Quantum Physics – Photons;
Light is *not* a wave?!

Text: Walker *etal. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition*
John Wiley & Sons Australia (HW)

With many thanks to Walter Kalceff for the use of his original lecture notes.

The Photon

In 1905, Einstein proposed that electromagnetic radiation (e.g. light) is quantised and exists in elementary amounts (quanta) that we now call photons.

A quantum of a light wave of frequency f has the energy

$$E = hf$$

where h is the Planck constant,

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$$
$$= 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$$

Recalling that $c = f\lambda$, we can also write:

$$E = \frac{hc}{\lambda}$$

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Example 1

A 100 W sodium vapour lamp is located at the centre of a large sphere that absorbs all the light reaching it. Assuming the emitted light has wavelength λ = 590 nm, calculate the rate at which photons are absorbed by the sphere.

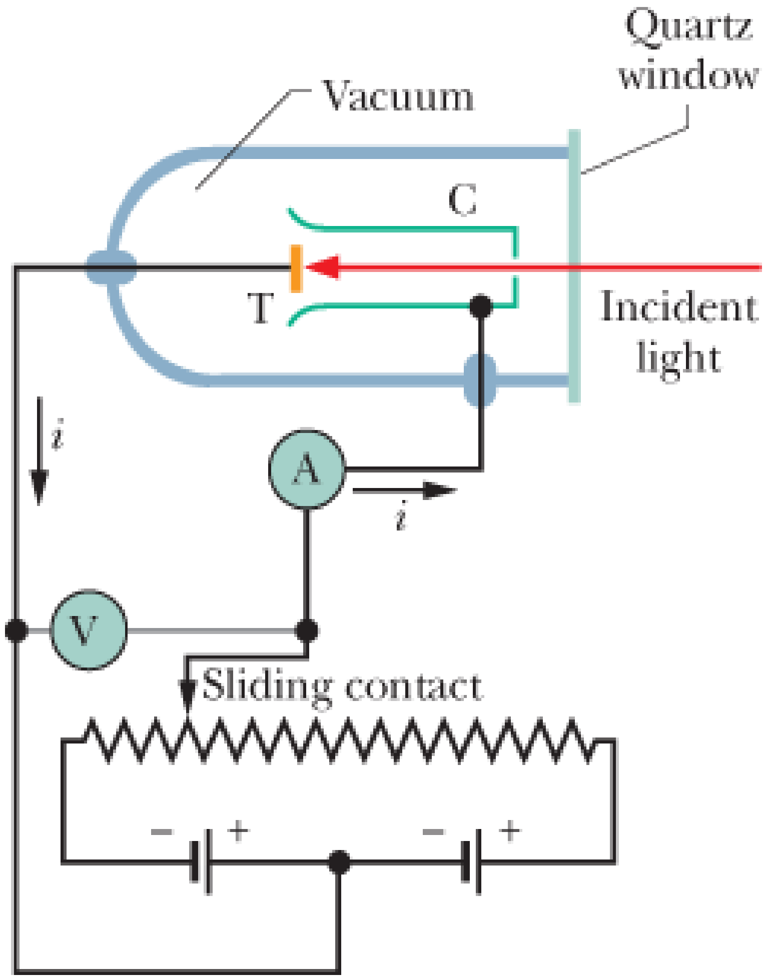
Photoelectric Effect:– Observation 1

Observation: Incident light shines on (a metal) target T, ejecting electrons.

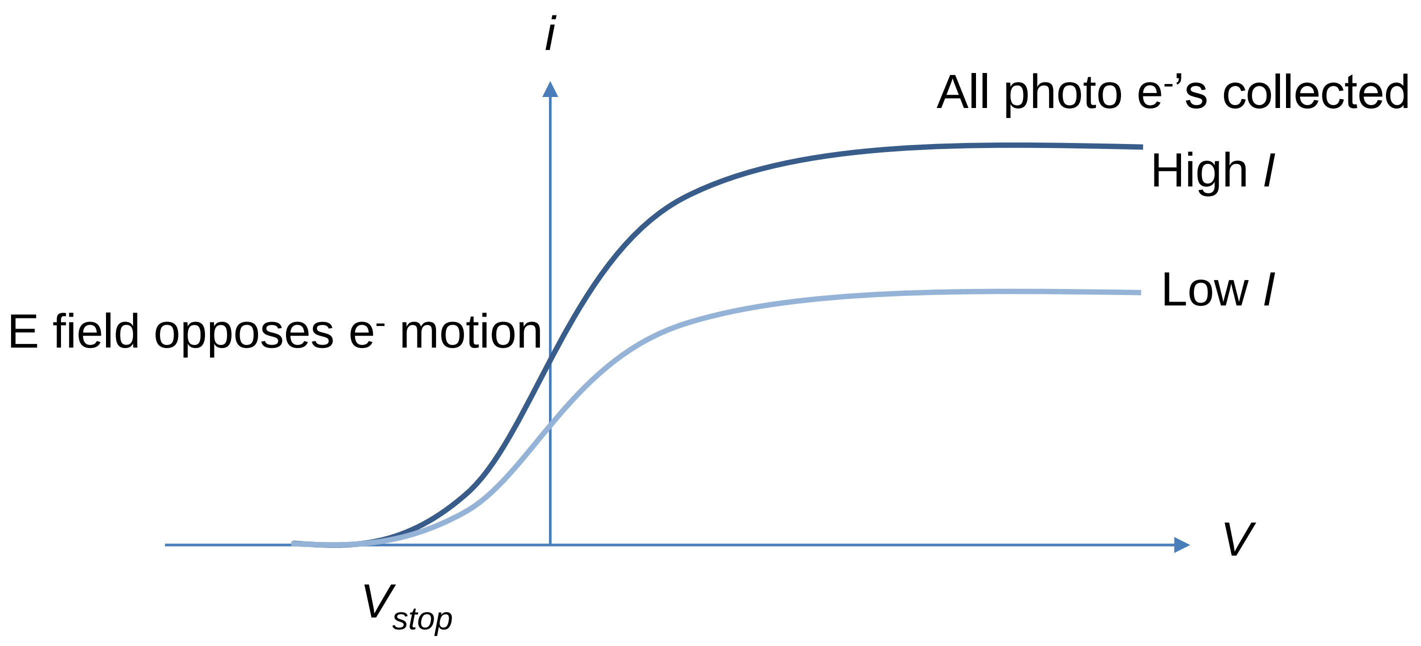
The electrons are collected by collector cup C and move in a direction opposite to the conventional current indicated.

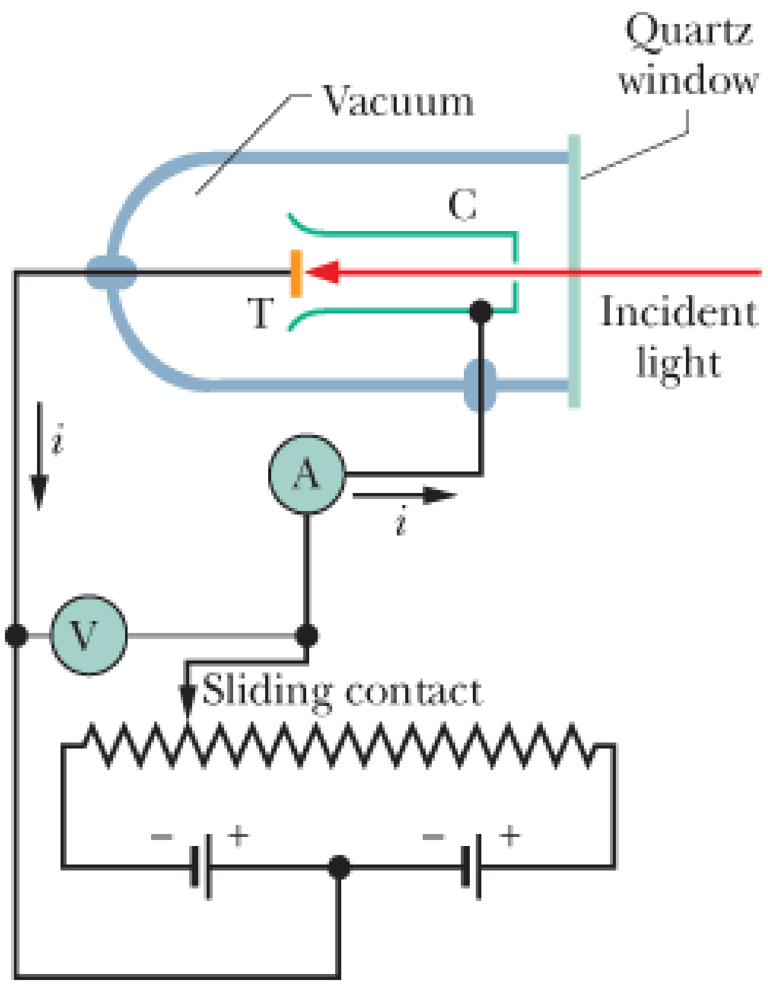
V is varied until it reaches a value called the stopping potential, V_{stop}, at which the current falls to zero.

(The batteries are used to produce and adjust the potential difference between target and collector.)



Photoelectric Effect – Observation 1





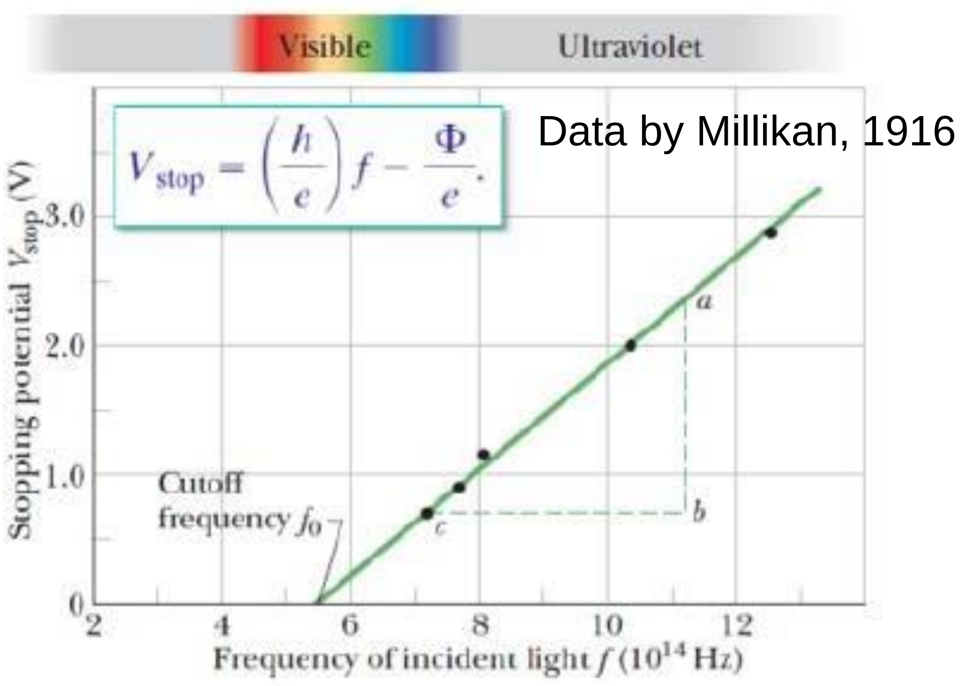
Explanation: When $V = V_{stop}$ the most energetic electrons are turned back just before reaching the collector. At this condition, K_{max}, the kinetic energy of these *most energetic* electrons is:

$$K_{max} = eV_{stop}.$$

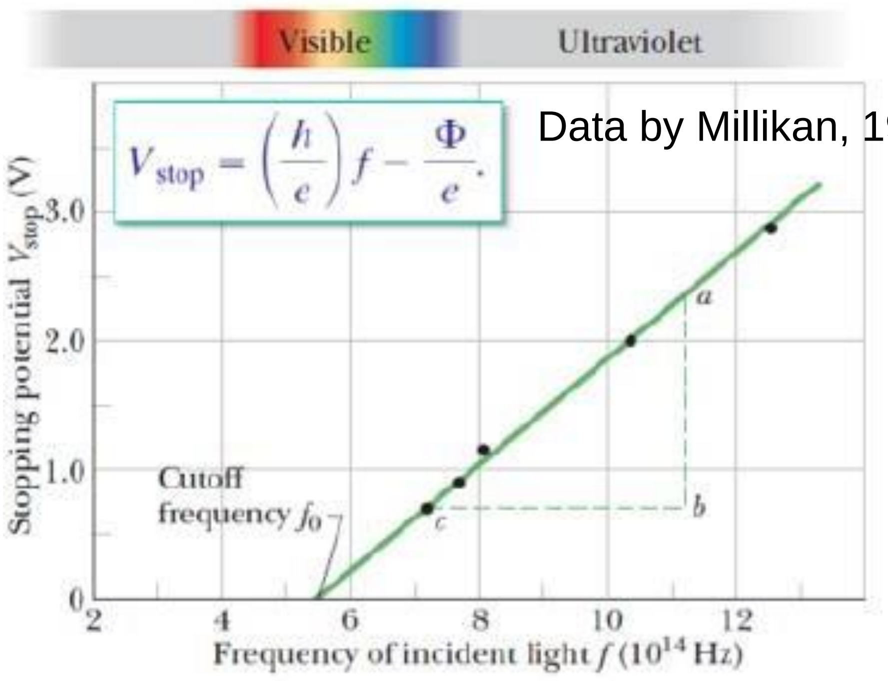
Photoelectric Effect:– Observation 2

Observation: If the frequency f of the incident light is varied and the associated stopping potential V_{stop} is measured, then the plot of V_{stop} versus f is obtained.

The photoelectric effect does not occur if the frequency is below a certain cut-off frequency f_0 (or, if the wavelength is greater than the corresponding wavelength $\lambda_0 = c/f_0$) *no matter how intense the light*.



Photoelectric Effect:– Observation 2



Data by Millikan, 1916

Explanation: Electrons within the target are held there by electric forces. To just escape from the target, an electron must pick up a certain minimum energy Φ, whose value is a property of the target material, called its *work function*.

If the energy transferred to an electron by light exceeds the work function of the material, the electron can escape the target.

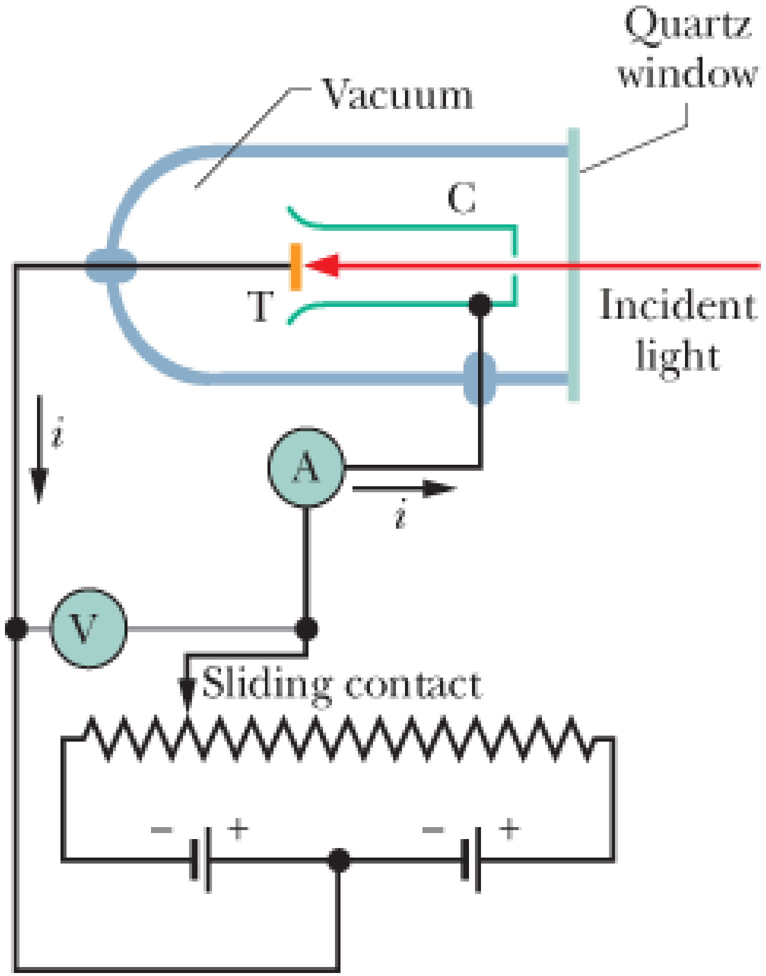
Photoelectric equation: $K_{\max} = hf - \Phi$

(Einstein's photon theory)

Photoelectric Effect:– Observation 3

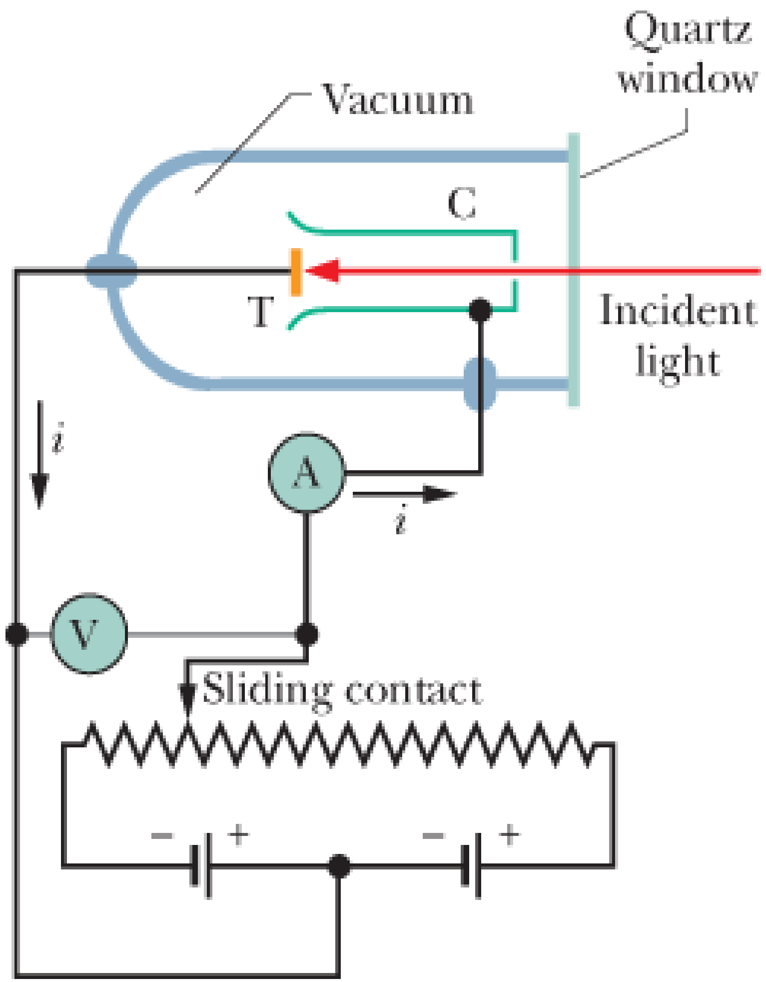
Observation: Electrons start flowing in the circuit within 10^{-10} s of the light falling on the surface, no matter how low is the incident light intensity.

Furthermore, the higher the incident light intensity, the larger is the number of electrons emitted.



Photoelectric Effect:– Observation 3

Explanation Light is behaving as a particle, the emission is an *all or none situation*:– if the photon energy is great enough, it will all be transferred to the electron causing emission *instantly*, otherwise the electron will remain bound to the metal surface.



If light is behaving as a wave, it *incrementally* imparts energy to surface electrons (at a rate proportional to intensity). Consequently, it should take much longer for an electron to absorb sufficient energy to `escape’ the metal surface.

Photoelectric Effect – summary of wave model failures

Stopping potential (V_{stop}), and hence K_{max} , is *independent of intensity* of light.

- wave model of light says K_{max} should *increase with intensity*!
(intensity is rate of energy delivered to target per area.)

Photoemission of electrons only occurs for light *above a threshold frequency*, f_0.

- wave model of light says photoemission should occur for *any frequency*!
(you just need enough intensity.)

Time before photoemission occurs is *independent of intensity*.

- wave model of light says time should *decrease with intensity*!
(Intensity is rate of energy given to electrons per area)

Photoelectric Effect – comparison of models

Quantum mechanical model

Amount of energy in a quantum (nondivisible lump) of EM field depends on frequency;

$$E = hf$$

Energy absorbed from light is:
- proportional to frequency
- happens in chunks of *hf*

Yes, that's the frequency of the *wave*

Classical (wave) model

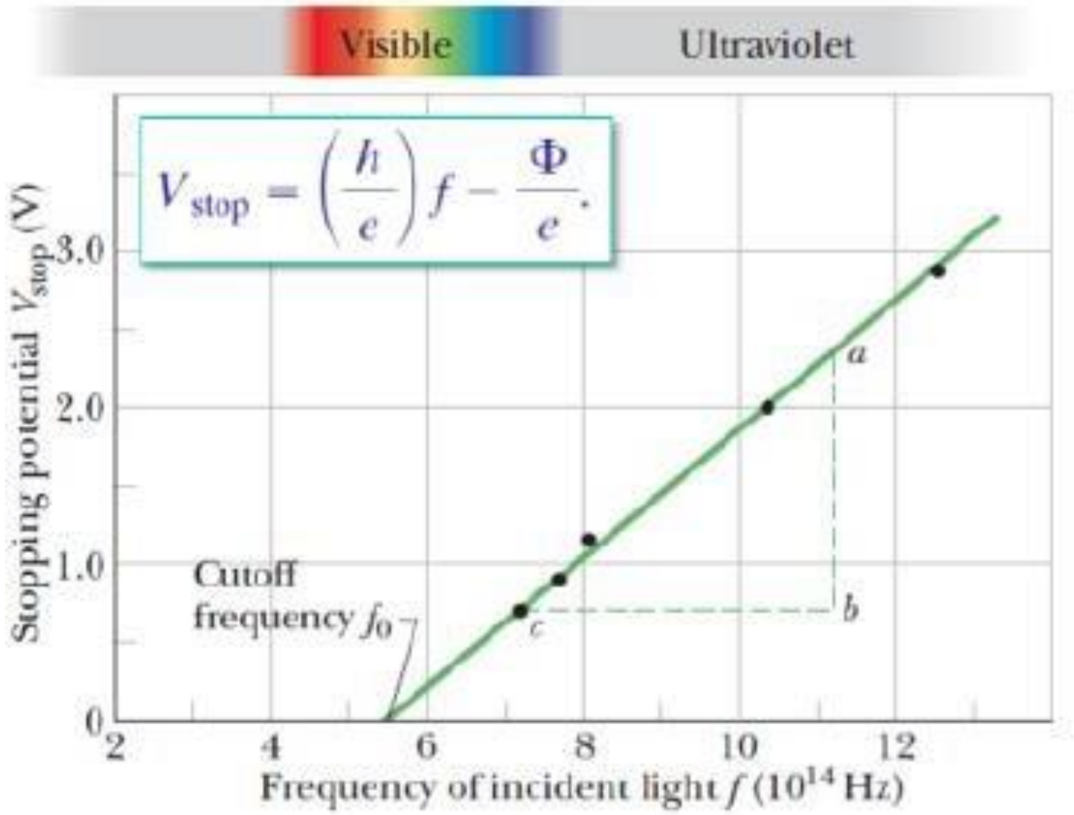
Intensity = rate of EM energy transported through an area/unit area;

$$I = \frac{E/t}{A} = \frac{E_{max}^2}{2\mu_0 c} \text{ (plane wave)}$$

Energy absorbed from a light wave is:
- proportional to Intensity
- gradual (incremental), dE step by step

Example 2

Determine the work function of sodium, using the plot below:



Photon Momentum

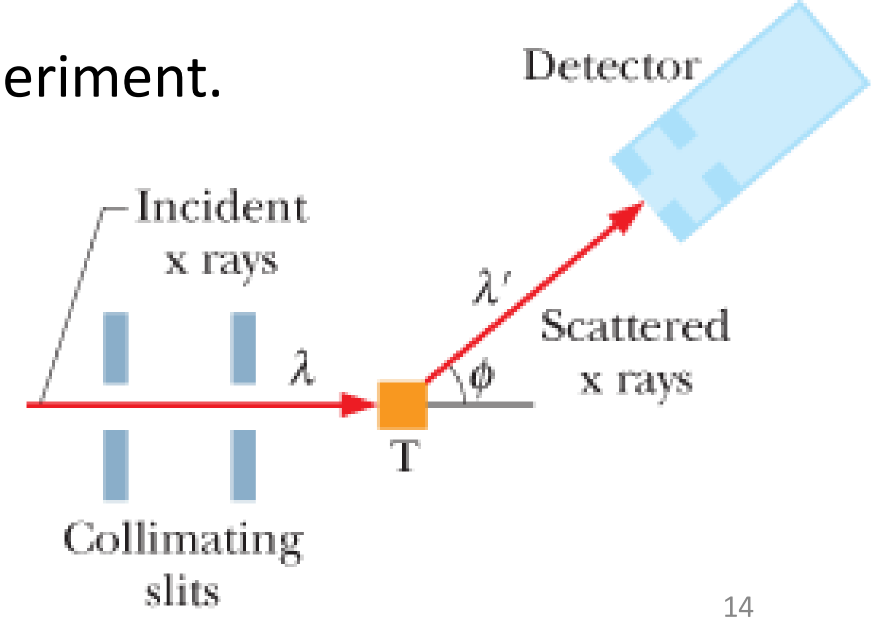
In 1916 Einstein proposed that a quantum of light has linear momentum, *p*:

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

When interacting with matter, photons transfer both energy and momentum.

In 1923, Arthur Compton demonstrated this in a classic experiment.

A beam of x-rays with λ = 71.1 pm was directed at a carbon target, T. Both the wavelengths and intensities of the scattered x-rays were measured.



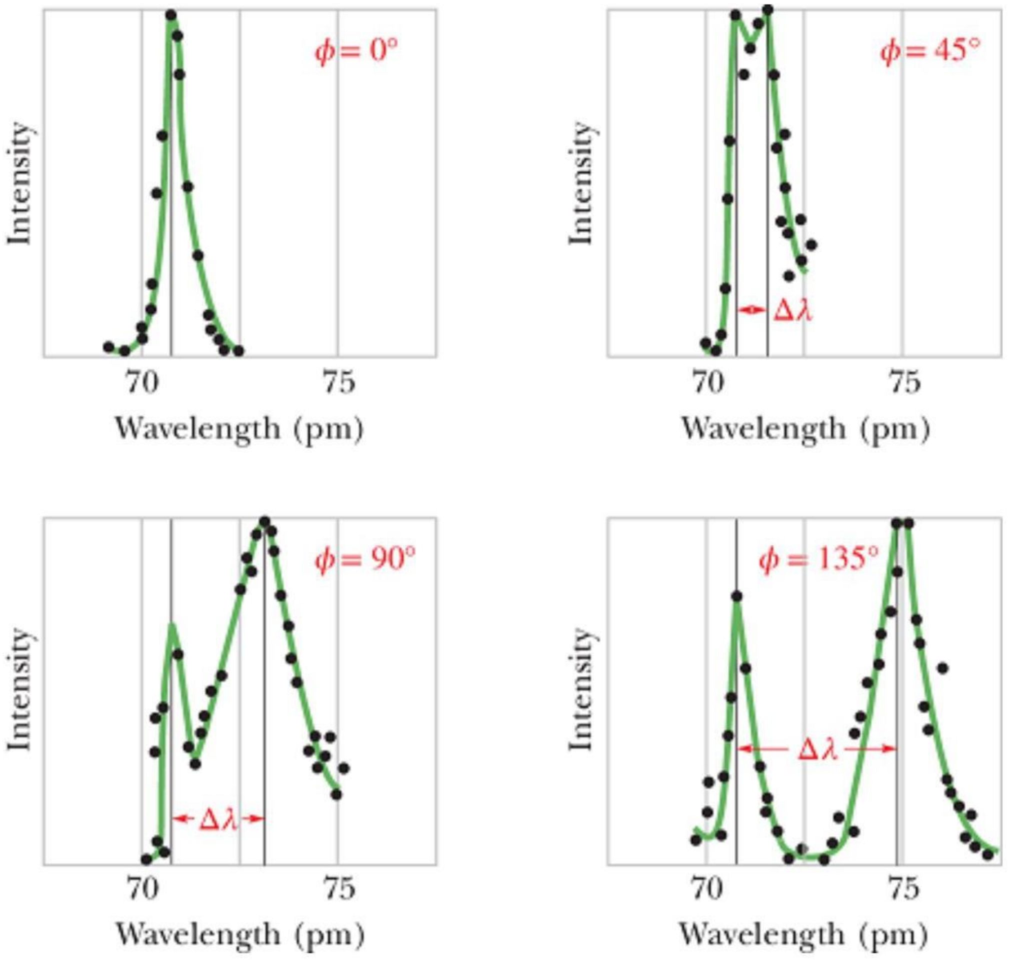
Compton Shift

Compton observed that the x-rays were scattered,

- the original wavelength,
- an extra peak with longer wavelength.

The difference in wavelength to the original is called the **Compton Shift**, $\Delta\lambda$.

This shift varies with the angle of scattering.



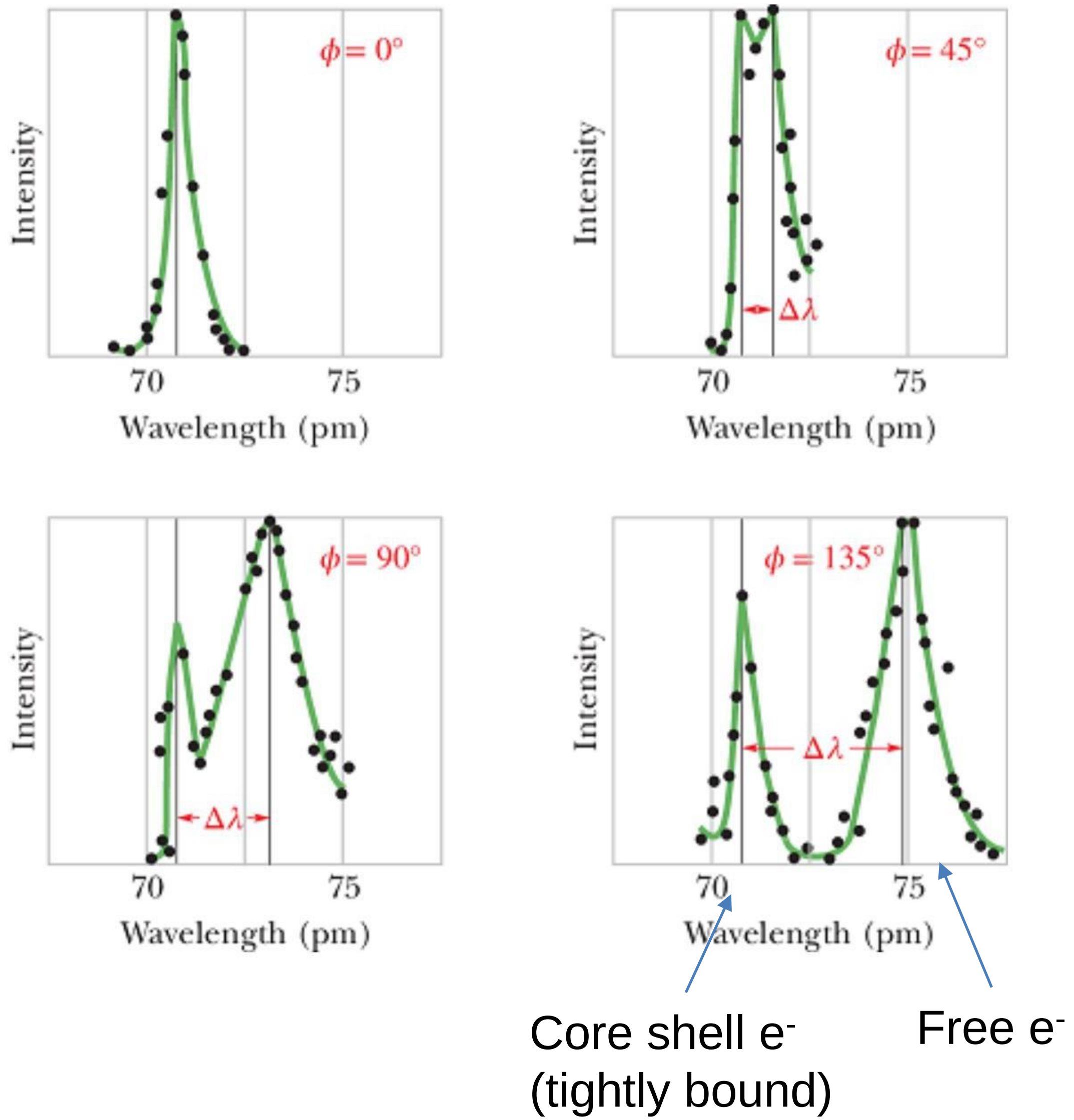
Compton Shift

Explanation

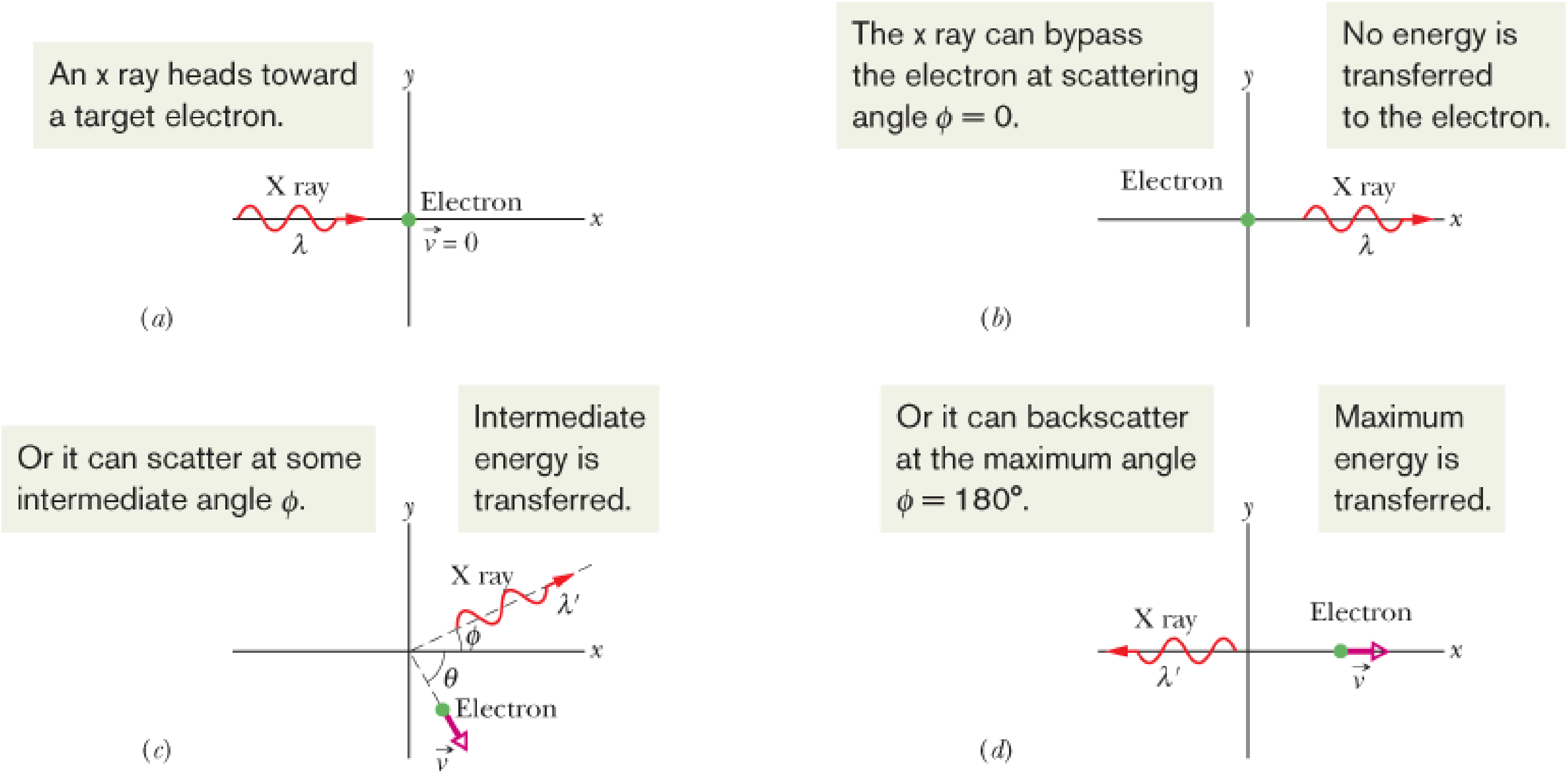
The classical (wave) model of x-rays predicts the oscillating E field causes the electrons to oscillate and re-radiate (i.e. reflect) waves at the *same frequency*. No shift is possible.

The only way to account for a frequency shift is if

- the electron absorbs all the incident photon energy
- the electron keeps of that energy and emits the remainder as a photon with lower energy



Compton scattering results



(isolated system)

By conservation of energy, $hf = hf' + K_e$. By conservation of momentum, $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + p_{e,x}$.

The Compton Shift, $\Delta\lambda$, is given by

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

A way of testing quantum equations is to let h -> 0 and see if they predict the classical result

Sec. 38.3 Figure 38-5

Example 3

X. rays of wavelength 25 pm (photon energy 49.6 keV) are scattered from a carbon target, and the scattered rays are detected at 85^{o} to the incident beam.

(a) What is the Compton Shift of the scattered x-rays?

(b) What percentage of the initial x-ray photon energy is transferred to an electron in such scattering?

Matter waves

- In 1924, Louis de Broglie said (in French): ``A beam of light is a wave, but it transfers energy and momentum to matter only at points, via photons. Why can't a beam of particles have the same properties? Why can't we think of a moving electron [or any other particle] as a matter wave that transfers energy and momentum to other matter at points?''
- de Broglie was appealing to the symmetry of nature...
- He suggested that $\lambda = h/p$ might apply not only to photons but also to electrons. We can use this equation to assign a wavelength, λ, called the de Broglie wavelength, to a particle with a momentum of p.

$$\text{de Broglie wavelength}, \lambda = \frac{h}{p} = \frac{h}{mv}$$

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Example 4

Calculate the de Broglie wavelength of an electron with a kinetic energy of 120 eV.

Example 5

1. Calculate the de Broglie wavelength of a 74 kg person walking at a speed of 5.0 m/s.
2. Why doesn't this runner diffract when walking through a 1.0 m wide door?
3. Calculate the de Broglie wavelength of a 50 eV electron.

Matter waves

- In 1989 Tonomura and co-workers from Hitachi set up a double slit apparatus illuminated by *individual electrons*. An interference pattern appeared to be built up electron-by-electron! (see images on next slide)

- Each electron somehow passed through both slits and interfered with itself!

(the travelling electron behaved as a wave)

- The interference then determined the *probability* that the electron would appear at a given position on the screen.

(When the electron was detected, it behaved as a particle)

- Many electrons appeared in regions corresponding to a bright fringe in optical interference and few electrons in regions corresponding to dark fringes.

Young’s experiment using electrons!

Approximate number of electrons involved:

(a) 7

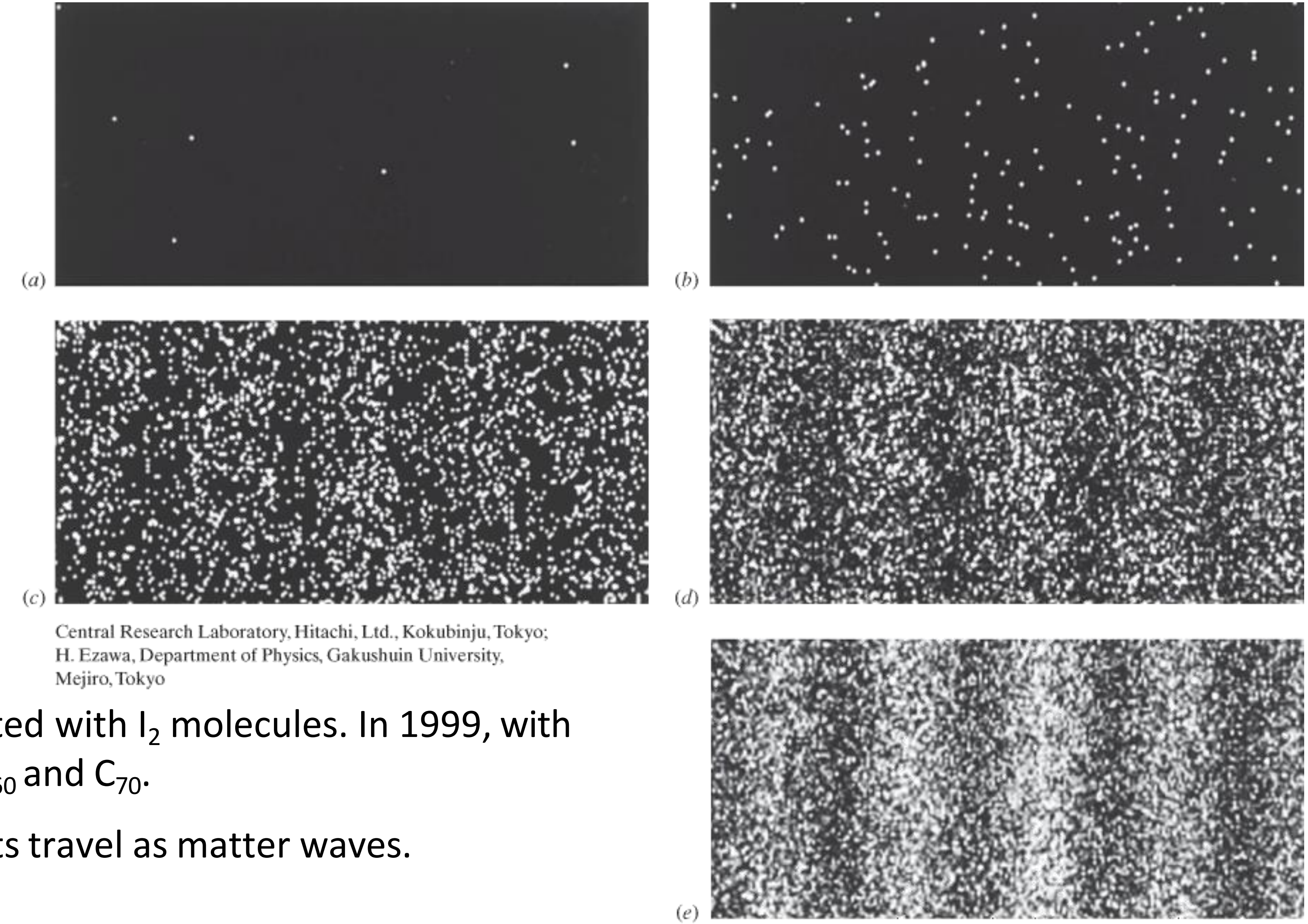
(b) 100

(c) 3000

(d) 20 000

(e) 70 000

Similar interference effects have been observed with protons, neutrons, and various atoms.



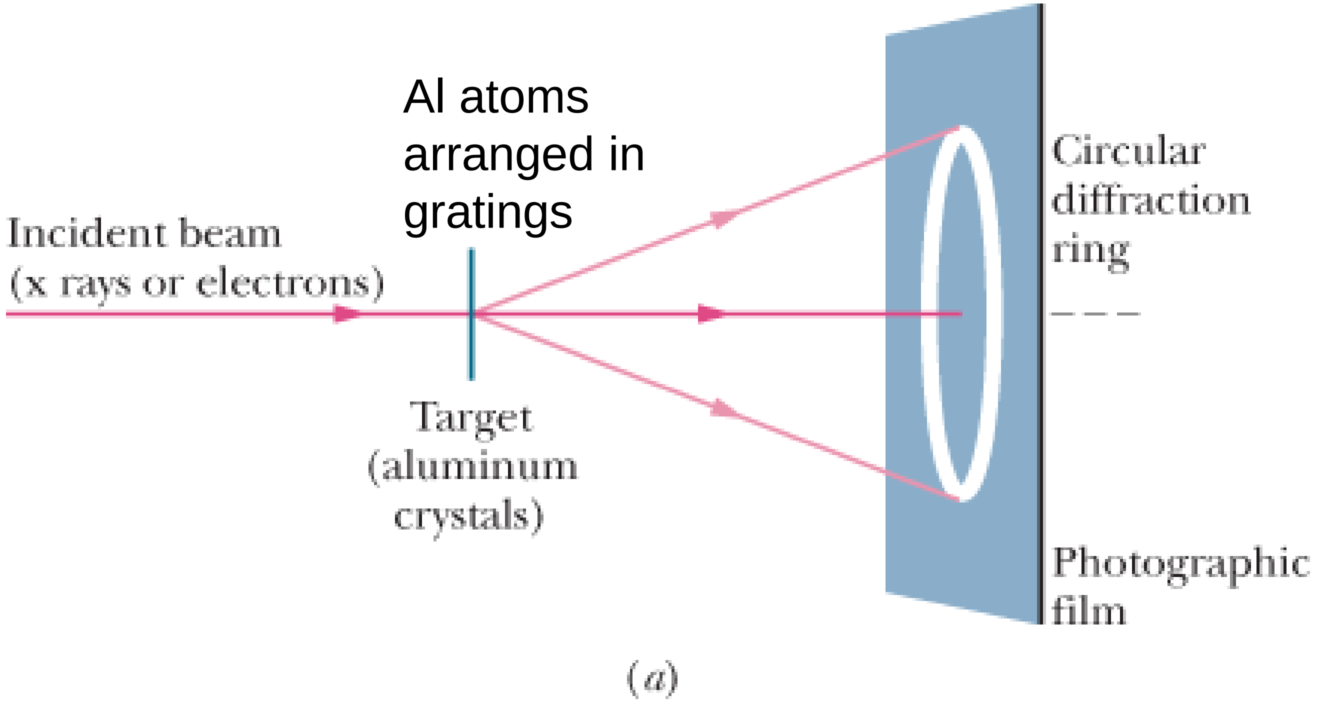
Central Research Laboratory, Hitachi, Ltd., Kokubinju, Tokyo; H. Ezawa, Department of Physics, Gakushuin University, Mejiro, Tokyo

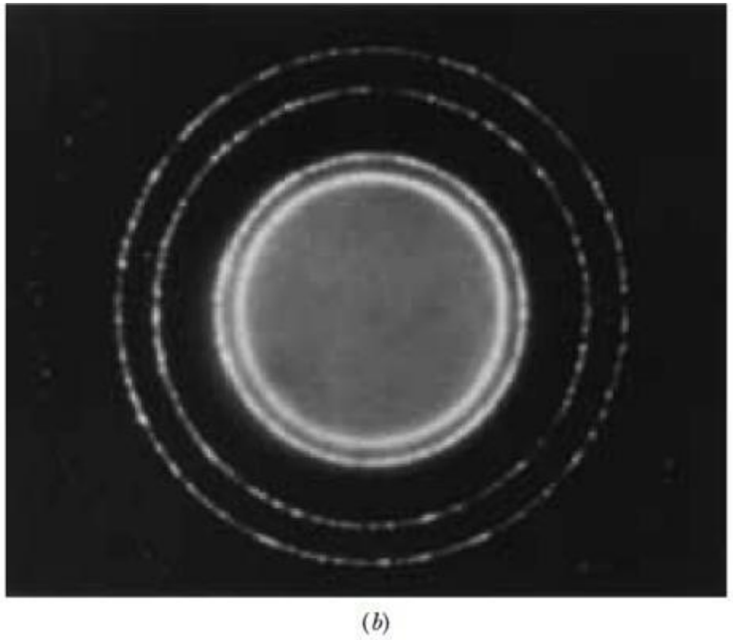
In 1994, was also demonstrated with I_2 molecules. In 1999, with fullerenes (aka buckyballs) C_{60} and C_{70}.

Apparently, such small objects travel as matter waves.

Sec. 38.5 figure 38-9

Diffraction using x-rays and electrons





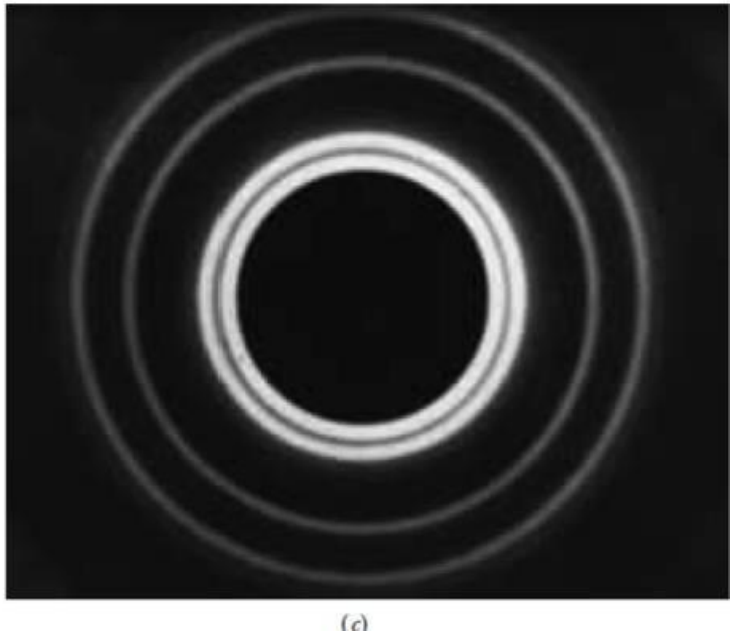


Figure 38-10 : (a) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of e^-.

Photographs of the diffraction patterns when the incident beam is (b) an x-ray beam (EM wave) and (c) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other.

Matter waves – what's waving?

What does the wavelength refer to? What is oscillating?

- We call the wave function Ψ (psi), it's analogous to $\vec{E}$ in EM waves
- We can't measure Ψ like we can $\vec{E}$
- We can measure $|\Psi|^2$

$$|\Psi|^2 = \begin{cases} \text{density of pcls at that point,} & n \gg 1 \\ \text{probability of pcl being at that point,} & n = 1 \end{cases}$$

How do we predict the wave's behaviour? Like using Newtonian mechanics to predict waves in a string?

- Quantum (wave) mechanics!

Quantum Mechanics – Statistics and particle location

- A wave function, $\Psi(x, y, z, t)$, can be used to describe matter waves, then its space and time variables can be grouped separately and can be written in the form

$$\Psi(x, y, z, t) = \psi(x, y, z)e^{-i\omega t}$$

$|e^{-i\omega t}|^2 = ?$

where $\omega = 2\pi f$ is the angular frequency of the matter wave.

- Suppose that a matter wave reaches a particle detector; then the probability that a particle will be detected in a specified time interval is proportional to $|\psi|^2$, where $|\psi|$ is the absolute value of the wave function at the location of the detector.

- $|\psi|^2$ is always both real and positive, and it is called the probability density. “The probability (per unit time) of detecting a particle in a small volume centred on a given point in a matter wave is proportional to the value of $|\psi|^2$ at that point.”

Schrödinger's Equation – QM's wave equation

Suppose a particle travelling in the x direction through a region in which forces acting on the particle cause it to have a potential energy $U(x)$. In this special case, Schrödinger's equation in one dimension can be written as:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U(x)]\psi = 0$$

$E - U(x)$ is the kinetic energy of the particle. If we assume $U(x)$ is constant (even zero for a free particle which has no force acting on it) we can write

$$E - U(x) = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2$$

Solution of Schrödinger's Equation

We also have the wave number: $k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k}$ so $E - U(x)$ becomes:

$$E - U(x) = \frac{1}{2m}\left(\frac{kh}{2\pi}\right)^2$$

Substituting we have:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(\frac{1}{2m}\left(\frac{kh}{2\pi}\right)^2\right)\psi = 0$$

Giving:

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution to this is:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

Here *A* and *B* are constants. This is the time-independent solution of Schrodinger's Equation

Solution of Schrödinger's Equation Cont….

The time dependent solution:

$$\Psi(x,t) = \psi(x)e^{-i\omega t}$$

The solution to this is:

$$\Psi(x,t) = \left(Ae^{ikx} + Be^{-ikx}\right)e^{-i\omega t}$$

$$\Psi(x,t) = Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}$$

The first term represents a wave travelling in the positive direction while the second represents a wave travelling in the negative direction of x.

Finding the probability density, $|\psi|^2$

Let's evaluate for a particle travelling in the positive only at time $t=0$:

$$\psi(x) = Ae^{ikx}$$

So,

$$|\psi|^2 = Ae^{ikx}Ae^{-ikx} = A^2e^{ikx-ikx} = A^2e^0 = A^2$$

Giving

$$|\psi|^2 = A^2$$

Indicating for a free particle with uniform potential energy (even zero), the probability density is a constant.

Solving the Schrödinger equation for a free particle travelling in one direction results in $|\psi|^2 = A^2$, a constant. This says that the particle has equal probability of being detected at all points along its path.

Sec. 38.6

Heisenberg's Uncertainty "Principle"

The result $|\psi|^2 = A^2$, is not general. It turns out that the (measured) values of a particle's position and momentum cannot be determined simultaneously with unlimited precision. Rather:

$$\Delta x \Delta p_x \geq \hbar$$

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\Delta x \, \Delta p_x \geq h$$

. Here Δx and Δp_x represent the uncertainties in
the measurements of x components of position vector **x** and momentum vector $\boldsymbol{p}$ (and similarly for y and z terms).

Even with the best measuring instruments, each product of the position uncertainty and momentum uncertainty will be greater than $h/2\pi$, never less!

Re: diffraction – the narrower the slit (defines y position), the wider the diffraction pattern and the worse the resolution (prob range of p_y)

Sec. 38.6

Example 6

A ball of mass 50 g travels with a speed of 30 m/s. If its speed is measured to an accuracy of 0.1%, what is the minimum uncertainty in its position?

Example 7

Fuzzy, the quantum duck*, lives in a world where $h = 1$ J·s. She has a mass of 2.0 kg and is initially known to be within a 1.0 m wide region. What is the minimum uncertainty in her speed?

*it says "quark"

Mr Tompkins in Wonderland, or, Stories of *c*, *G*, and *h* (1940) by physicist Gamow.