UTS

SEAT NUMBER:

STUDENT NUMBER:

SURNAME:
(FAMILY NAME)

OTHER NAMES:

This paper and all materials issued must be returned at the end of the examination.
They are not to be removed from the exam centre.

Examination Conditions:

It is your responsibility to fill out and complete your details in the space provided on all the examination material provided to you. Use the time before your examination to do so as you will not be allowed any extra time once the exam has ended.

You are **not** permitted to have on your desk or on your person any unauthorised material. This includes but not limited to:

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- Smart watches and bands
- Electronic devices
- Draft paper (unless provided)
- Textbooks (unless specified)
- Notes (unless specified)

You are **not** permitted to obtain assistance by improper means or ask for help from or give help to any other person.

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You are not permitted to leave your seat (including to use the toilet) during the final 15 mins.

During the examination **you must first seek permission** (by raising your hand) from a supervisor before:

- Leaving early
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Misconduct action will be taken against you if you breach university rules.

Declaration: I declare that I have read the advice above on examination conduct and listened to the examination supervisor's instructions for this exam. In addition, I am aware of the university's rules regarding misconduct during examinations. I am not in possession of, nor do I have access to, any unauthorised material during this examination. I agree to be bound by the university's rules, codes of conduct, and other policies relating to examinations.

Signature:

Date:

37181 Discrete Mathematics

Time Allowed: 120 minutes.

Reading time: 10 minutes.

Reading time is for reading only. You are not permitted to write, calculate or mark your paper in any way during reading time.

Restricted Open Book
No Calculators Permitted

Permitted materials for this exam:

3 pages A4 double-sided handwritten or printed notes.

Materials provided for this exam:

None.

Students please note:

Write all answers on this exam paper. At the end of the exam, check that you have clearly marked exactly ONE response for each multiple choice question in Part A.

Write your name and student ID number on the top of each page of your 3 pages of notes.

Do not open your exam paper until instructed.

Rough work space

Do **not** write your answers on this page.

PART A

Choose the correct answer and indicate your choice by circling. Each question is worth 1 mark.

1. Let A, B be finite sets, with $|A| = 5$ and $|B| = 6$. The number of one-to-one functions from A to B is

   A. 6^5

   B. 6!

   C. 5!

   D. 0

   E. none of the above.

2. Let A, B be finite sets, with $|A| = 4$ and $|B| = 2$. The number of onto functions from A to B is

A. 4^2

B. 2^4

C. $2^4 - 2$

D. $2^4 - 1$

E. none of the above.

3. Let p, q be statements. The statement

$$(p \to (q \vee \neg p)) \wedge q$$

is logically equivalent to

A. $\neg p$

B. q

C. $q \wedge \neg p$

D. F

E. none of the above.

4. The string

$$+ \div c + b \times 3\, a\, 6$$

is the preorder traversal encoding of the arithmetic expression

A. $\left(\dfrac{b+3a}{c}\right)+6$

B. $6+\left(\dfrac{3}{b+ac}\right)$

C. $\dfrac{3+a+b}{6c}$

D. $\left(\dfrac{c}{b+3a}\right)+6$

E. none of (A)–(D).

5. Define a function $B : \mathbb{N}^2 \to \mathbb{N}$ using the following recursive definition.

$$\begin{aligned} B(0,n) &= n^2 && n \geq 0, \\ B(m,0) &= B(m-1,1) && m > 0, \\ B(m,n) &= B(m-1, B(m,n-1)) && m,n > 0. \end{aligned}$$

Then $B(1,2)$ is equal to

A. 1

B. 3

C. 5

D. 9

E. none of (A)–(D).

6. Consider the following segment of pseudocode:

```
procedure (n int)

while n>20
      n := n - floor(n/10)
      print n
endwhile
```

On input $n = 100$, how many times is the command `print n` executed?

A. 16

B. 17

C. 18

D. 19

E. none of (A)–(D).

7. Consider the following segment of pseudocode:

```
procedure (n int)

sum := 0
for i := 1 to n do
      for j := 1 to i*n do
            sum := sum + 1
print sum
```

Define the time-complexity function $f(n)$ to be the number of times the statement

```
sum := sum + 1
```

is executed.

Which of the following statements is true?

A. $f(n) \in O(n)$

B. $f(n) \in O(n \log n)$

C. $f(n) \in O(n^2)$

D. $f(n) \in O(n^2 \log n)$

E. none of (A)–(D).

8. Alice constructs an RSA system and publishes $n = 77$ and $e = 43$. Bob then sends Alice the encoded message $c = 67$. What was Bob's intended message?

   A. $m = 64$

   B. $m = 67$

   C. $m = 69$

   D. $m = 27$

   E. none of (A)–(D).

9. Define a relation $\mathscr{T}$ on the set of all finite length binary strings by $b_1 \mathscr{T} b_2$ if

   - the sum of the digits in b_1 is strictly less than the sum of the digits in b_2, or
   - b_1 is obtained from b_2 by deleting one digit from b_2.

   Which of the following is true?

   A. $\mathscr{T}$ is reflexive

   B. $\mathscr{T}$ is antisymmetric

   C. $\mathscr{T}$ is transitive

   D. $0110 \mathscr{T} 11$

   E. none of (A)–(D).

10. Using repeated squaring, or otherwise, the remainder of 2^{85} on division by 31 is

A. 1

B. 2

C. 3

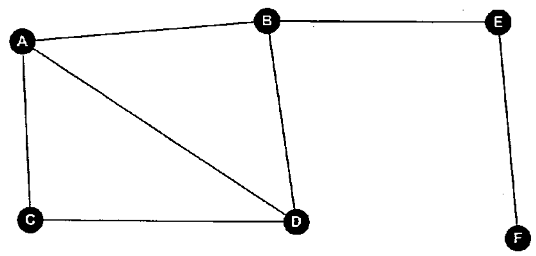
D. 4

E. none of (A)–(D).

PART B

Write full solutions in the space provided. Show all steps. Clearly strike out with a single line any mistakes. Each question is worth 3 marks.

11. Consider the following graph



Draw the following, or explain why it is not possible:

(a) three non-isomorphic spanning trees

(b) a Hamiltonian path

(c) an Euler path

12. Prove or disprove: for all $n \in \mathbb{N}$,
$$11^n - 6$$
is divisible by 5.

13. Fill in the missing boxes in the following proof from the options below. (You may use options more than once, and not all options fit).

Lemma 1. *Let $m \in \mathbb{Z}$. If m^3 is even then m is even.*

Proof. Suppose m is [ ].

Then there exists $p \in \mathbb{Z}$ such that [ ].

Thus $m^3 =$ [ ] $=$ [ ].

so m^3 is [ ].

The statement then follows by [ ]. □

(a) direct proof

(b) contrapositive

(c) false

(d) $m = 2p$

(e) $m = 2p + 1$

(f) $m = 3p + 1$

(g) odd

(h) even

(i) $(2p + 1)^3 = 8p^3 + 6p^2 + 6p + 1$

(j) $(2p)^3 = 8p^3$

(k) $(2p + 1)^2 = 4p^2 + 4p + 1$

(l) $(2p + 1)^3 = 8p^3 + 12p^2 + 6p + 1$

(m) $(2p)^2 = 4p^2$

(n) $2(4p^3)$

(o) $2(4p^3 + 3p^2 + 3p) + 1$

(p) $2(4p^3 + 6p + 3p) + 1$

14. Recall: $f \in O(g)$ if $\exists k \in \mathbb{N}_+, m \in \mathbb{R}_+$ such that $|f(n)| \leq m|g(n)|$ for all $n \in \mathbb{N}, n \geq k$.

Prove that $5n + 7 \in O(n^2)$, stating clearly your values for k and m.

15. Recall the function $B : \mathbb{N}^2 \to \mathbb{N}$ from Question 5, defined recursively by

$$\begin{aligned} B(0,n) &= n^2 && n \geq 0, \\ B(m,0) &= B(m-1,1) && m > 0, \\ B(m,n) &= B(m-1, B(m,n-1)) && m,n > 0. \end{aligned}$$

Prove or disprove:

$$\text{for all } x, y \in \mathbb{N}, \quad B(x,y) \neq 1 \text{ implies } x = 0.$$

Extra space for working

Extra space for working

Extra space for working

END OF EXAMINATION