Current, Resistance and Electromotive Force

These lectures slides based on lectures by Dr. Danica Solina

1

CONDUCTOR

In ordinary metal e.g. copper, some electrons free to move, and do so randomly ($v > 10^6$ ms^{-1}) having collisions with atoms. A similar situation exists in ionic liquids, plasmas…

The electrons are attracted to the positive ions of the material therefore do not escape so there is no net flow (or current) of electrons.

APPLY A STEADY ELECTRIC FIELD INSIDE THE CONDUCTOR

Charged particles are then subject to a constant force

$$\vec{F} = q\vec{E}$$

There are still collisions with random direction changes but together with the field there is a **slow net motion in the direction of the field**.

2

Drift Velocity, $\vec{v}_d$

Term for the net motion of the particles

Random average motion ~ 10^6 ms^{-1}

Drift velocity ~ 10^{-4} ms^{-1}

This field is set up in and around the wire with the speed of light ~ 10^8 ms^{-1} .

The electrons all start to move at ~ the same time even though slow therefore instantaneously turned 'on' regardless of the speed.

3

Work and Energy : Field and Current

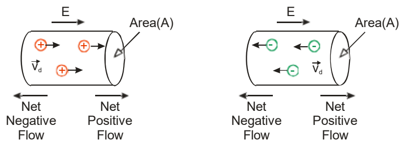
- $\vec{E}$ does work on the moving charges.
- Moving charges have kinetic energy.
- Kinetic energy transferred to conductor materials by means of collisions with ions.
- Ions vibrate about their equilibrium positions.
- Energy transfer results in an increase in temperature
- Most of the work done by the $\vec{E}$ goes into heating the conductor, not into moving the charges faster.

4

Direction Current Flow

Moving particles ⟶ positive or negative
Negative ⟶ metals
Both ⟶ ionized gas or ionic solution
Holes and Electrons ⟶ semiconductors

Direction of Charge Flow



5

Conventional Current

Current (I) is defined to be in the direction of flow of positive charge called conventional current.

This is historical but the sign of the moving charges is of little importance in electric circuit analysis.

Current

Current is defined as the net charge (dQ) through the area per unit time:

$$I = \frac{dQ}{dt}$$ (A- amperes)

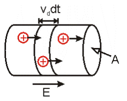
6

Current, Drift Velocity and Current Density

- Assume free charge is positive, then $\vec{v}_d$ is in the direction of $\vec{E}$.
- Let n = moving charged particles per unit volume (m^{-3}).
- Assume all move $v_d \Rightarrow s_d = v_d dt$ in the interval dt.
- Volume $dV = Av_d dt$ Number Particles $n\,dV$

Now:

$$dQ = nAv_d dt\, q \Rightarrow I = \frac{dQ}{dt} = nAv_d q$$



7

Current Density (J)

- Current per unit cross-sectional area.

$$J = \frac{I}{A} = nv_d |q|$$

- This is independent of the charge sign.

8

Current Density Vector ($\vec{J}$)

- Current per unit cross-sectional area.

$$\vec{J} = nq\vec{v}_d$$

- This is dependent on the sign of the charge but when q is positive, $\vec{v}_d$ is in the direction of E.
- When q is negative, $\vec{v}_d$ is in opposite direction of E.
- Both cases $\vec{J}$ is positive and in the direction of E.

9

Example

Copper wire of nominal diameter 1.00 mm carries a constant current of 1.50 A. The free electron density is 8.5×10^{28} electrons per m^3.

Find the magnitude of (a) current density (b) drift velocity.

10

Resistivity (ρ)

At a given temperature

$$\vec{J} \; \alpha \; \vec{E} \qquad \vec{J} = \frac{1}{\rho}\vec{E}$$

Ratio

$$\text{constant} = \frac{\vec{E}}{\vec{J}} = \rho$$ (units: Ω.m)

The greater the resistivity the larger field needed to produce a given current density.

Metals: small ρ ~10^{-14} Insulators: > by × 10^{22}

N.B. $\sigma = \frac{1}{\rho} = \frac{\vec{J}}{\vec{E}}$ This is conductivity (units: $\Omega^{-1}.m^{-1}$)

11

Conductors

•Good electrical conductors are usually good heat conductors e.g. silver, copper, tungsten

- Poor electrical conductors are usually poor heat conductors e.g. ceramic, plastic

Resistivity and Temperature

$$\rho(T) = \rho_0\big(1 + \alpha(T - T_0)\big)$$

where α is the temperature coefficient of resistivity

- This is valid over a temperature range of ~ 100 °C.

12

Temperature Coefficients of Resistivity, α ((°C)$^{-1}$)
(near room temperature)

Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon(graphite)	-0.0005	Mercury	0.0088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

HW : Sec. 26.3 Electromotive Force and DC Circuits 13

13

Resistance (R)

$$\vec{E} = \rho \cdot \vec{J}$$

Now $E = \frac{V}{L}$ and $J = \frac{I}{A}$

Substitute: $\frac{V}{L} = \rho \frac{I}{A} \Rightarrow V = \rho \frac{L}{A} I$

when ρ = constant then V α I. Let resistance (R) be

$$R = \frac{V}{I} = \rho \frac{L}{A}$$

Units: Ω - Ohms

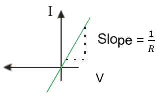
HW : Sec. 26.3 Electromotive Force and DC Circuits 14

14

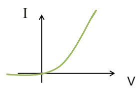
Ohm's Law (as we know it)

$$V = IR$$

Ohmic resistor



Semiconductor diode



Also because ρ α T and $R = \rho \frac{L}{A}$ then $R(T) = R_0\,[1 + \alpha(T - T_0)]$

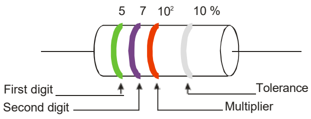
HW : Sec. 26.4 Electromotive Force and DC Circuits 15

15

Resistor

Circuit device with specific resistance value.

Symbol:



$57 \times 10^2\,\Omega \pm 10\%$
$5.7\,k\Omega \pm 10\,\%$

16

Resistor Colour Table

Colour	Digit Value	Multiplier Value
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Tolerance:		
	no band	± 20 %
	silver band	± 10 %
	gold band	± 5 %

17

Example

For the previous example with copper wire ($d = 1.00$ mm, $I = 1.50$A, $n = 8.5 \times 10^{28}\ m^{-3}$) and given $\rho = 1.72 \times 10\text{-}8\ \Omega m$ for copper at 20 °C find

a. The magnitude of E within the wire,

b. The potential difference if L = 50.0 m

18

Example cont……

c. The resistance of the wire given $\rho = 1.72 \times 10^{-8}\ \Omega\,m$ for copper at 20 °C

d. Given the temperature coefficient of resistivity, α, is 0.00393 $°C^{-1}$, what is the resistance at 0 °C and 100 °C?

Electromotive Force and DC Circuits 19

19

Points of Importance

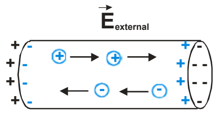
- Resistance is a property of the device/object.
- Resistivity is a property of the material from which the device/object is made.

Electromotive Force and DC Circuits 20

20

Electromotive Force and Circuits

- Steady current flow requires a closed circuit.
- Why? Let's look at a piece of wire in isolation



$\vec{E}_2$ forms in opposite direction owing to accumulated charge

$$\Rightarrow \quad \vec{E}_{tot} = \vec{E}_{ext} + \vec{E}_2 \quad = 0 \Rightarrow \quad \vec{J} = 0$$

- So current stops thus no steady motion occurs in an incomplete (open) circuit

HW : Sec. 27. 1

Electromotive Force and DC Circuits 21

21

How is a steady current maintained in a closed circuit?

Think fountain – water falls into the basin
- At the top: High PE → Low PE
- Work by gravitational field
- Water is then **pumped** back up against gravitational field
- Work by Pump: Low PE → High PE

Electric field force – push charge from high PE to low PE but Electromotive Force, aka EMF, is the pump that pushes the charge from low PE to high PE.

N.B. EMF is not a real force
but **energy per unit charge**

http://www.openclipart.org/detail/382/fountain-by-johnny_automatic

22

Electromotive Force (EMF), ε

Units: V or J/C

Sources : batteries, electric generators, solar cells, fuel cells which convert energy of some form to electrical energy.

Ideal Source: maintains constant potential difference between terminals

$$V_{ab} = \varepsilon = IR$$

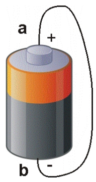
23

Internal resistance, *r*

A real source has **internal** resistance i.e. charge encounters resistance in the "pump". If ohmic then *r* is constant.

As current travels through *r*, it moves through a decrease in potential of *Ir*. Therefore as charge goes from negative terminal b to positive terminal a the potential difference between them is V_{ab}

$$V_{ab} = \varepsilon - Ir = IR$$



Where R is the **external** resistance in the circuit.
The current in the circuit is then given by

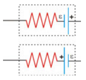
$$I = \frac{\varepsilon}{R + r}$$

24

Revision Symbols for Circuit Diagrams

Conductor with negligible resistance

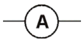
Resistor



Source of emf with internal resistance

Source of emf

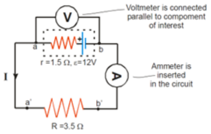
Voltmeter – used to measure potential difference between terminals. A voltmeters has large resistance and is connected in parallel.



Ammeter – measures current through section. Has very low resistance and is connected in series.

Example

What are the voltmeter and ammeter readings?



Potential Change around a Circuit

This is for ohmic resistors only.

$$V_{ab} = \varepsilon - Ir = IR$$

$$\Rightarrow \varepsilon - Ir - IR = 0$$

$$or$$

$$\varepsilon = Ir + IR$$

Energy and Power in Electric Circuits

There is a steady state flow therefore no increase in kinetic energy occurs but there is transfer of energy into the circuit elements e.g. heat for toaster element, or out of circuit e.g. battery.

$$-dW_{field} = dU = dQV_{ab} \quad \text{and} \quad dQ = Idt$$

$$(dW_{element} = -dW_{field}) \; P = \frac{dW}{dt} = \frac{dQ}{dt}V_{ab} = V_{ab}I$$

$$\boxed{P = VI}$$

Where does this energy come from?

- The electric field that is set up along the circuit element
- The magnetic field that goes around the circuit element (more later)

Rate energy delivered to/extracted from element.

HW : Sec. 27. 1 Electromotive Force and DC Circuits 28

28

Power (P)

$$P = V_{ab}I \quad \text{in J/s or W}$$

But $V_{ab} = IR$ for a resistive element.

$$\Rightarrow \quad P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R}$$

Power Out of a Source:

$$P = V_{ab}I = I\varepsilon - I^2r$$

HW : Sec. 27. 1 Electromotive Force and DC Circuits 29

29

Example

For the previous example, calculate the power dissipated (a) in the 3.5 Ω resistor and (b) in the internal resistance.

Electromotive Force and DC Circuits 30

30

Example

A storage battery has an emf of 25 V and an internal resistance of 0.20 Ω. Find its terminal voltage when

- **a.** it is delivering 8.0 A
- **b.** it is being charged with 8.0 A.

31