Electromagnetic Induction

Text: Walker *etal. (2021), Halliday's Fundamentals of Physics – First Australian and New Zealand Edition*
John Wiley & Sons Australia (HW)
Walker, Jearl (2014), *Fundamentals of Physics – Halliday & Resnick 10th Edition*
Wiley and Sons Inc, Australia(HRW)

With many thanks to Walter Kalceff for allowing the use of his notes as the basis of these notes.

Introduction – Electromagnetic Induction.

Last week we finished with Ampere’s and Biot-Savart Law. Both are used to find the size of the magnetic field.

What is so important about the magnetic field?

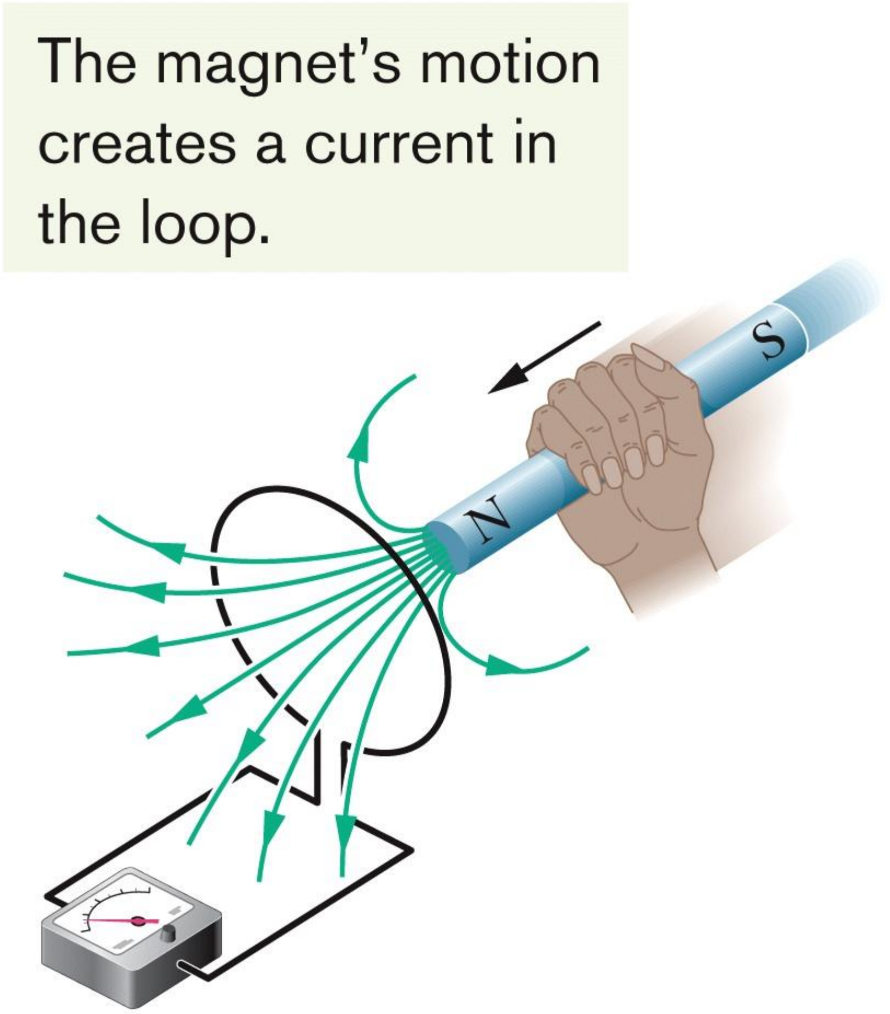
- Credit card
- Washing machine,
- Power drill
- Metal detector
- Generators
- Motor

History – Electromagnetic Induction.

In 1830s in England Michael Faraday and USA Joseph Henry did several pioneering experiments.

First experiment: Conducting loop connected to a galvanometer. Bar magnet **moved** toward results in current. When bar magnet moved away, current direction changes. No current when magnet does not move.

From: (HRW) Figure 30.1, Page 865.

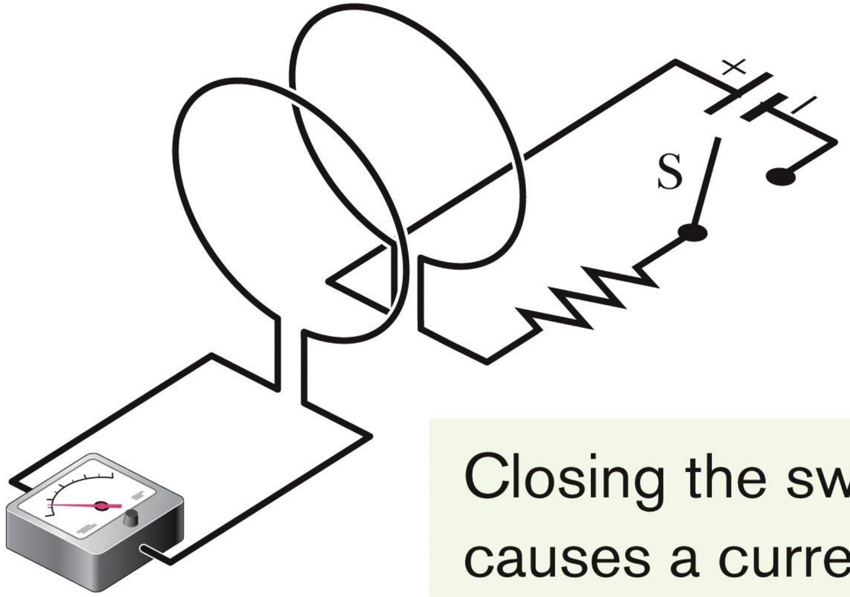


HW : Ch. 30.1

History – Electromagnetic Induction.

In 1830s in England Michael Faraday and USA Joseph Henry did several pioneering experiments.

Second experiment: Conducting loop connected to a galvanometer. When a current is turned ON in a coil it induces a current in the second coil connected to the galvanometer. Once the current reaches a constant value no current is induced, turning off the current, current is once again induced in the opposite direction.



Closing the switch causes a current in the left-hand loop.

From: (HRW) Figure 30.2, Page 865.

The work done to induce the current is called an **induced emf** and the process of producing this current is called **induction**.

HW : Ch. 30.1

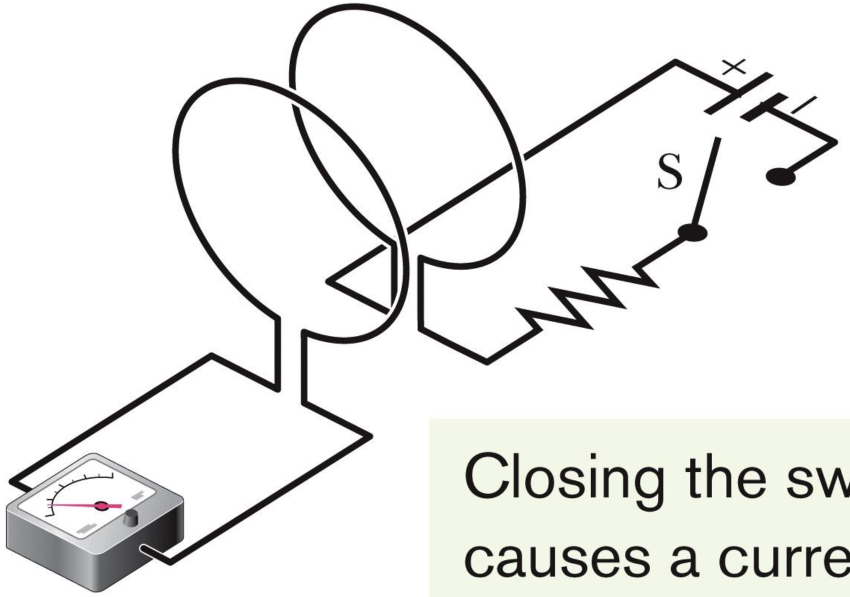
History – Electromagnetic Induction.

If the coil with galvanometer was moved toward the coil with current a current would also be induced.

Faraday realized that the common element in all induction is changing the amount of magnetic field passing through the loop or the change in "**magnetic flux, Φ_B**."

This is the foundation behind this week's lecture topic. This simple phenomena is the basis behind power generation and lead to the way we live today…..meaning electricity!

Let's look at magnetic flux.



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From: (HRW) Figure 30-2, Page 865.

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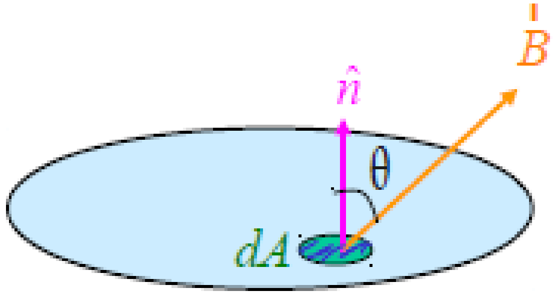
Magnetic Flux, Φ_B .



The magnetic flux through a surface that borders a loop is determined as follows:

We divide the surface that has the loop at its border into area elements of area *dA.* For each element we calculate the magnetic flux, Φ_B, through it by: $d\Phi_B = \vec{B} \cdot d\vec{A} = B_{\perp} dA$

Where $B_{\perp}$ is the perpendicular component of the magnetic field: $d\Phi_B = B dA \cos\theta$

We integrate (sum) all the terms:

$$\Phi_B = \int_{area} B \cos\theta \, dA = \int_{area} \vec{B} \cdot d\vec{A} = BA\cos\theta$$

where the SI unit of magnetic flux is in Weber, Wb = Tm^2 .

If $\vec{B}$ is uniform over a flat area $\vec{A}$ then $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

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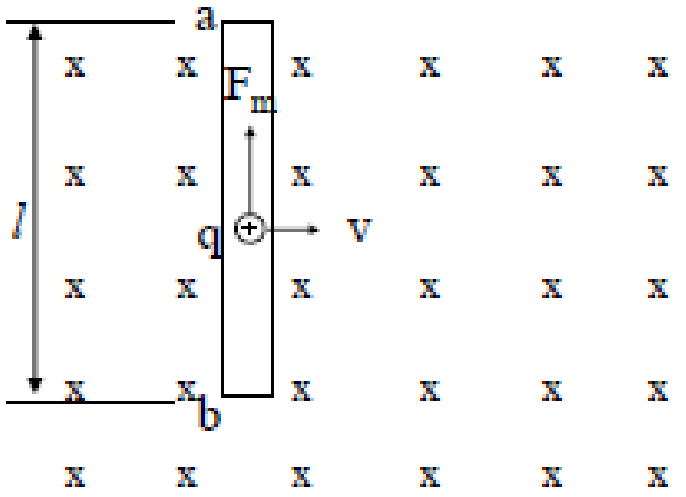
Motional emf

Suppose a wire of length l is swept through the magnetic field at velocity $\vec{v}$ as shown. We know that:



$$\vec{F}_m = q\vec{v} \times \vec{B}$$

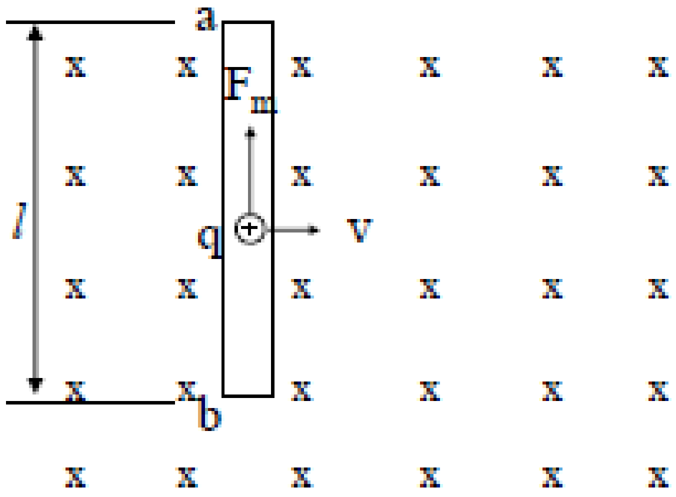
Therefore, free charges move in the direction of $\vec{F}_m$until the accumulation of excess charges at the ends establishes an electric field such that the resultant force on every charge in the conductor is zero.

Suppose a charge q moves from *a* to *b*. The work done by a force $\vec{F}_m$ in so doing is

$$W = \vec{F}_m \cdot \vec{l} = \left(q\vec{v} \times \vec{B}\right) \cdot \vec{l}$$

HW : Ch. 30.1

Motional *emf* cont.



Now, emf is the work per unit charge.

$$\varepsilon = \frac{W}{q} = \frac{q\vec{v} \times \vec{B} l}{q} = \vec{v} \times \vec{B} l$$

where θ is the angle between $\vec{v}$ and $\vec{B}$ the magnitude is:

$$\varepsilon = vBl \sin\theta$$

This is called the induced motional *emf*. Alternatively,

$$\vec{F}_E = q\vec{E} = \vec{F}_B = q\vec{v} \times \vec{B}$$

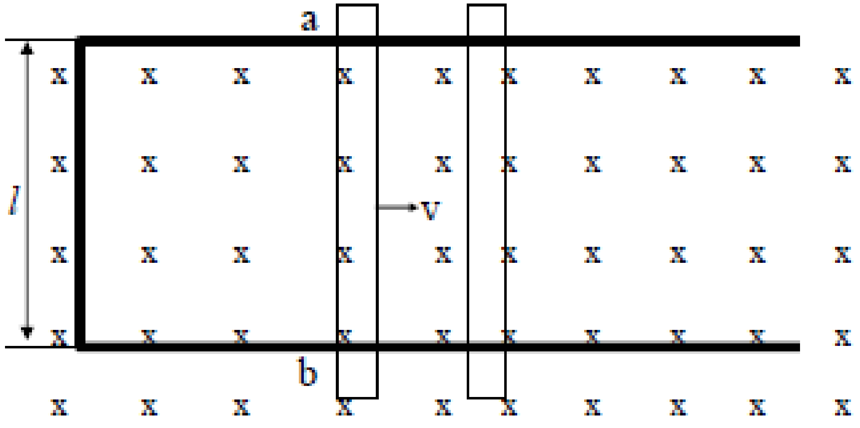
$$\vec{E} = \vec{v} \times \vec{B}$$

therefore the potential difference between a and b is $V = El = vBl \sin\theta$.

HW : Ch. 30.1

Motional *emf* cont.

Suppose now the moving conductor slides along a stationary U-shaped conductor (this also known as a slide wire generator):



If the resistance of the closed loop is $R\Omega$, a current, will flow in the circuit.

$$I = \frac{\varepsilon}{R} = \frac{vBl\sin\theta}{R}$$

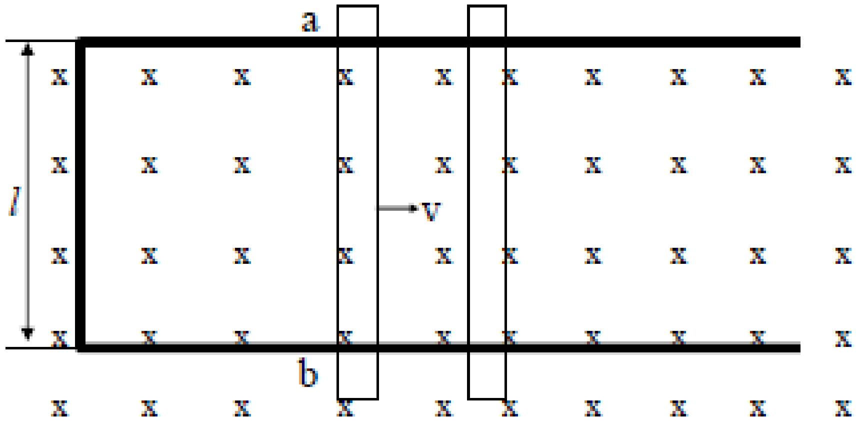
will flow in the circuit.

Example 1

Suppose B=0.1 Tesla, v=1m/s, l=0.25m, R=0.5Ω (assumed constant).

1. What current flows if $\boldsymbol{v} \perp \boldsymbol{B}$ as above?

2. What force is required to keep the rod moving at constant velocity?



Faraday’s Law

While the rod moves dS to the right, the area enclosed by abcd increases by

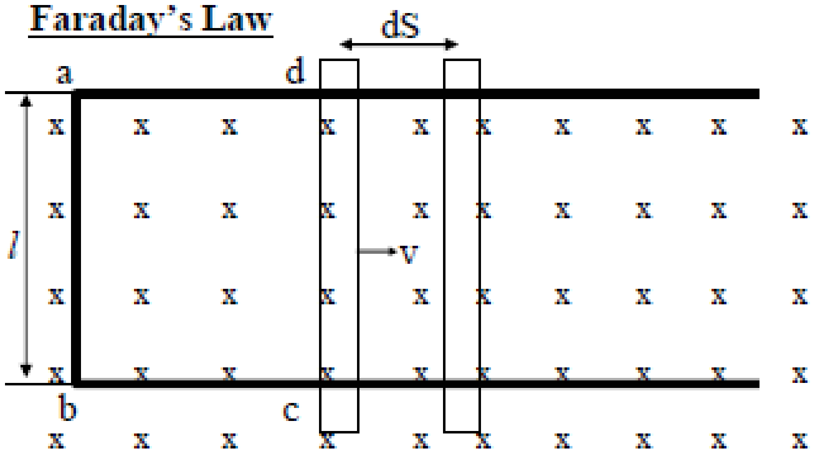
$$\mathrm{d}A = l\mathrm{d}S$$

and the corresponding change in magnetic flux through the circuit is

$$\mathrm{d}\Phi = B\mathrm{d}A = Bl\mathrm{d}S$$

The rate of change of flux is

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)Bl = vBl$$



But $vBl = \varepsilon$, the induced emf so this equation states that:

The induced emf in a circuit of N number of loops is numerically equal to the rate of change of the magnetic flux through it.

$$\varepsilon = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

- Faraday’s Law

HW : Ch. 30.1

Note:

$$\frac{d\Phi}{dt} = \frac{d}{dt}\left(\vec{B} \cdot \vec{A}\right)$$

If A is fixed, but B changes, we can still get an induced emf.

$$\frac{d\Phi}{dt} = A\frac{dB}{dt}$$

If B is constant and A changes:

$$\frac{d\Phi}{dt} = B\frac{dA}{dt}$$

No things constant:

$$\frac{d\Phi}{dt} = A\frac{dB}{dt} + B\frac{dA}{dt}$$

Relative motion is not always required
e.g. transformer

Sir Robert Peel, PM of England, asked Michael Faraday "What possible use can electromagnetic induction have?"

Faraday replied, "I know not, but I wager your government will tax it!"

HW : Ch. 30.1

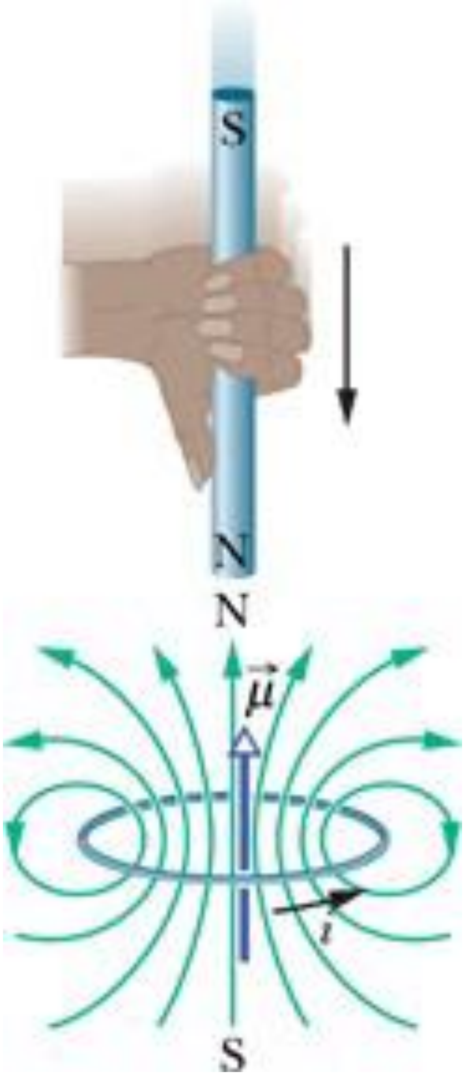
Lenz’s Law

The direction of an induced current is such as to oppose the change producing it.

$$\varepsilon = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

The change referred to can be a motion, increase or decrease in magnetic field, etc.

In the earlier example of the rod being moved to the right, the induced emf will be such as to set up a current which will result in a force on the rod urging it to the left.

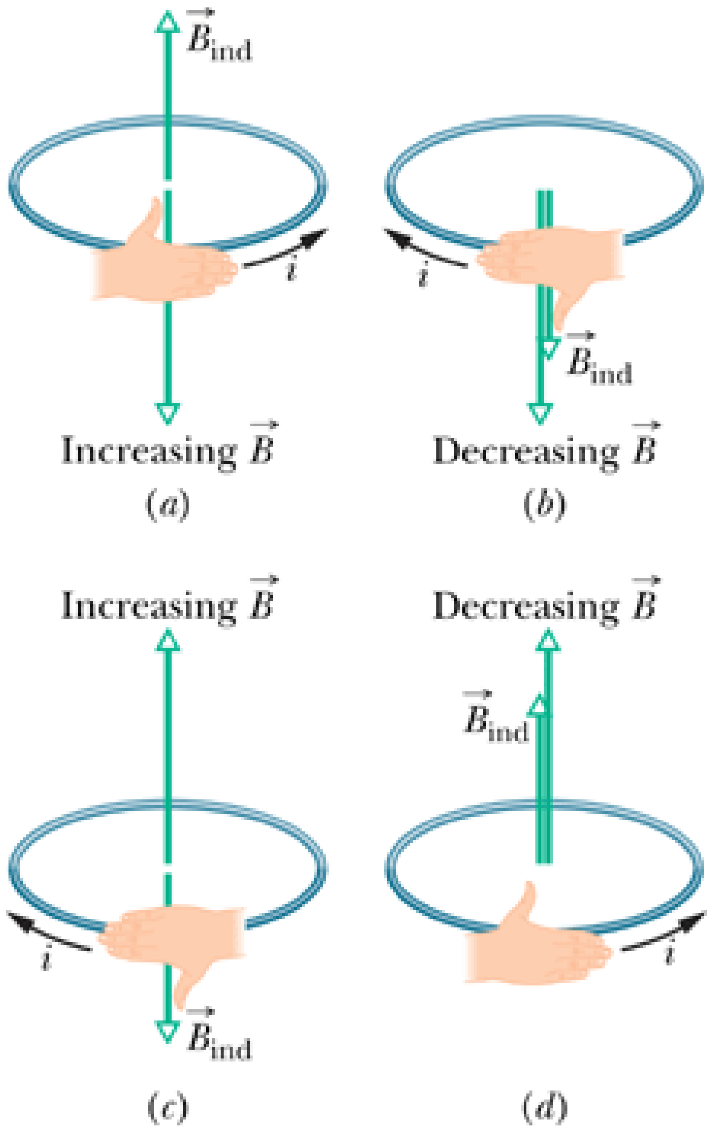


HW : Ch. 30.1

Right hand rule and Lenz's law

If flux increases, point thumb opposite to direction of B. The fingers then curl in the direction of the induced current.

If flux decreases, point thumb in the direction of B. The fingers then curl in the direction of the induced current.



HW : Ch. 30.2

Loop pulled through a field

Suppose a loop is pulled from a region of magnetic field with restricted region.

- As the loop is pulled through the field the amount of flux through the loop changes.
- When pulling, a force must be applied to keep the loop moving at constant *v* as the induced current results in a magnetic force on the loop.
- Top and bottom wires have opposing F which cancel out. Left arm has force that acts inwards owed to the induced current.

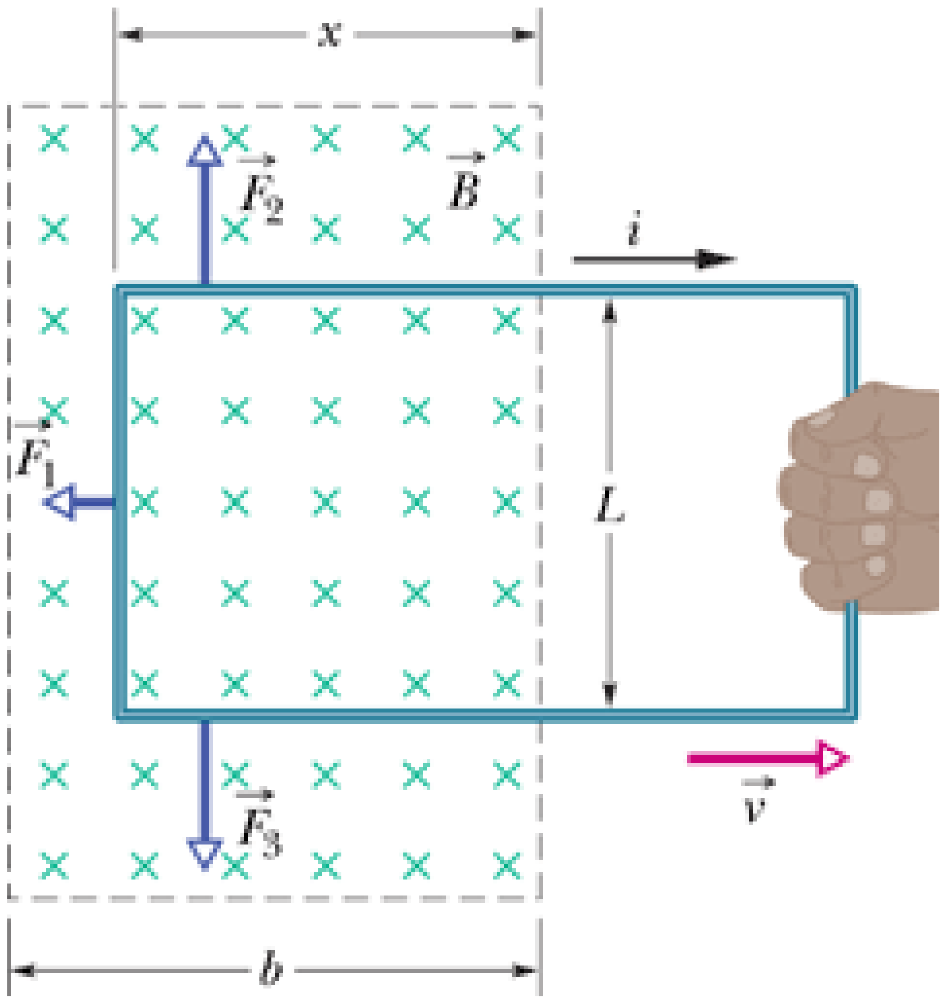
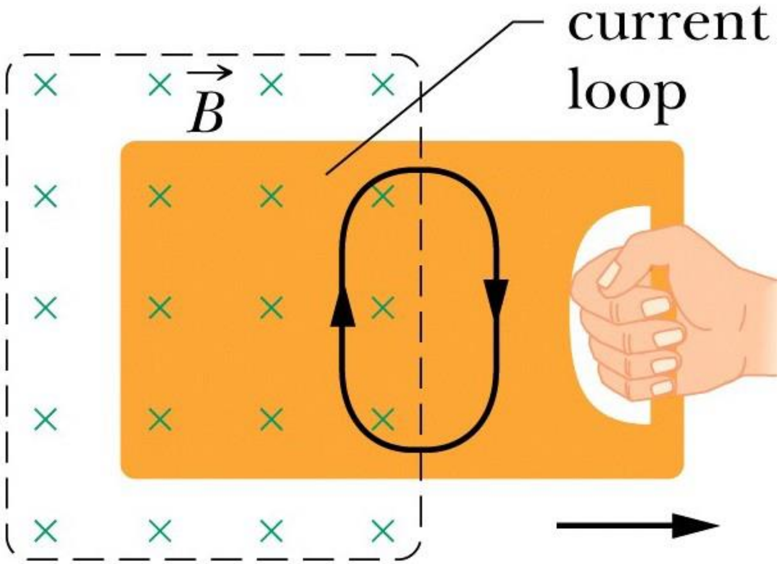


Diagram p872 Figure 30-8 HRW

HW : Ch. 30.2

Eddy Currents



Similar to the loop in previous example but replacing with solid conducting plate.

The relative motion of field and conductor induces a current -> opposing force therefore work is needed to pull the plate out of the field.

Unlike the wire loop, the electrons making up the current in the plate follow many paths and they swirl about in the plate as if they were caught in an 'eddy' of water. This type of current is called an **Eddy Current**. The current induced in the plate results in the mechnical energy dissipated as thermal energy.

Interaction between the eddy currents and field produces a breaking action! Others uses of eddy currents: stop rotation of circular saw when power is turned off, breaking mechanism in rapid-transit vehicles (train), induction heating, metal detectors. Not all good effects…..in transformers with iron core, eddy currents waste energy therefore a layering system is used to reduce path of eddy currents.

HRW p. 874 Figure 30-10

HW : Ch. 30.2

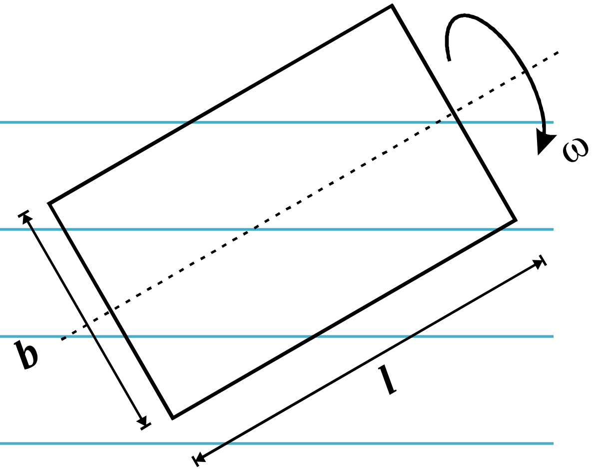
Example 2

A coil of wire has 1000 loops of radius 2cm. It is placed $\perp$ to a uniform magnetic field which increases at 0.2T/s.

What is the magnitude of the induced emf produced?

Example 3

A rectangular coil containing 400 turns has a length $l = 80\text{ mm}$ and width $b = 50\text{ mm}$. The coil is rotated as shown with an angular velocity $\omega = 8\text{ rad/s}$. The axis of rotation is normal to the direction of a uniform magnetic field of induction 0.1 T.



(a) Find the emf induced in the coil as it rotates.

(b) What would be the induced emf if the coil were fixed normal to a field $B = 0.1 \sin 8t\ \text{ Tesla}$?

Example 3 cont.

A rectangular coil containing 400 turns has a length $l = 80\ \text{mm}$ and width $b = 50\ \text{mm}$. The coil is rotated as shown with an angular velocity $\omega = 8\ \text{rad/s}$. The axis of rotation is normal to the direction of a uniform magnetic field of induction $0.1\ \text{T}$.

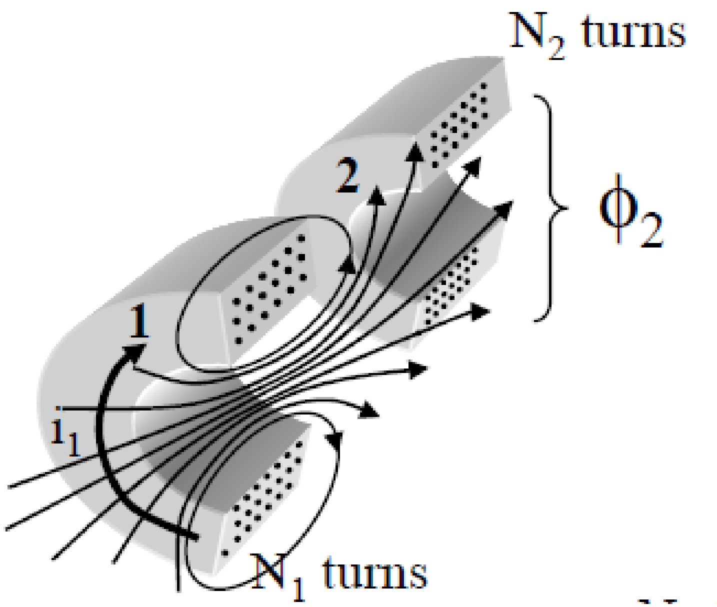
(a) Find the emf induced in the coil as it rotates.

(b) What would be the induced emf if the coil were fixed normal to a field $B = 0.1 \sin 8t\ \ \text{Tesla}$?

Example 4

An airplane with a total wing span of 30 m flies at a speed of 200 m/s normal to a magnetic field of 2×10^{-5} T directed vertically downwards. Find the emf induced between the opposite wing tips and state which one is positive with respect to the other.

Mutual inductance



Consider two closely wound coils near each other.

Current i_1 flows in coil 1 with N_1 turns, setting up a magnetic field.

field lines pass through coil 2 with N_2 turns, producing a flux Φ_2.

$$B \propto i_1 \qquad \therefore \qquad \Phi_2 \propto i_1 \qquad \text{also} \qquad N_2\Phi_2 = Mi_1$$

where M is a constant of proportionality. If i_1 changes with time, so does Φ_2. This changing flux induces an emf ε_2 in coil 2.

$$\varepsilon_2 = N_2\frac{d\Phi_2}{dt} = M\frac{di_1}{dt}$$

M depends on the geometry (size, shape, number turns, and orientation of the coils) of the two coils. It is called their mutual inductance.

$$M = \frac{N_2\Phi_2}{i_1} \qquad (\text{Henry, H})$$

HW : Ch. 30.9

Electric Toothbrush and transformers

Electric toothbrush: Base contains a coil supplied with alternating current, which induces an emf in a coil within the brush which is used to recharge the toothbrush battery.

Another use of mutual inductance is the transformer which is used to raise or lower voltages by changing the geometry of the second coil (turns).

https://www.oralb.com.au/en-au/products/pro-4000-crossaction-rechargeable

HW : Ch. 30.9

Example 5

Two coils have a mutual inductance of M Henry. Starting from the equation

$$E_{secondary} = -M\frac{dI_{primary}}{dt}$$

show that if a charge Q passes in the secondary circuit then the current in the primary circuit has changed by an amount

$$I_p = -\frac{R_s Q}{M}$$

where R_s is the resistance of the secondary circuit.

HW : Ch. 30.9

Example 6

Two coils have a mutual inductance of M Henry. If the primary current is

$$I_p = I_0 \sin \omega t$$

what will be the emf induced in the secondary coil?

Mutual Inductance and Self-Inductance

Mutual inductance: we have considered two coils or independent circuits where a changing current in one coil produces a changing flux which cuts the other coil and induces a current/emf in the other.

An important effect also occurs in single circuits. When a current is present, a field is set-up causes a magnetic flux in the same circuit. This flux changes when the current changes. As such any circuit that has a changing current has an induced emf by the variation in its own magnetic field called a **self-induced emf**.

Lenz's Law: this induced emf opposes the change in current that caused the emf making current variation more difficult.

Self-inductance is usually referred to as just inductance.

HW : Ch. 30.4

Self-Inductance, *L* or simply Inductance

Suppose a coil has N turns of wire and a flux Φ passes through each turn. The self inductance, L, of the coil is defined as

$$L = \frac{N\Phi}{i} \quad \mathrm{H} \qquad \text{or} \qquad N\Phi = Li$$

Differentiating with respect to time:

$$N\frac{d\Phi}{dt} = L\frac{di}{dt}$$

Since

$$\varepsilon = N\frac{d\Phi}{dt}$$

If follows that

$$\varepsilon = L\frac{di}{dt}$$

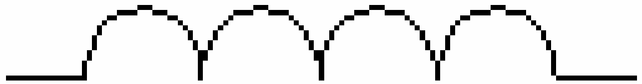
In words, the self inductance of a circuit is the self induced emf per unit rate of change of current. The direction of induced emf is given by Lenz's law.

$$\varepsilon = -L\frac{di}{dt}$$

HW : Ch. 30.5

Inductors as Circuit Elements

A device with particular inductance is called and **inductor**, or a *choke* and has the symbol:



The inductor’s purpose is to oppose any variation in current through the circuit. In a DC circuit it helps to maintain a steady current when there might be some variation in the emf. In an AC circuit it suppresses variations in current that are more rapid than desired.

In Fluorescent lights the inductor (magnetic ballast) is used to make sure current does not get too high and makes it possible for the tube to work with the AC of household wiring to stop the plasma from rapidly deionizing owed to AC.

HW : Ch. 30.4

Inductance of a solenoid

We have seen that

$$L = \frac{N\Phi}{I} \quad \mathrm{H}$$

For an air-core solenoid,

$$B = \mu_0 nI \quad \text{where} \quad n = \frac{N}{l}$$

Assuming the field is uniform for the entire cross-section, *A*, the total flux through the solenoid is

$$\Phi = BA = \left(\mu_0 \frac{N}{l} I\right) A$$

Thus,

$$L = \frac{N\Phi}{I} = \mu_0 N^2 \frac{A}{l}$$

This value depends ***only on the physical characteristics of the solenoid*** (*N*, *l*, *A* and the material within it).

If the coil is wound on a core of magnetic material with permeability μ, then μ_0 has to be replaced by μ. If the material is ferrite, then $\mu \approx 15000\ \mu_0$ and so the self inductance increases 15,000-fold!

The induced emf also increases by this factor, as

$$\varepsilon = -L\frac{dI}{dt} = \mu_0 N^2 \frac{A}{l}\frac{dI}{dt}$$

HW : Ch. 30.5

Energy Storage in an inductor

Suppose an inductor of inductance *L* carries zero initial current.

Now suppose at some later time *t* the current is *i* and it changes at a rate *di/dt*.
The terminal voltage of a source supplying this current is then

$$V_{AB} = L\frac{di}{dt}$$

and the instantaneous power supplied by the source is

$$P = V_{AB}i = Li\frac{di}{dt}$$

The energy *dU* supplied to the inductor in a tiny time interval *dt* is

$$dU = Pdt = Lidi$$

Thus, the total energy supplied while the current increases from 0 to *I* is

$$U = L\int_0^I idi = \frac{1}{2}LI^2$$

HW : Ch. 30.7

Energy Storage in an inductor

It turns out that the current builds up to a maximum steady value, *I*, at which time di/dt= 0.

The energy

$$U = \frac{1}{2}LI^2$$

is stored in the magnetic field of the coil. (c.f. capacitor, where the energy $U = \frac{1}{2}CV^2$ is stored in the electric field between plates.)

Sometimes we refer to the ***energy density*** of an inductor

$$\text{Energy density } (u) = \frac{\text{Energy stored in inductor } (U)}{\text{Volume of inductor } (V)}$$

For a solenoid of length *l* and cross sectional area *A*,

$$u = \frac{U}{V} = \frac{\frac{1}{2}LI^2}{Al}$$

but

$$L = \frac{NBA}{I} \quad \text{and} \quad B = \mu\frac{N}{l}I$$

so

$$u = \frac{\frac{1}{2}\frac{NBA}{I}I^2}{Al} = \frac{\frac{1}{2}NBI}{l} = \frac{1}{2\mu}B\left(\mu\frac{N}{l}I\right)$$

giving the energy density of a solenoid:

$$u = \frac{B^2}{2\mu}$$

HW : Ch. 30.7, 30.8