Moving Charges and Resulting Magnetic Fields in Conductors

These lectures slides were prepared by
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Text: Walker *etal. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition*
John Wiley & Sons Australia (HW)

Many thanks to Walter Kalceff whose notes formed the basic of these lectures….If I have seen further it is on the shoulders of giants!

The Magnetic Field:

Recall how the interaction of two charges was described through an electric field:

(1) one charge sets up an electric field $\vec{E}$ in the space around it

(2) the electric field exerts a force $\overrightarrow{F_E} = q\vec{E}$ on a charge within the field.

Charges in motion:

(1) a moving charge (current) sets up a magnetic field in the space around it.

(2) the magnetic field exerts a force on a moving charge (current) in it.

Magnetic field is denoted by the symbol $\vec{B}$. It too is a vector quantity. The field is directed from North to South outside a magnet and has the units Tesla (T).

What force is experienced by a charge in a magnetic field?

It depends on:

– magnitude and sign of the moving charge, q (C)

– velocity of the charge, $\vec{v}$ (m/s)

– magnitude of the magnetic field, $\vec{B}$ (T).

The force (N) is given by:

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B}$$

$\overrightarrow{F_B}$ is always perpendicular to both $\vec{B}$ and $\vec{v}$.

It has magnitude:

$$\overrightarrow{F_B} = q|\vec{v}|\left|\vec{B}\right|\sin\theta$$

where θ is the angle between the direction of the velocity and magnetic field.

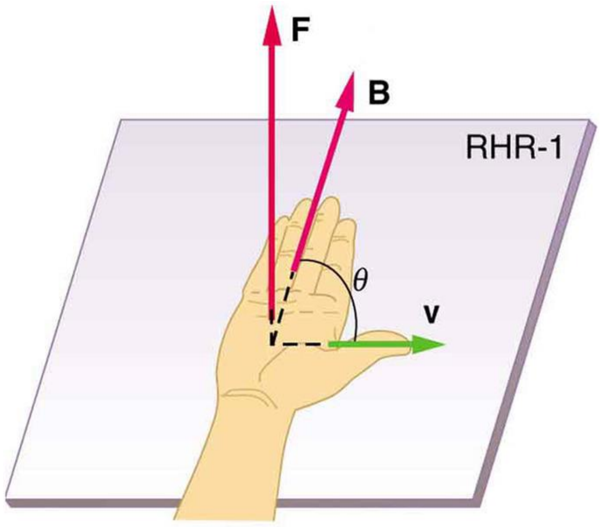
HW : Ch. 28.1

Direction force on a charged particle in magnetic field

- The right hand push rule can be used to find the direction of the force on a positive charge travelling in a magnetic field. The fingers point in the direction of the magnetic field, the thumb indicates direction of velocity of the POSITIVE particle and the palm the direction of the force on the particle.

- The left hand can be used for NEGATIVE particles with fingers still pointing in the direction of the magnetic field and thumb the direction of the velocity of the NEGATIVE particle and the palm the direction of the resulting force on the particle.

What if the particle entered an extended uniform magnetic field always perpendicular to its path?

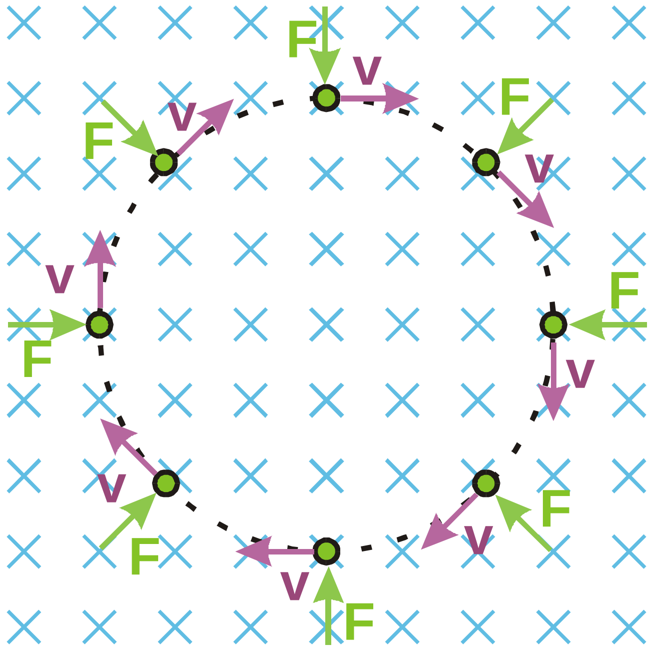


$F = qvB \sin\theta$

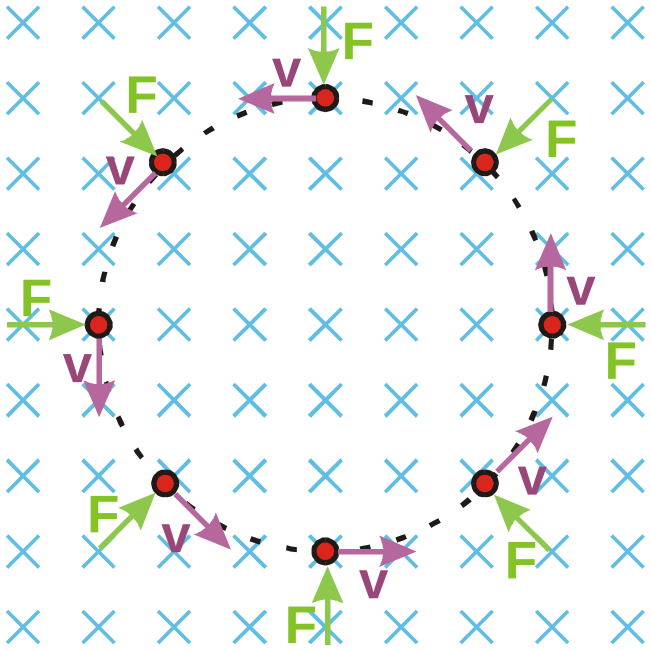
$\mathbf{F} \perp$ plane of $\mathbf{v}$ and $\mathbf{B}$

https://www.boundless.com/physics/textbooks/boundless-physics-textbook/magnetism-21/magnetic-force-on-a-moving-electric-charge-157/direction-of-the-magnetic-force-the-right-hand-rule-553-10931/

Charged Particle in Magnetic Field



Electron in Magnetic Field



Proton in Magnetic Field

- Circle :if the motion is in a plane.
- Spiral with fixed radius if there is also a vertical vector.

Magnetic Force

- It is the basis of many devices, including
  - ammeters, voltmeters
  - TV picture tubes (back in the day before plasma)
  - magnetic bottles for plasma containment in fusion
  - mass spectrometers

- Magnetic forces are difficult to picture, because $\vec{F}$ is perpendicular to both $\vec{B}$ and $\vec{v}$.

Units of $\vec{B}$

Given:

$$B = \frac{F}{qv}$$

the units of magnetic field are: $NC^{-1}m^{-1}s = NA^{-1}$ = Tesla, T.

[In cgs units, $\vec{B}$ is in Guass, G. 1 Tesla = 10^4 G.]

Typical Values for Magnetic Fields:

Earth's field = several G (i.e. 10^{-4} T);

Strong permanent magnet ~ 0.4 T; large laboratory magnet ~ 1 – 3 T;

Superconducting magnet ~ 8 T; pulsed magnet up to 70 T.

HW : Ch. 28.1

Example

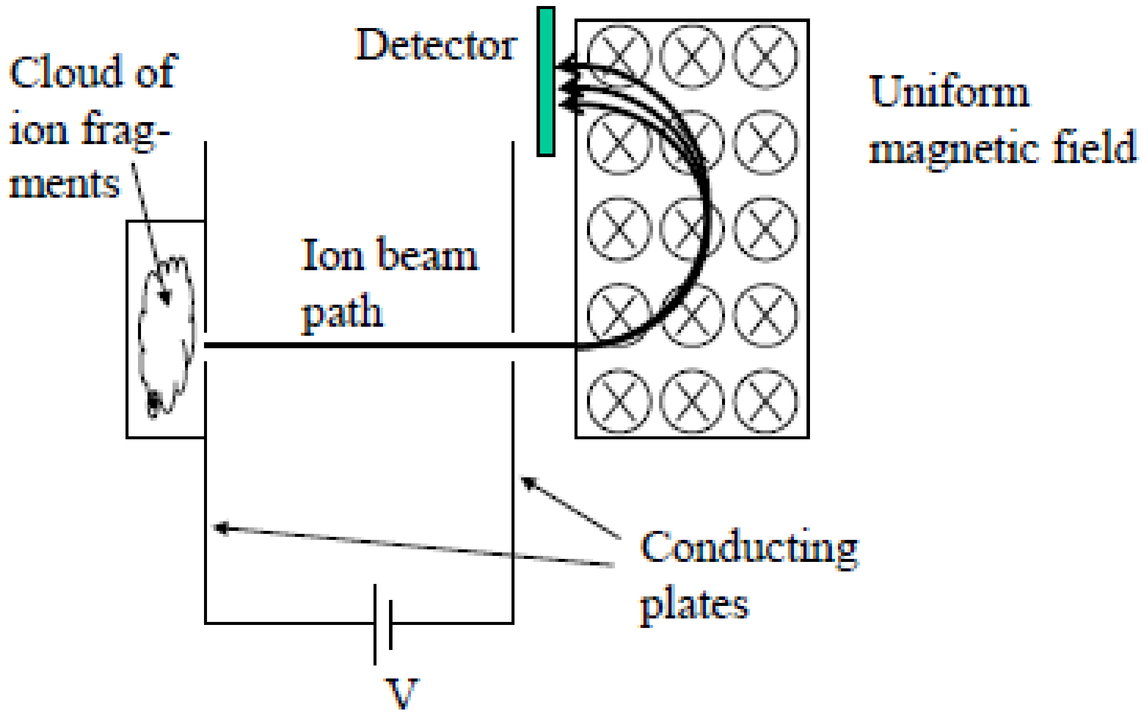
1. What is the magnetic force on an electron whose velocity is 2×10^{6} m/s in a magnetic field of 2×10^{-2} T (200 G) directed perpendicular to the velocity?
2. How large an electric field would be required to produce a force of the same strength?

Example

What is the force on an electron traveling at 3.5×10^6 m/s to the right in a magnetic field of 0.054 T out of the page at an angle of 45^O to the field?

The mass spectrometer

- uses the Lorentz force- both electric and magnetic fields
- unknown molecule in vapour phase is ionised and dissociated by a high energy electron beam
- some ion fragments emerge through a small aperture into a uniform electric field between two parallel flat plate conductors with a large potential difference between them
- here they are accelerated to form a beam of ions which emerge through a second aperture, after which they pass through a uniform magnetic field perpendicular to their line of flight, before being detected.



HW : Ch. 28.4

Analysis of mass spectrometer operation

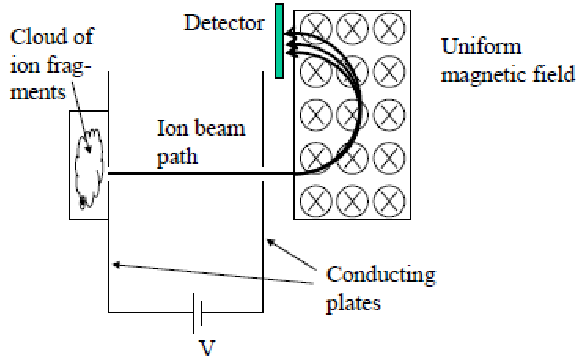
On emerging from $\vec{E}$ field, kinetic energy of the ionised fragments is qV.
However,

$$KE = \frac{1}{2}mv^2 \qquad\qquad \therefore \quad v = \sqrt{\frac{2qV}{m}}$$

where m is the mass of fragment, q is the charge on the fragment, V the potential difference between the plates.

Within the magnetic field region, $\vec{B}$ and $\vec{v}$ are mutually $\perp$, so

$$F = qvB = B\sqrt{\frac{2q^3V}{m}}$$

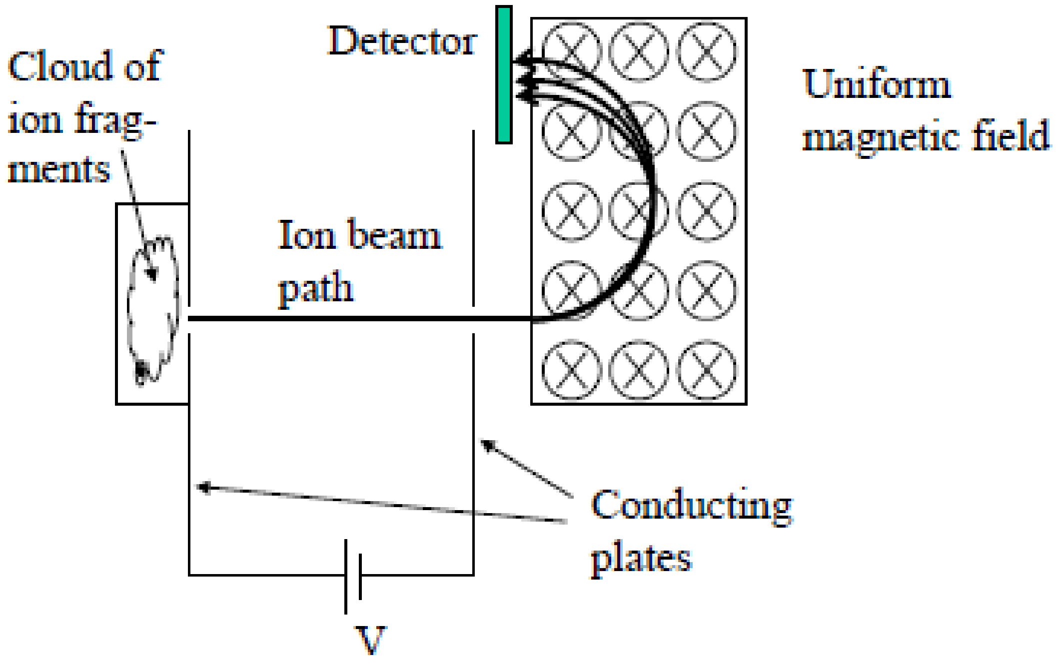


HW : Ch. 28.4

Analysis of mass spectrometer operation

$\vec{F}$ is at right angles to both $\vec{v}$ and $\vec{B}$, so the fragments must follow a circular path of radius r given by:

$$F = \frac{mv^2}{r} = B\sqrt{\frac{2q^3V}{m}}$$



Hence radius of path taken by fragments is

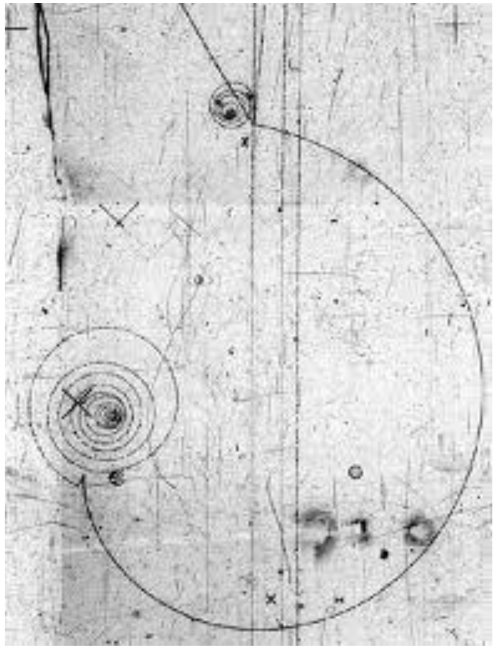
$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

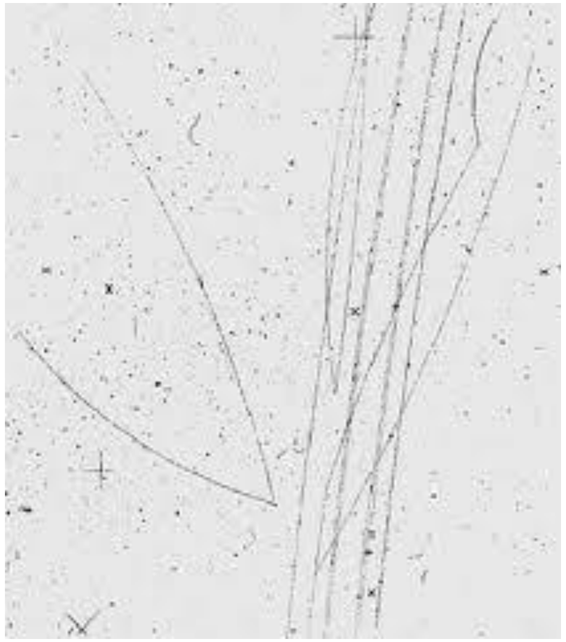
Thus, by recording the position, r, where fragments are detected, their mass can be measured directly.

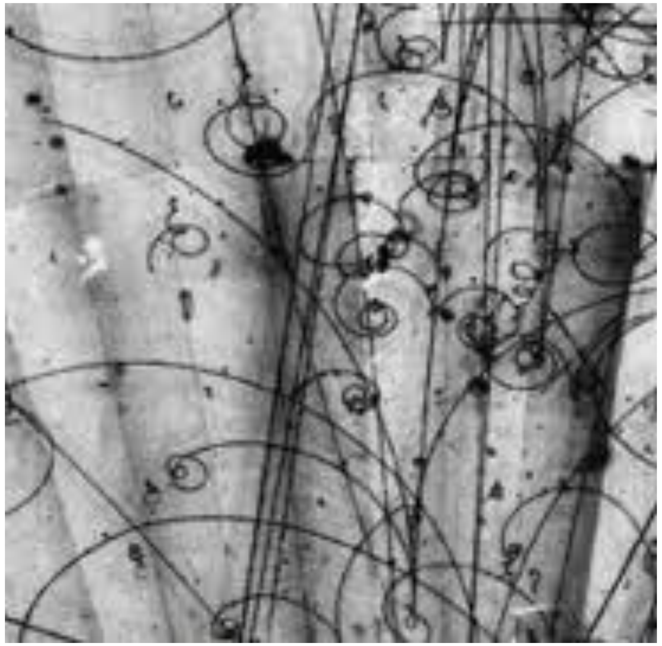
HW : Ch. 28.4

Other applications of Lorentz force

- Observation of fundamental particles in **bubble chambers**
  - tiny bubbles form along path of energetic charged particles, because molecules of supersaturated vapour become ionised and act as nucleation sites for bubbles
  - if the chamber is in a magnetic field, the direction of the tracks indicates the sign of the charge, while the radius is a measure of their momentum.
  - http://www-outreach.phy.cam.ac.uk/camphy/sweepnik/sweepnik1_1.htm



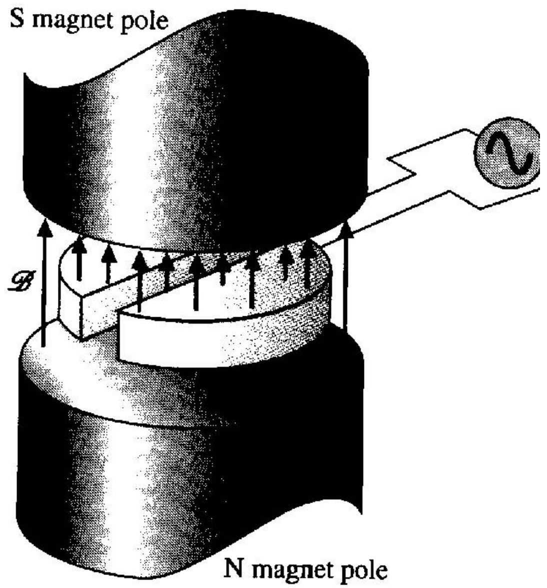




Other applications of Lorentz force

Cyclotron (type of particle accelerator)

- Charged particles can be injected into the hollow "Dees" near their centre. A uniform magnetic field from the pole pieces of an electromagnet causes them to move in a circular path.
- As the particles come to the gap between the Dees, they are given a "kick" by a large $\vec{E}$ field. They accelerate and move into a larger radius orbit. The $\vec{E}$ field is arranged to reverse direction when the particles next cross it, again accelerating them.
- By the time they reach the edges of the Dees, the particles are ready to be extracted and used as required in various ways.



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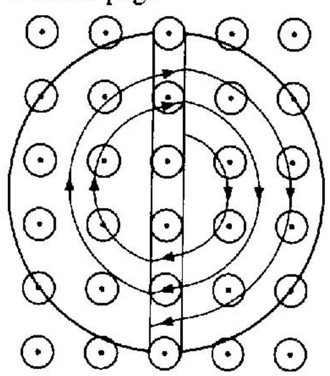
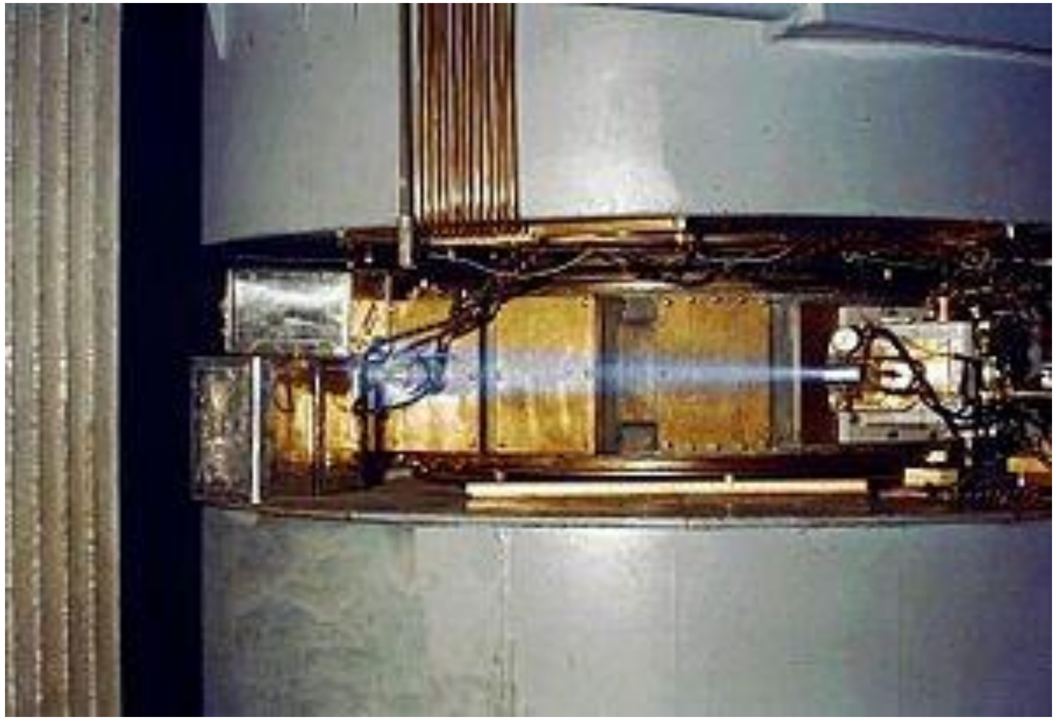


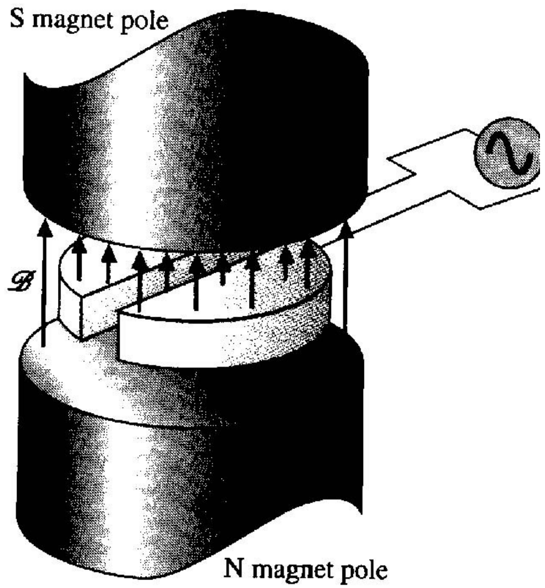
HW : Ch. 28.5

Other applications of Lorentz force

Cyclotron (type of particle accelerator)

60-inch cyclotron, circa 1939, showing a beam of accelerated ions (likely protons or deuterons) escaping the accelerator and ionizing the surrounding air causing a blue glow





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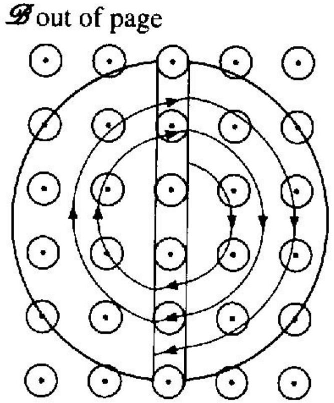


HW : Ch. 28.5

Other applications of Lorentz force

Synchrotrons

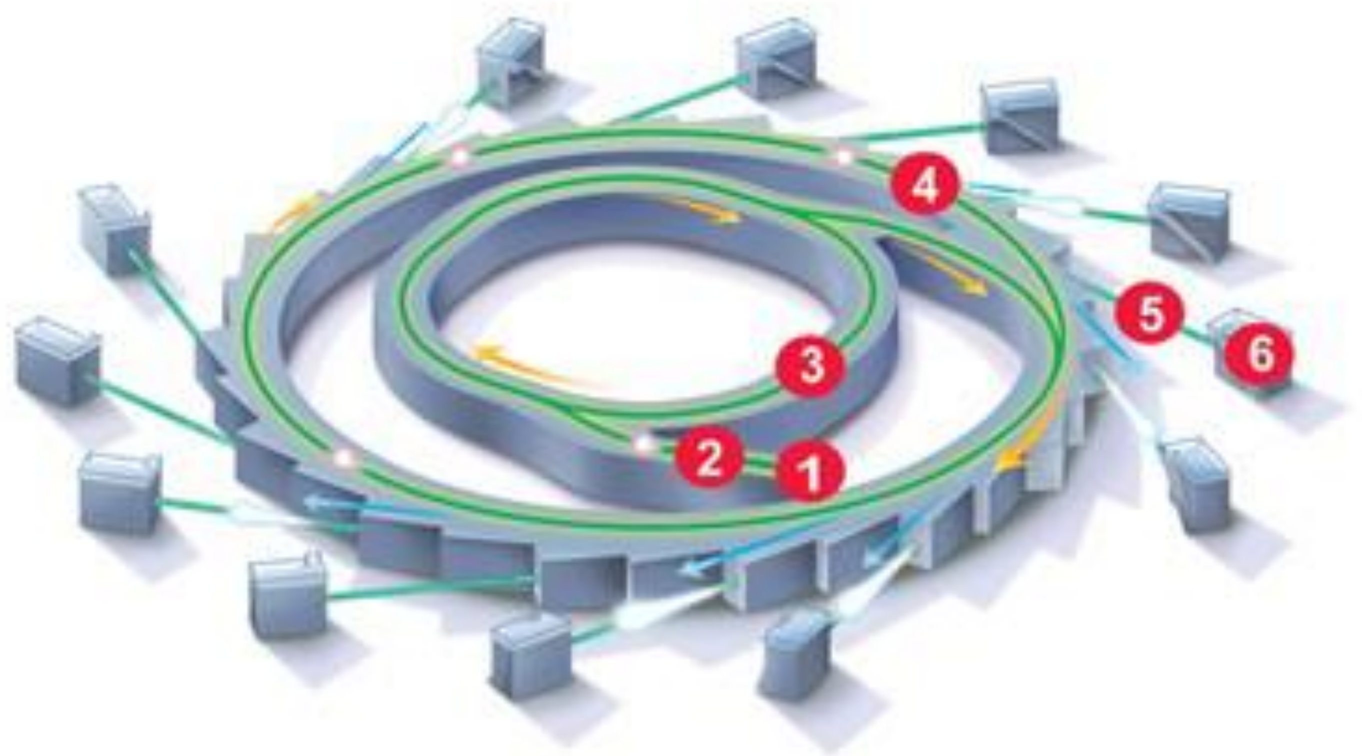
- A synchrotron is a very large, circular instrument about the size of a football field that produces an intense beam of light (brighter than the sun!).
- Electrons are generated in the centre (electron gun) and accelerated to 99.9997% of the speed of light by the linear accelerator (linac).
- The electrons are then transferred to the booster ring, where they are increased in energy. They are then transferred to the outer storage ring.
- The electrons are circulated around the storage ring by a series of magnets separated by straight sections. As they are deflected through the magnetic field created by the magnets, they give off electromagnetic radiation, so that at each bending magnet a beam of synchrotron light is produced.
- The intense light produced is filtered and adjusted to travel into experimental workstations, where the light reveals the innermost, sub-microscopic secrets of materials under investigation, from human tissue to plants to metals and more.

HW : Ch. 28.5

Other applications of Lorentz force

Synchrotrons

1. electron gun
2. linac
3. booster ring
4. storage ring
5. beamline
6. end station

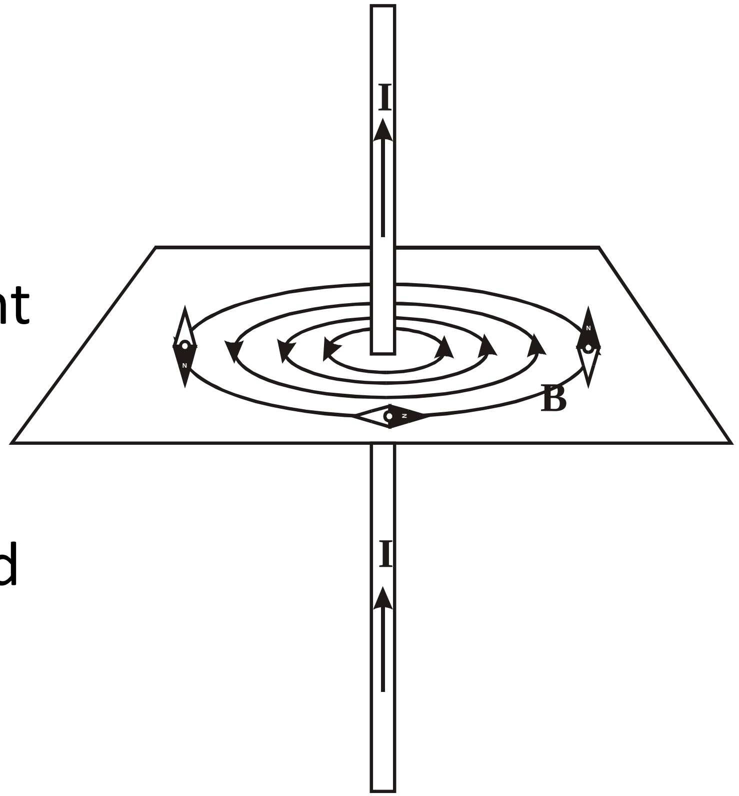


HW : Ch. 28.5

A little history on Magnetism:

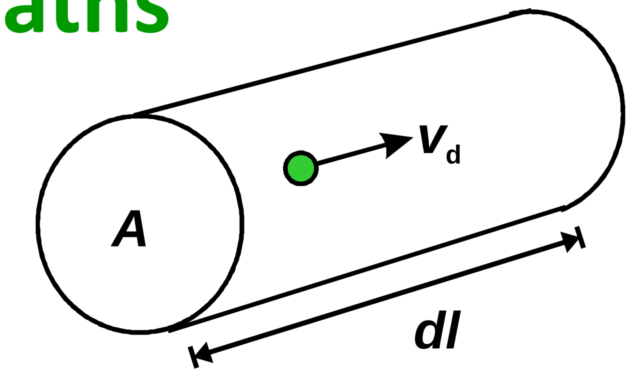
Oersted found the deflection of the compass needle is :

- Increased when the compass is placed closer to the current carrying wire.
- increased with larger current in the wire.
- unaffected by putting material (e.g. glass, wood, water and various rocks) between the wire and the compass.



A current carrying wire behaves like a magnet.

Force on a current in a magnetic field – the maths



Recall current density:

$$J = nqv_d$$

were n is the number of charges per unit volume.

The current through A is

$$I = JA = nqv_dA$$

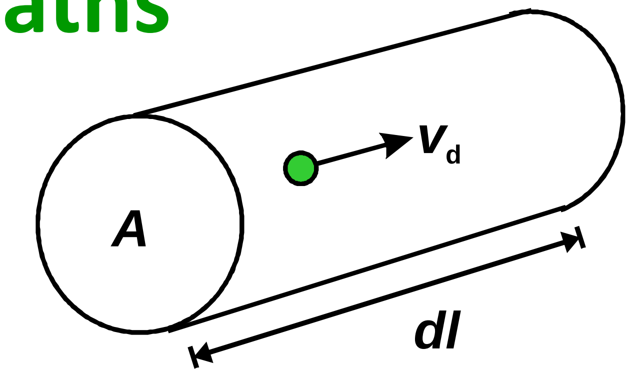
Suppose a wire of length $\mathrm{d}l$ is placed in a region of uniform magnetic field, $\vec{B}$, and current I is flowing in the wire. As the electrons are constrained to remain in the wire, the wire experiences a force.

The total number of charge carriers, moving at velocity v_d is

$$N = nA\mathrm{d}l$$

HW : Ch. 28.3

Force on a current in a magnetic field – the maths



The force on any of these charges is

$$\vec{F} = q\vec{v}_d \times \vec{B}$$

Thus, the force on the wire is

$$\mathrm{d}\vec{F} = nAdlq\vec{v}_d \times \vec{B} = I\mathrm{d}\vec{l} \times \vec{B}$$

Note: $\mathrm{d}\vec{l}$ has the direction of **positive** charge flow.

When the angle between the field and wire is θ then the magnitude of the force is:

$$\left|\vec{F}\right| = I\left|d\vec{l}\right|\left|\vec{B}\right|\sin\theta$$

HW : Ch. 28.3, 28.6

Directions and Force:

What would happen if, in this diagram, we reversed

a) the current direction?

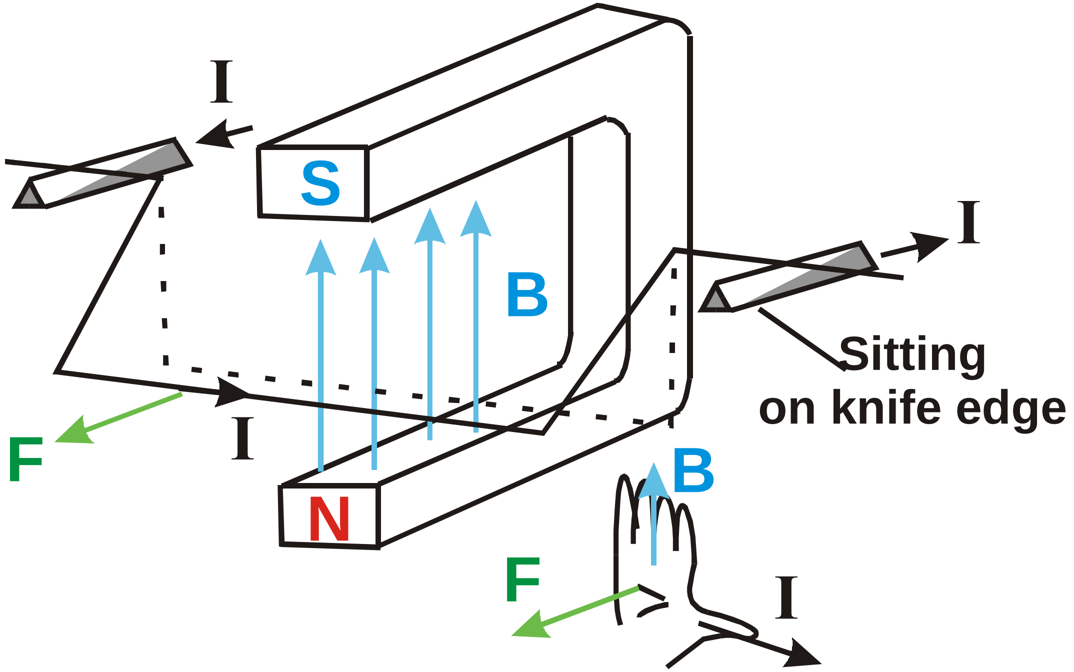
The force would be inward.

b) the magnetic poles?

The force would be inward.

c) both the current direction and the magnetic poles?

The force would be outward.

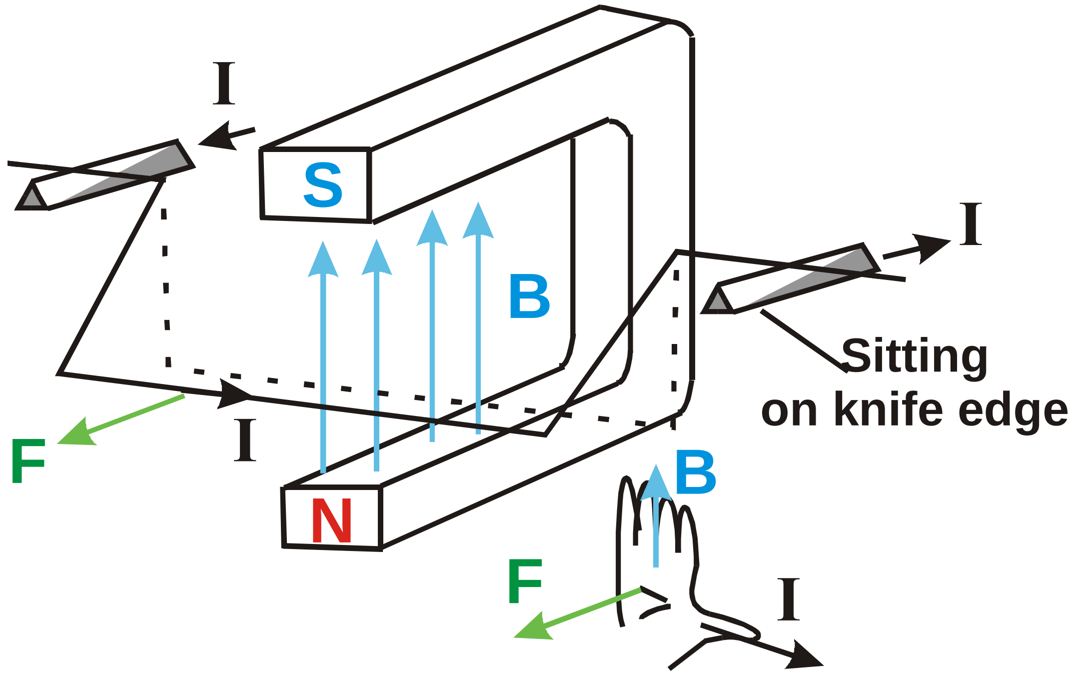


HW : Ch. 28.3 , 28.6

Example

What force will a conductor of length 6.0 cm experience if it carries a current of 300 mA and is placed perpendicular to a magnetic field of strength 0.50 T as shown?

Solution

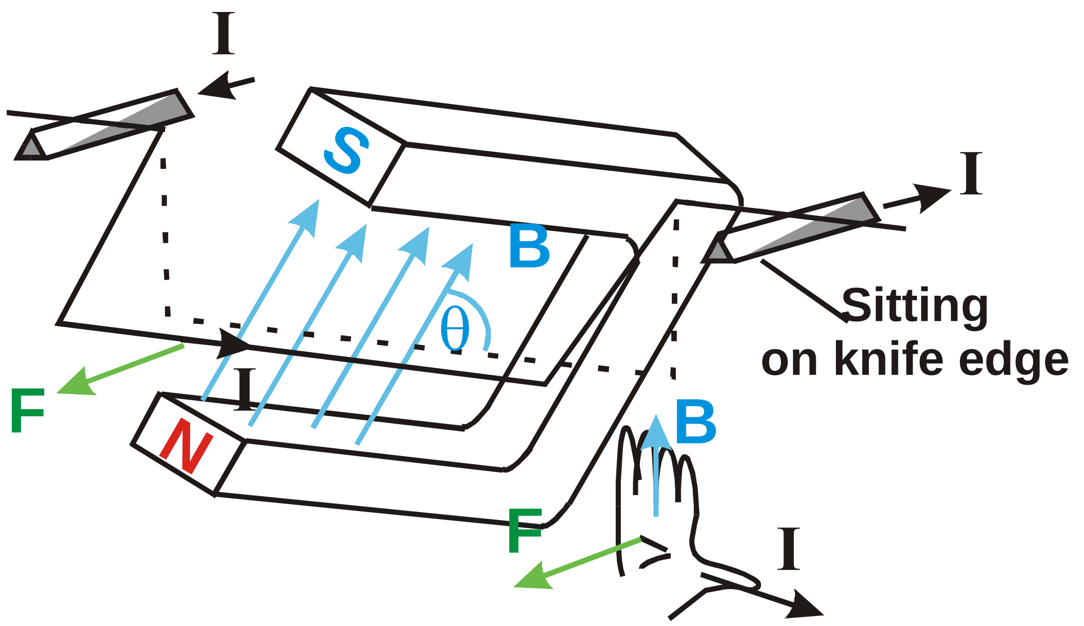


HW : Ch. 28.3 , 28.6

Example

If we rotate our field in the previous example so that it is 60^o to the conductor what force will the conductor now experience?

Solution



HW : Ch. 28.3 , 28.6

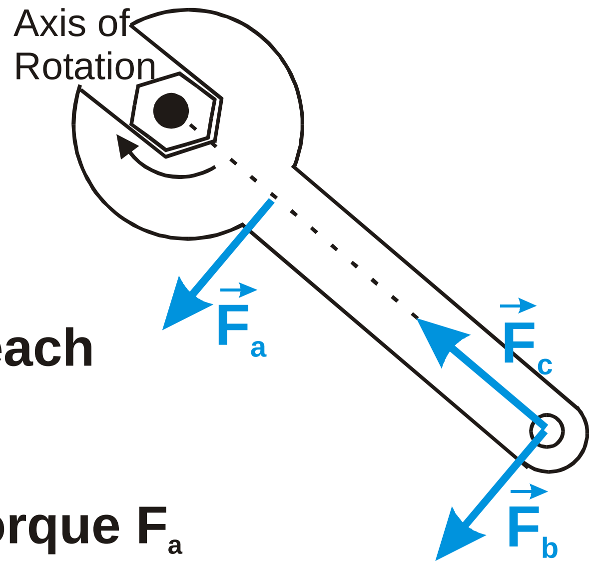
Torque (Γ in Nm)

Torque (or moment) of a force is a measure of its tendency to cause a body to rotate.

$|F_a|=|F_b|=|F_c|$

but torque for each force is:

torque for F_b > torque F_a and torque for F_c = 0



$\vec{F}_a$ **: small effect**

$\vec{F}_b$ **: large effect**

$\vec{F}_c$ **: no effect**

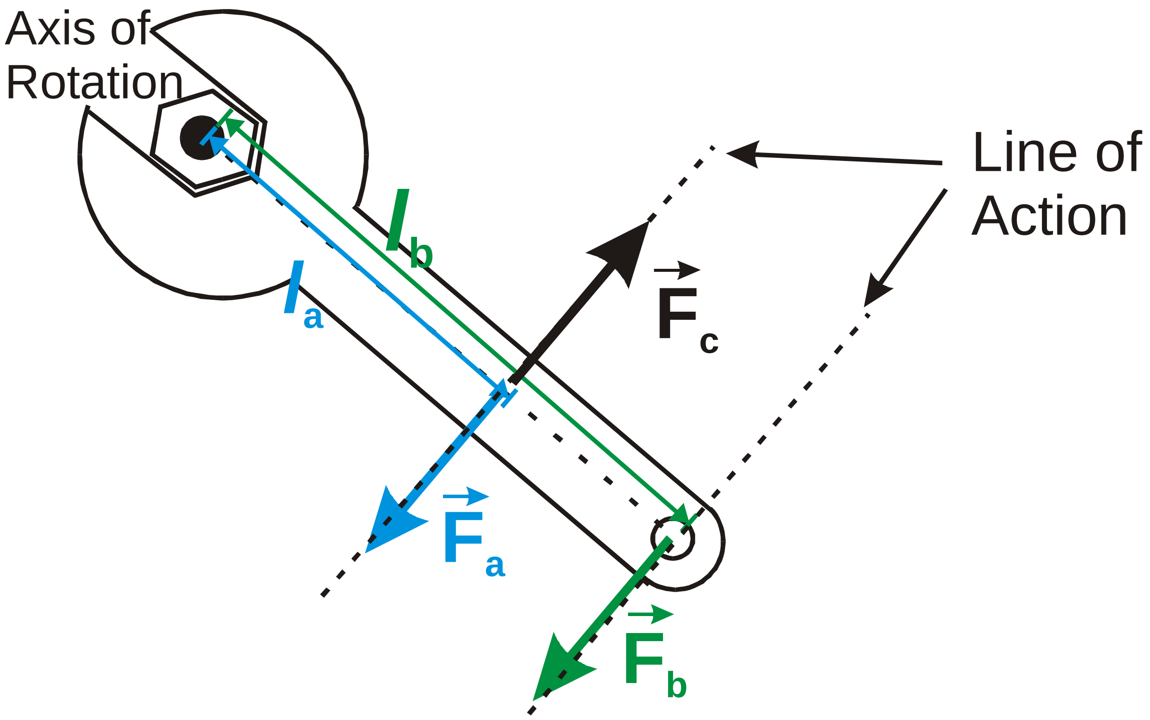
Torque (Γ in Nm)

Torque is defined as the product of the force acting and the perpendicular distance, l, between the axis of rotation and the line of action.

$\Gamma_a = F_a l_a$ clockwise (-ve)

$\Gamma_c = F_c l_a$ counterclockwise (+ve)

The perpendicular distance, l, is also known as the 'lever arm' or 'moment arm'.



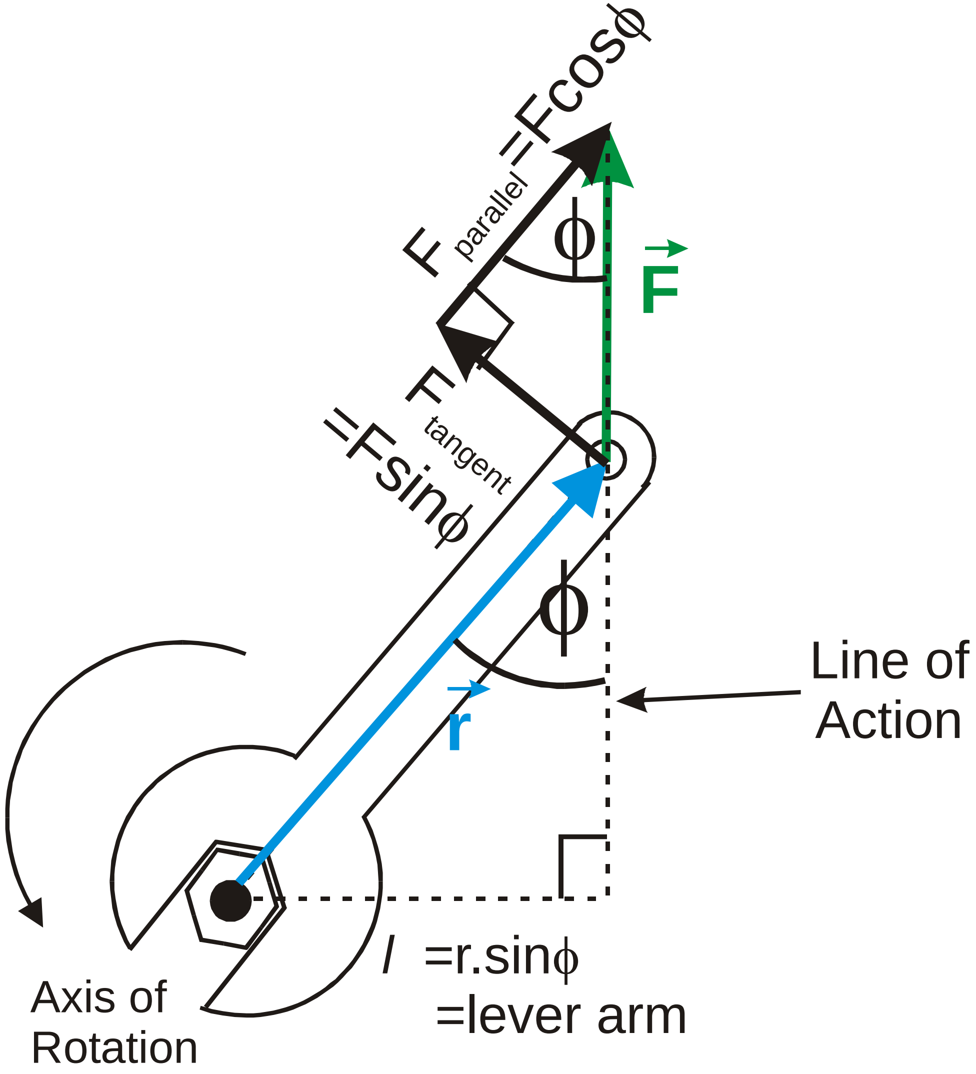
Torque (Γ)

$$\Gamma = Fl$$

$$= Fr\sin\phi$$

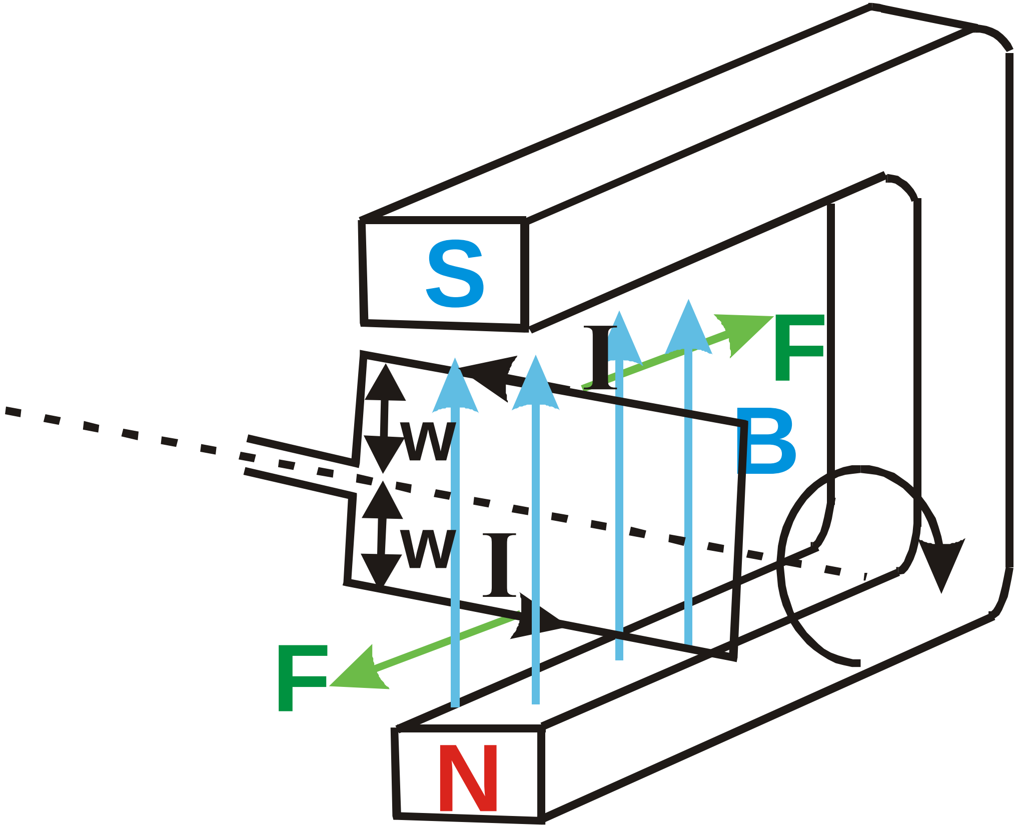
$$= F\sin\phi r$$

$$= \vec{r} \times \vec{F}$$



What if……..

– instead we place n rectangular loops in our magnetic field?



$$F = BIL\sin\theta$$

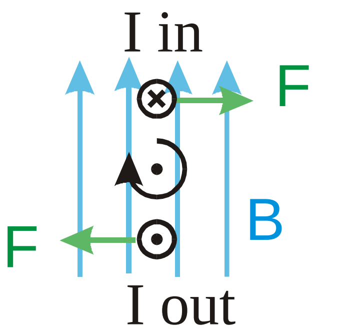
$$F = nBIL\sin\theta$$

Experiences Torque –
Rotates ->
The simple DC motor

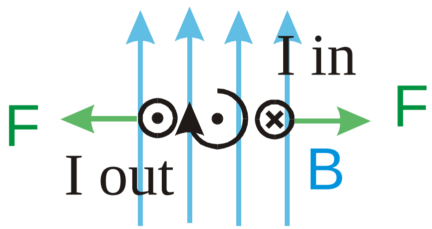
HW : Ch. 28.7

DC Motor and Torque

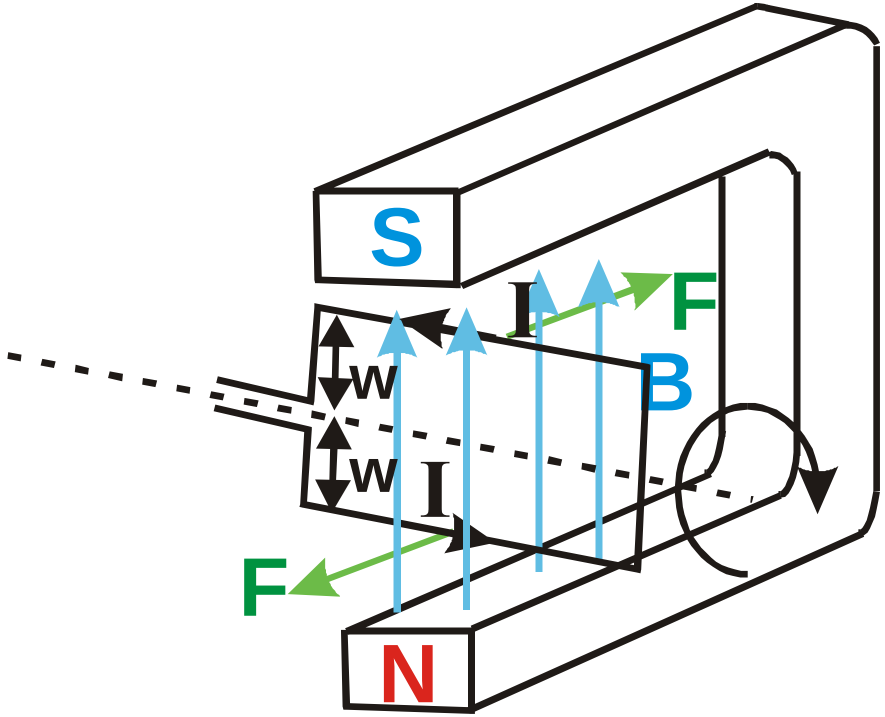
The wire is always perpendicular to the field but, as it rotates, the torque it experiences changes as shown below:



F perpendicular to moment arm - Maximum Torque sin ϕ= sin 90 = 1



F parallel to moment arm - Zero Torque sin ϕ= sin 0 = 0



Note: Momentum keeps our loop rotating in the field.

HW : Ch. 28.7

Calculating Torque

Using the equation for torque and substituting for force and the perpendicular distance from the rotation axis ‘l’ with w:

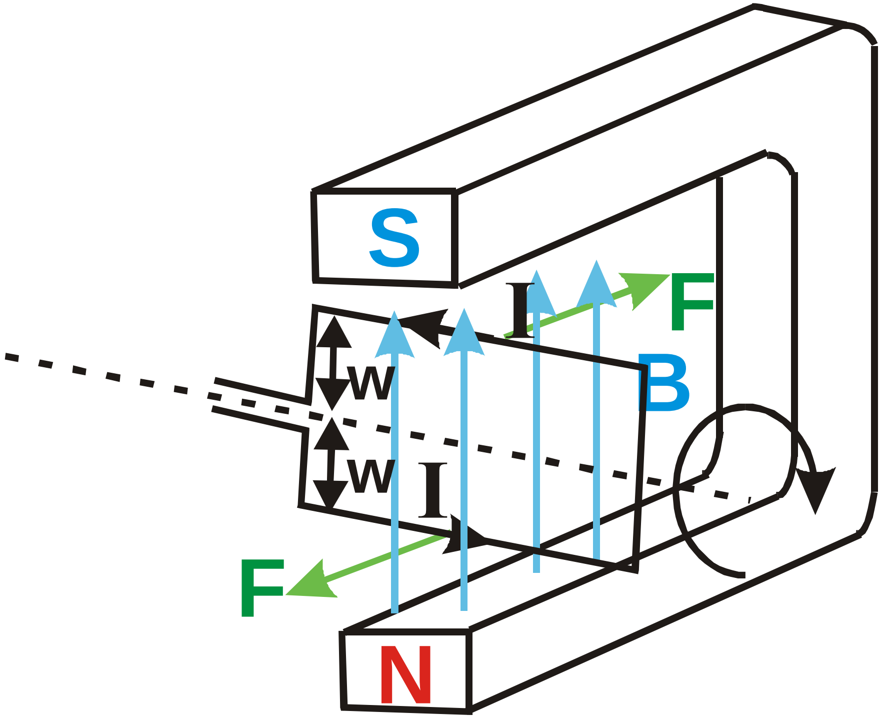
$$\Gamma = Fl$$

$$= nBILw\cos\theta + nBILw\cos\theta$$

$$= 2nBILw\cos\theta$$

$$= nBIA\cos\theta$$

Here θ is taken as the angle between the plane of our coil and the field and A = 2*Lw* = area of coil in the field.

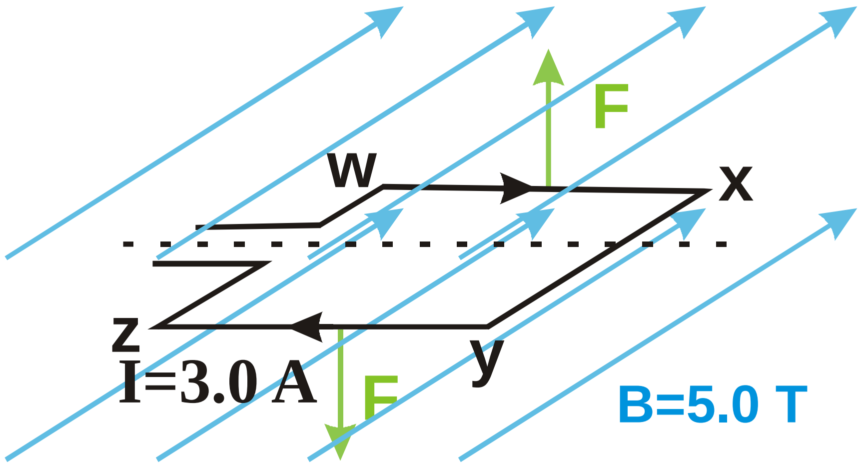


HW : Ch. 28.7

Example

A rectangular coil (wxyz) with 150 turns lies with its plane parallel to a uniform magnetic field of strength 5.0 T as shown. A DC current of 3.0 A flows from a battery along the path wxyz.

i. Show the direction of the force on the sides wx and yz.
ii. If the side xy is 30 mm long and the sides wx and zy are 50 mm, what are the forces acting on sides wx, xy and yz?
iii. What is the torque on the coil?



HW : Ch. 28.7

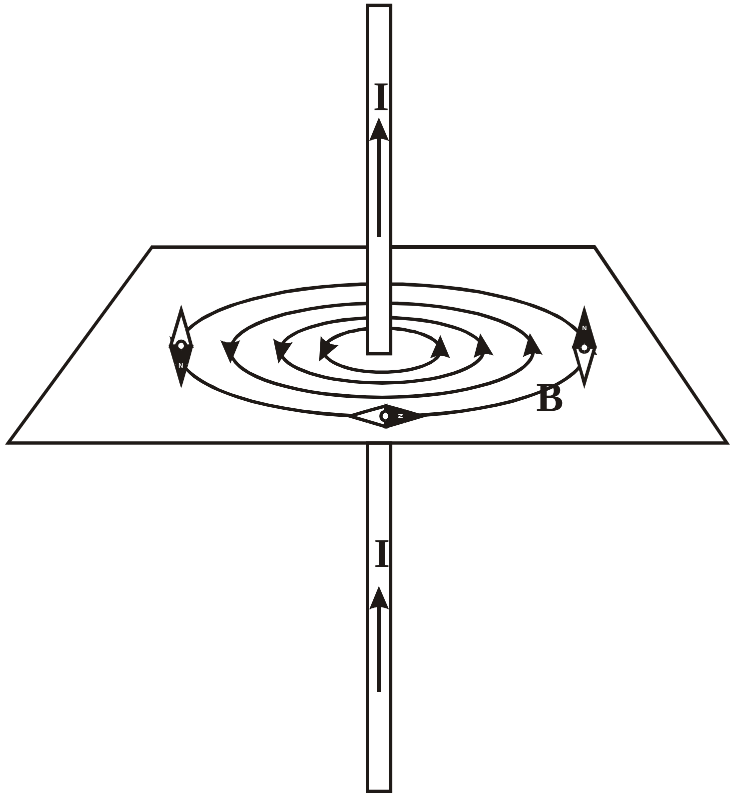
Example cont:

ii. If the side xy is 30 mm long and wx and zy are 50 mm, what are the forces acting on sides wx, xy and yz?
iii. What is the torque on the coil?

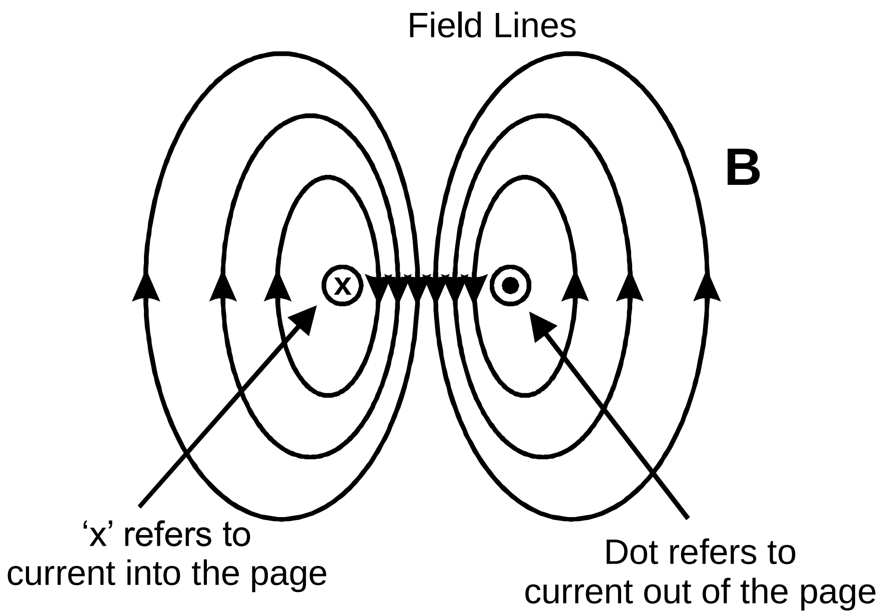
HW : Ch. 28.7

Magnetic field of a straight conductor

Magnetic field lines are continuous. Unlike electric field lines, they do not originate at one location and end at another. The direction of magnetic field lines is defined by the right hand grasp rule. The thumb points along the wire in the direction of conventional current flow and the fingers curl around the wire in the direction of the magnetic field.



Field pattern of a current loop



HW : Ch. 29.1

The Biot-Savart Law

The Biot-Savart law allows magnetic fields to be determined near conductors. This law is the magnetic “equivalent” of the superposition principle. The law expresses the magnetic field as the sum of tiny sections of a current-carrying conductor.

The magnitude of the contribution to the magnetic field at point P due to a current element $\mathrm{d}\vec{l}$ (which points in the direction of current flow) is given by:

$$dB = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}l\sin\phi}{r^2}$$

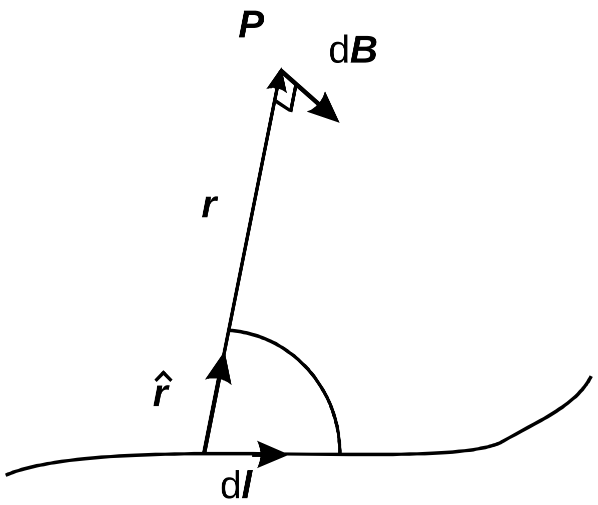
Where μ_0 is a universal constant, called the permeability of free space and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{kgm/C^2}$ exactly.

HW : Ch. 29.1

The Biot-Savart Law

In vector form:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l} \times \underline{\hat{r}}}{r^2}$$



Path of current flow, I

The direction of $\vec{B}$ is not along $\vec{r}$ but ⊥ (perpendicular) to a plane containing both $\vec{r}$ and $\vec{l}$.

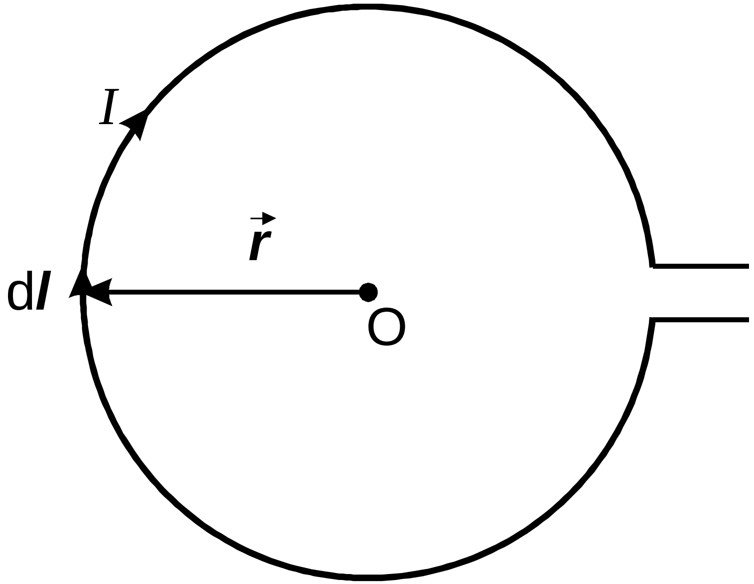
The Biot-Savart ("bee-oh") law allows the magnetic field allows the magnetic field $\vec{B}$ to be calculated at any point in space due to a current in a complete circuit. It is given by the vector sum of all the contributions $\mathrm{d}\vec{B}$ from the current elements $\mathrm{d}\vec{l}$. In integral form:

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{\text{current path}} \frac{I\mathrm{d}\vec{l} \times \underline{\hat{r}}}{r^2}$$

HW : Ch. 29.1

Example

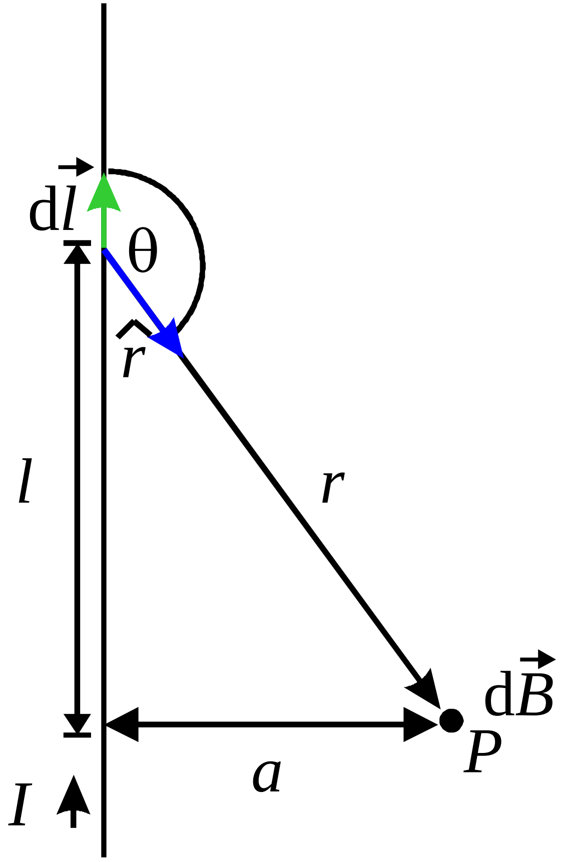
What is the magnetic field at the centre of a circular conducting loop?



HW : Ch. 29.1

Example

What is the magnetic field at a point P near a long, straight, current carrying wire?



HW : Ch. 29.1

A little history on Magnetism:

- **1821**: Andre-Marie Ampere (France), reasoned that if a current exerts a magnetic force on a compass needle, then two such wires should interact magnetically with each other. He showed in a series of experiments that
  - parallel (straight) currents attract
  - anti-parallel currents repel
  - the force is inversely proportional to the square of the distance between the wires.

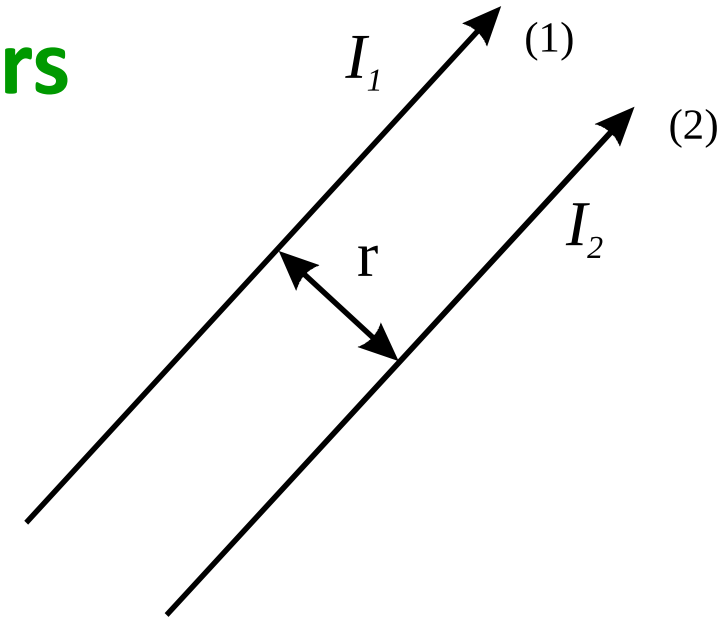
Force between long parallel conductors

We have seen that the force $\mathrm{d}\vec{F}$ on a wire $\mathrm{d}\vec{l}$ carrying a current I in a magnetic field $\vec{\boldsymbol{B}}$ is

$$\mathrm{d}\vec{F} = I\mathrm{d}\vec{l} \times \vec{B}$$

Now consider two long conductors separated by distance $\vec{r}$ and carrying currents I_1 and I_2 respectively. The magnetic field set up by (2) at the position of (1) is

$$B = \frac{\mu_0 I_2}{2\pi r}$$



The force on a length l of conductor (1) in this field is then

$$F = I_1 l \frac{\mu_0 I_2}{2\pi r}$$

Since $\vec{B} \perp \vec{l}$ i.e.

$$F = \frac{\mu_0}{2\pi} \frac{l I_2 I_1}{r}$$

HW : Ch. 29.2

Basis of the definition of the Ampere:

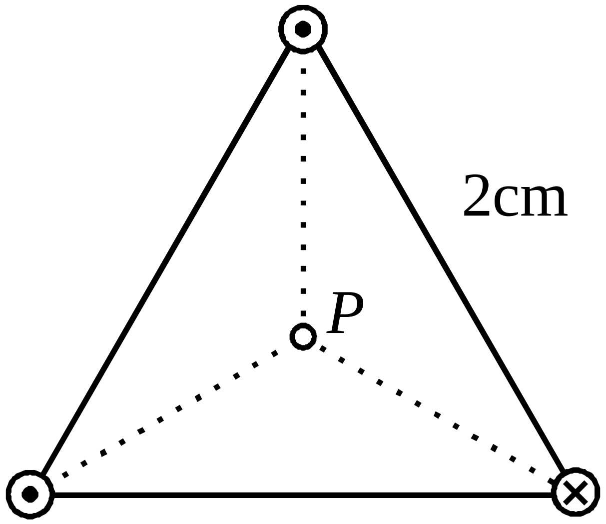
An ampere is the current which must flow in each conductor for a force of 2 × 10^{-7} N to exist between them at a separation of 1m

$$F = \frac{\mu_0}{2\pi}\frac{lI_2I_1}{r} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{1 \times 1 \times 1}{1} = 2 \times 10^{-7}\mathrm{N}$$

HW : Ch. 29.2

Example

Three infinite wires, each carrying a current of 5A, are arranged at the vertices of an equilateral triangle with 2 cm sides as shown below.



Calculate the magnitude and direction of the magnetic field at *P*, the centre of the triangle.

HW : Ch. 29.2

Ampere's Law

Allows $\vec{B}$ at a point near a current distribution to be calculated.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

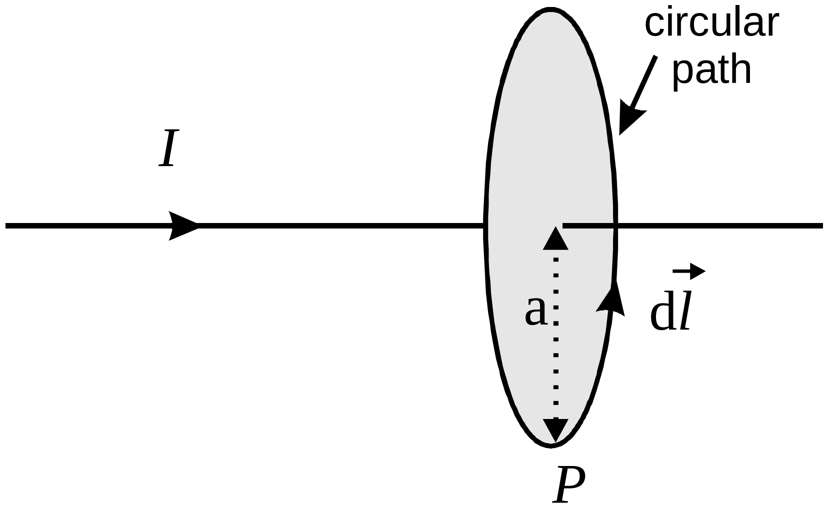
In words:

The line integral of the magnetic field $\vec{B}$ around any closed path is equal to μ_0 times the net current piercing the area bounded by the path.

HW : Ch. 29.3

Example

What is the magnetic field near a long straight conductor?



HW : Ch. 29.3

In applying Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

If $\vec{\boldsymbol{B}}$ is everywhere parallel (tangent) to the integration path, and $|\vec{B}|$= constant at every point on the path, then the line integral is just $|\vec{B}|$ multiplied by the circumference of the path.

If $\vec{\boldsymbol{B}}$ is perpendicular to the path at any point(s), those parts of the path make no contribution to the line integral.

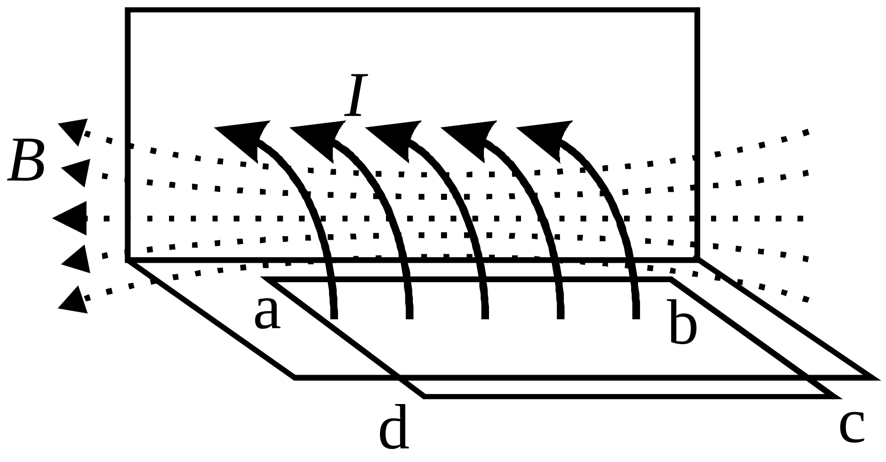
In $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$, B is the total magnetic field at each point of the path.

Choose the path carefully. The point(s) at which B is to be determined must lie on the path. The path must have enough symmetry to be evaluated.

HW : Ch. 29.3

Example

What is the magnetic field at the centre of a solenoid?



HW : Ch. 29.4