Family Name:		Student ID:
Given Name:		
Tutorial:	Wed Thur Fri	
	10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm	
Tutor:	Cahit Jerry Jie Murray Roumani Sherwin Tim Tom	

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 2

INSTRUCTIONS. 40 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC1 before 7:59pm Tuesday 8 March 2022.
Name your file as `LPC2-LastName-StudentID.pdf`. Show all relevant working and steps.
You may refer to your personal class notes and the Table on page 3 of this document.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites. We are checking your learning progress, not somebody else's.

1. (1.5 marks)
   (a) Prove or disprove: for all $x, y \in \mathbb{R}$, if $x + y \geqslant 300$ then $x \geqslant 150$ or $y \geqslant 150$.

   (b) Your method in part (a) is

   A. direct proof

   B. proof by contradiction

   C. proof via contrapositive

   D. counterexample

   E. none of (A)–(D).

Date: Tuesday 8 March 2022.

2. (1.5 marks)
   (a) Prove or disprove: if $k^2 \in \mathbb{Z}$ is divisible by 4 then k is divisible by 4.

   (b) Your method in part (a) is

   A. direct proof

   B. proof by contradiction

   C. proof via contrapositive

   D. counterexample

   E. none of (A)–(D).

3. (2 marks) Simplify the following logical expression using the rules from Table 1 (next page). Give the reason for each simplification next to each step. The first step is partially done for you, as well as a final answer.

$$\neg\left((\neg p \wedge \neg q) \rightarrow (\neg q \rightarrow \neg p)\right)$$

$\neg\left((\neg p \wedge \neg q) \rightarrow (\neg q \rightarrow \neg p)\right)$

$\equiv$ De Morgan's law

$\equiv$

$\vdots$

$\equiv F$

END OF LPC2

Some tautologies with names:

logic rule (tautology)	name
$\neg(\neg p) \quad \leftrightarrow \quad p$	double negative
$\neg(p \vee q) \quad \leftrightarrow \quad \neg p \wedge \neg q$ $\neg(p \wedge q) \quad \leftrightarrow \quad \neg p \vee \neg q$	De Morgan
$p \vee q \quad \leftrightarrow \quad q \vee p$ $p \wedge q \quad \leftrightarrow \quad q \wedge p$	commutative
$p \vee (q \vee r) \quad \leftrightarrow \quad (p \vee q) \vee r$ $p \wedge (q \wedge r) \quad \leftrightarrow \quad (p \wedge q) \wedge r$	associative
$p \vee (q \wedge r) \quad \leftrightarrow \quad (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \quad \leftrightarrow \quad (p \wedge q) \vee (p \wedge r)$	distributive
$p \vee p \quad \leftrightarrow \quad p$ $p \wedge p \quad \leftrightarrow \quad p$	idempotent
$p \vee F \quad \leftrightarrow \quad p$ $p \wedge T \quad \leftrightarrow \quad p$	identity
$p \vee (p \wedge q) \quad \leftrightarrow \quad p$ $p \wedge (p \vee q) \quad \leftrightarrow \quad p$	absorption
$p \to q \quad \leftrightarrow \quad \neg p \vee q$	useful one
$p \vee \neg p \quad \leftrightarrow \quad T$ $p \wedge \neg p \quad \leftrightarrow \quad F$	inverse
$p \vee T \quad \leftrightarrow \quad T$ $p \wedge F \quad \leftrightarrow \quad F$	domination
$p \to q \quad \leftrightarrow \quad \neg q \to \neg p$ $(\neg p \to F) \to p$	contrapositive contradiction