Family Name: Student ID: _ _ _ _ _ _ _ _
Given Name:
Tutorial: Wed Thur Fri
10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm
Tutor: Cahit Jerry Jie Murray Roumani Sherwin Tim Tom

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 4

INSTRUCTIONS. 40 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC1 before 7:40pm Tuesday 22 March 2022.
Late uploads will not be accepted by Canvas.
Name your file as `LPC4-LastName-StudentID.pdf`. Show all relevant working and steps.
Handwrite on blank paper, printout or tablet (**do not type**).
You may refer to your personal class notes, and a basic (non-programmable) calculator.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites.

1. (1.5 marks) Fill in the missing lines of the following proof by induction.

Lemma 1. *For all $n \in \mathbb{N}_+$*

$$1 \cdot 2 \cdot 3 \ + \ 2 \cdot 3 \cdot 4 \ + \ \dots \ + \ n(n+1)(n+2) \ = \ \frac{n(n+1)(n+2)(n+3)}{4}.$$

Proof. Let $P(n)$ be the statement $\boxed{}$

Then $P(\boxed{})$ is true since $\boxed{}$

Assume $P(k)$ for $k \geqslant \boxed{}$. Then

$\boxed{}$

Thus by PMI $P(n)$ is true for all integers $n \geqslant \boxed{}$. □

Date: Tuesday 22 March 2022.

2. (2 marks) Consider the following segment of pseudocode: [1]

```
procedure (n int, j int)

if j < 50
        j := j+50

print j

while n<1000
        n := n + 2*j
        print n
endwhile
```

(a) Which of the following statements does not satisfy the property that if it is true before one iteration of the `while` loop, then it is true after one iteration (*i.e.* is a loop invariant)?

A. $n + j$ is even

B. nj is odd

C. $n > j$.

D. $n + j$ is odd

E. $nj > 0$

F. $j < 50$

G. none of (A)–(F).

Explain your reasoning:

(b) Let $d_1d_2d_3d_4d_5d_6d_7d_8$ be your student ID number. Write out the output of the pseudocode given above on input $n = d_1d_2d_3$ and $j = d_7d_8$. [2]

[1] the syntax `n int, j int` means the inputs are integers.
[2] Eg. if your ID is 12300321 then $n = 123$ and $j = 21$.

3. (1.5 marks) Let $P(n)$ be the statement

$$5n + 7 \leqslant n^2$$

(a) Find the smallest value $c \in \mathbb{N}$ so that $P(c)$ is true.

(b) Prove that $P(n)$ is true for all integers $n \geqslant c$ for some integer $c \in \mathbb{N}$. [3]

END OF LPC4

[3] Hint: use PMI. Set out your proof as per the template.