Family Name: Student ID: _ _ _ _ _ _ _ _

Given Name:

Tutorial: Wed Thur Fri

10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm

Tutor: Cahit Jerry Jie Murray Roumani Sherwin Tim Tom

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 3

Instructions. 40 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC1 before 7:59pm Tuesday 15 March 2022.
Name your file as `LPC3-LastName-StudentID.pdf`. Show all relevant working and steps.
You may refer to your personal class notes, and a basic (non-programmable) calculator.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites. We are checking your learning progress, not somebody else's.

1. (1.5 marks) Your student ID number consists of eight digits $d_1d_2d_3d_4d_5d_6d_7d_8$.
Use the Euclidean algorithm to find

$$\gcd(d_1d_2d_3d_4, d_5d_6d_7),$$

showing all steps. [1]

Date: Tuesday 15 March 2022.

[1] For example, if my ID is 12345678 then I need to compute $\gcd(1234, 567)$, and if my ID is 34560789 then I need to compute $\gcd(3456, 78)$. We are treating the ID as a string of digits, so that each student has a different problem to solve.

2. (1.5 marks) Let A, B, C be sets in some arbitrary universal set $\mathscr{U}$. Show that

$$\overline{(A \cup C)} \cup B = \overline{(A \cap C)} \cup (B \cup \overline{C})$$

is false in general by giving an example using $\mathscr{U} = \{1, 2, 3, 4, 5\}$. [2]

$\mathscr{U} =$ $\{1, 2, 3, 4, 5\}$ $B =$

$A =$ $C =$

$\overline{(A \cup C)} \cup B =$

$\overline{(A \cap C)} \cup (B \cup \overline{C}) =$

3. (1 mark) Prove or disprove: [3]

If $A \subseteq B$ then $A \cap \overline{B} = \emptyset$.

[2]You may find it useful to draw a Venn diagram first, but final answer should be the explicit sets as a counterexample.

[3]Hint: direct? contrapositive? contradiction? counterexample? Remember a Venn diagram is not a proof, so any proof should look like "let $x \in \ldots$". A counterexample should be some explicit sets $A = \{\ldots$.

4. (1 mark) Let $\mathbb{N}_+$ denote the set of positive natural numbers $\{1, 2, 3, 4, \ldots\}$.
Let $x, d \in \mathbb{Z}$ with $d > 0$.

Define
$$M = \{x - qd \mid q \in \mathbb{Z}\}.$$

Prove that $M \cap \mathbb{N}_+$ is non-empty. [4]

END OF LPC3

[4] Hint: cases. Think about $x = 0, x > 0, x < 0$.