Family Name:		Student ID:	_ _ _ _ _ _ _ _
Given Name:			
Tutorial:	Wed Thur Fri 10am 10:30am 11am 11:30am 12:30am 1pm 2pm 2:30pm 3pm 3:30pm 4pm 4:30pm 5pm		
Tutor:	Cahit Jerry Jie Murray Roumani Sherwin Tim Tom		

37181 DISCRETE MATHEMATICS
LEARNING PROGRESS CHECK 7

INSTRUCTIONS. 40-60 minutes.
Upload **as a single PDF file** on Canvas/Assignments/LPC7 before 7:59pm Tuesday 26 April 2022. (Recommended: 7:40pm)
Late uploads after 7:59pm will not be accepted by Canvas.
Name your file as `LPC7-LastName-StudentID.pdf`. Show all relevant working and steps.
You may refer to your personal class notes, and a basic (non-programmable) calculator.
Work on this on your own without discussing with anyone or using Discord/WeChat/any websites including paid homework sites.

1. (1 mark) If $d_1d_2d_3d_4d_5d_6d_7d_8$ is your student ID number, let $n = 100 + d_7d_8$ and $a = d_5d_6$. [1]

   Find $d \in \mathbb{Z}_n$ such that $d \cdot a \equiv 1 \mod n$, that is, the multiplicative inverse mod n for a,
   or
   explain why no such d exists. Show all steps. [2]

Date: Tuesday 26 April 2022.
[1]for example, if your ID is 12345678 then $n = 100 + 78 = 178$, and $a = 56$.
[2]Hint: Euclidean algorithm backwards.

2. (1.5 marks) Recall that a *factor* of a string v is a string u so that $v = xuy$ for some strings x, y. For example, 001 is a factor of 010011 where $x = 01$ and $y = 1$.

   Define a relation $\mathcal{T}$ on the set of all finite length (including 0) binary strings by

   $$a\mathcal{T}b \quad \text{if} \quad a \text{ is a factor of } b.$$

   (a) Prove that $\mathcal{T}$ is a *partial order* on the set of all finite length binary strings. [3]

   (b) Draw the *Hasse diagram*[4] for $\mathcal{T}$ on the set of all binary strings of length $0, 1, 2$ and 3.

[3]Hint: to *prove* that u is a factor of v, you should say what x and y are. For example u is a factor of u since $u = \lambda u \lambda$ where λ is the string of length 0 (the *empty string*).

[4]this was in Lecture 9.

3. (1.5 marks) Let f_n be the number of binary strings of length n which do not contain a factor 111 and have final digit 0.

(a) What is f_0, f_1, f_2, f_3, f_4?

(b) Find a recursive formula for f_n. [5]

[5] Hint: every string counted by f_n must end in 0, so think about all the possible endings: $00, 10, \ldots$. Hint 2: if a string ends in 10, then it ends in either 010 or 110. If it ends in 110, the digit before must be ....

4. (1 mark) (a) Find the multiplicative inverse mod 26 for $\alpha = 19$. Show all steps.[6]

(b) In a scene from the Netflix series *Discrete Game*, players have to decode the following message sent to them using an affine cipher with $\alpha = 19$ and $\beta = 8$. What is the message? Show all steps. [7]

`U G V F`

END OF LPC7

[6]Hint: Euclidean algorithm backwards

[7]Hint:

a	b	c	d	$\dots$	z
0	1	2	3	$\dots$	25