Family Name:	Student ID: _ _ _ _ _ _ _ _
Given Name:	
Tutorial:	Wed Thur Fri 10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm
Tutor:	Cahit Jerry Jie Murray Roumani Sherwin Tim Tom

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 6

INSTRUCTIONS. 40-60 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC6 before 7:40pm Tuesday 5 April 2022.
Late uploads will not be accepted by Canvas.
Name your file as `LPC6-LastName-StudentID.pdf`. Show all relevant working and steps.
You may refer to your personal class notes, and a basic (non-programmable) calculator.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites.

1. (1 mark) Use induction to prove that $n \leqslant 30^n$ for all $n \in \mathbb{N}_+$.
   Set out your proof as in the induction template.

Date: Tuesday 5 April 2022.

2. (2 marks) Recall: $f \in O(g)$ if $\exists k \in \mathbb{N}_+, m \in \mathbb{R}_+$ such that $|f(n)| \leqslant m|g(n)|$ for all $n \in \mathbb{N}, n \geqslant k$.

(a) Prove that $5n + 7 \in O(n^2)$, stating clearly your values for k and m, and justifying each step.

$m =$ $k =$

(b) Prove that $n + \log_{30} n \in O(n)$, stating clearly your values for k and m, and justifying each step.

$m =$ $k =$

3. (1 mark) Fill in the missing boxes in the following proof from the options below. (You may use options more than once, and not all options fit.)
Please write both the letter and the phrase (eg. direct proof, (a)).

Lemma 1. *Let $m \in \mathbb{Z}$. If m^3 is even then m is even.*

Proof. Suppose m is [ ].

Then there exists $p \in \mathbb{Z}$ such that [ ].

Thus $m^3 =$ [ ]

$=$ [ ].

so m^3 is [ ].

The statement then follows by [ ]. →[1] □

(a) direct proof

(b) contrapositive

(c) false

(d) $m = 2p$

(e) $m = 2p + 1$

(f) $m = 3p + 1$

(g) odd

(h) even

(i) $(2p + 1)^3 = 8p^3 + 6p^2 + 6p + 1$

(j) $(2p)^3 = 8p^3$

(k) $(2p + 1)^2 = 4p^2 + 4p + 1$

(l) $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$

(m) $(2p)^2 = 4p^2$

(n) $2(4p^3)$

(o) $2(4p^3 + 3p^2 + 3p) + 1$

(p) $2(4p^3 + 6p^2 + 3p) + 1$

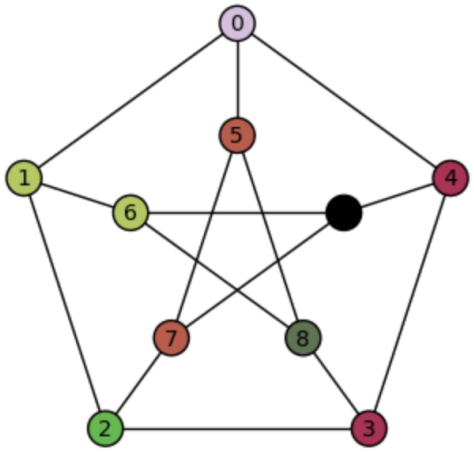
(q) magic

(r) induction

[1] This is the end of proof symbol, not a box to fill.

4. (1 mark) In a deleted scene from the Netflix series *Squid Game*, players have to play the following game.

   One at a time, they have to walk around the following network:



   The player starts on any node they choose (a *node* is one of the coloured/numbered circles in the picture).

   Each 30 seconds, a bell sounds and the player must move along an edge to another node. If the player ever visits the same node twice, or does not move when the bell sounds, they are instantly shot and killed.

   After 5 minutes, the game ends. What is the chance a player survives this game? Explain/justify your answer (maybe using PHP).

[2]

END OF LPC6

[2] If your justification uses the (generalised) pigeonhole principle, state clearly what are the pigeons, what are the boxes, and the rule for places a pigeon into a box.