Family Name:	Student ID: _ _ _ _ _ _ _ _
Given Name:	
Tutorial:	Wed Thur Fri
	10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm
Tutor:	Cahit Jerry Jie Murray Roumani Sherwin Tim Tom

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 5

INSTRUCTIONS. 40-60 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC1 before 7:40pm Tuesday 29 March 2022.
Late uploads will not be accepted by Canvas.
Name your file as `LPC4-LastName-StudentID.pdf`. Show all relevant working and steps.
You may refer to your personal class notes, and a basic (non-programmable) calculator.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites.
Recall $\mathbb{N}$ is the set of natural numbers including 0 in this subject.

1. (2 marks) Let $\mathscr{R}$ be the relation on $\mathbb{N}$ defined by "$a\mathscr{R}b$ if $\frac{a}{b} \in \mathbb{N}$ or $5 \mid (b-a)$".

   So for example $(6,2) \in \mathscr{R}$ and $(1,5) \notin \mathscr{R}$.

   Decide if each of the following are *true* or *false*, and briefly justify each answer.

   (a) $\mathscr{R}$ is reflexive

   (b) $\mathscr{R}$ is symmetric

   (c) $\mathscr{R}$ is antisymmetric

   (d) $\mathscr{R}$ is transitive

Date: Tuesday 29 March 2022.

2. (1.5 marks) Define a function $B : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ using the following recursive definition.

$$\begin{array}{lcll} B(0,n) & = & n^2 & n \geqslant 0, \\ B(m,0) & = & B(m-1,1) & m > 0, \\ B(m,n) & = & B(m-1, B(m,n-1)) & m,n > 0. \end{array}$$

(a) Compute $B(1,1)$. Show your working.

(b) Compute $B(2,2)$. Show your working.

(c) Prove or disprove: for all $x, y \in \mathbb{N}$, $B(x,y) = 1$.

3. (0.5 marks) Let A be a set and $\mathscr{P}(A)$ the power set of A.

   Complete the following proof that there is *no* onto function from A to $\mathscr{P}(A)$.

   Proof: Suppose (for contradiction) that $f : A \to \mathscr{P}(A)$ is an onto function.

   Consider the set $B = \{x \in A \mid x \notin f(x)\}$.

   If f is onto, then for all $C \in \mathscr{P}(A)$,
   there must be an element $x \in A$ so that $f(x) = C$.

   $B \subseteq A$ so $B \in \mathscr{P}(A)$, so there is an element $x \in A$ so that $f(x) = B$.

   Question: is $x \in B$ or $x \notin B$?

4. (1 mark) Prove that for all $n \in \mathbb{N}$, $7^n + 2$ is divisible by 3. [1]

END OF LPC5

[1]Hint: $2 = 14 - 12$.