occur?

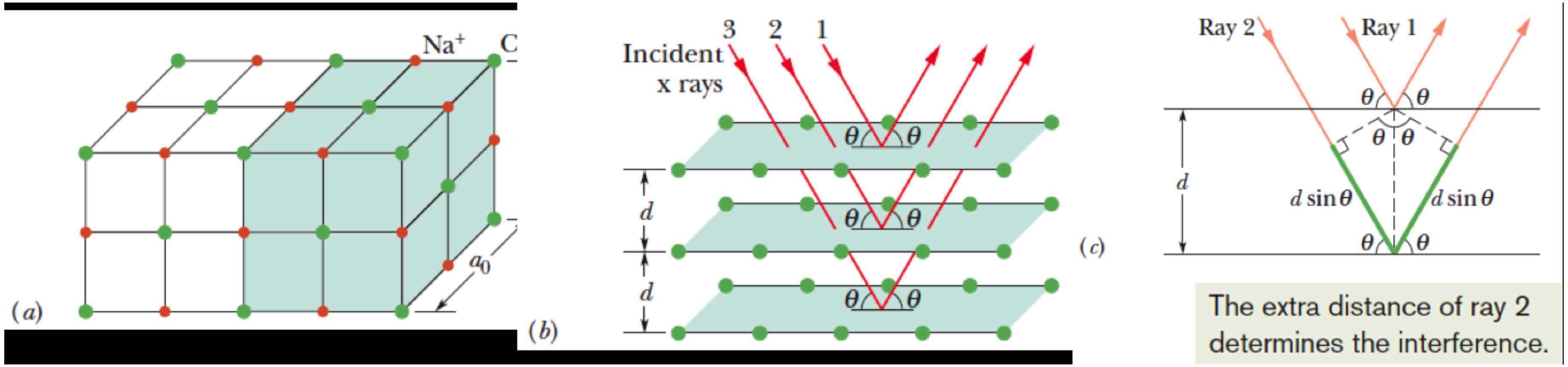
Example 6

For example 5 use the grating dispersion to calculate the angular separation between the two lines (in the first order).

Example 7

In example 5 what is the smallest number of lines that must be illuminated to resolve the doublet in first order?

X-ray Diffraction



•Incident x-rays undergo diffraction by the lattice.

•The x-rays are diffracted as if from a family of parallel planes with $\theta_{incident} = \theta_{reflection}$, both measured *relative to the planes.*

•The path length difference between waves effectively reflected by two adjacent planes is $2dsin\theta$ therefore for x-ray diffraction intensity maxima

$$2d\sin\theta = n\lambda \qquad \text{with } n = 0, 1, 2, \ldots.$$