DISCRETE MATH 37181 HOMEWORK SHEET 1

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Instructions. Try these sometime after your tutorial and before the next lecture and LPC (on your own, or with some study friends). Set aside some time each week to keep up with the homework. Partial solutions at the end of the PDF.

1. Draw truth tables for the following statements.

   (a) $((p \to q) \land (q \to r)) \to (p \to r)$

   (b) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

   (c) $s \leftrightarrow (p \to ((\neg p) \lor s))$

   (d) $(p \to (r \to s)) \leftrightarrow ((p \to r) \to s)$

   (e) Which of the statements in parts (a)—(d) are tautologies?

2. The statement

$$\neg\,(p \land (q \lor r))$$

   is logically equivalent to [1]

   A. $p \land (\neg q \land \neg r)$

   B. $\neg p \land (\neg q \lor \neg r)$

   C. $\neg p \lor (\neg q \land \neg r)$

   D. $\neg(p \lor \neg q) \land \neg r$

   E. none of the above.

3. The statement

$$\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}\,[(xy > 0) \lor (x = 0) \lor (y = 0)]$$

   is [2]

   A. true

   B. false

   C. only true for some values of x and y

   D. not a statement

   E. none of the above.

Date: Week 1.

[1]To answer this, draw a truth table the questions and for each multiple choice answer, or ... is there an easier way to do this?

[2]You think its true? Then *prove it* – show it is true for every x and y. You think its false? Give a *single example* of x, y for which it is not true. More on how to prove and disprove statements in maths next week. For now, can you read the symbols and understand the meaning?

4. Consider the following pseudocode:

```
procedure (n positive integer)
while n>0
      n := floor(n/2)
      print n
```

(a) On input $n = 50$ the output to the code is [3] [4]

A. 25 12 6 3 1 0

B. 0 0 0 0 0 0

C. 50 25 12 6 3 1

D. 25 13 6 3 1 0

E. none of the above.

(b) On input $n = 7$ what is the output? [5]

5. (a) Express the following sentence using the ideas learned this week: "All senior leaders at UTS are white".

(b) Find the negation of the statement and express in English.

(c) Is it true?

[3] Question 4 is a preview of future topics, so if you never saw "pseudocode" before (*eg.* you are not a comp sci major) don't worry, can you try to guess how the procedure operates? What does `floor` mean?

[4] if you *are* a comp sci major, pseudocode is not a real programming language, its half-way between some programming language(s) and English (and mathematics).

[5] Questions 2,3,4(a) were on the 37181 final exam 2017

Brief solutions:

1 (a)

p	q	r	$((p \to q) \wedge (q \to r)) \quad \to \quad (p \to r)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

This is a tautology. It is called the *Law of Syllogism* in some textbooks.

(b)

p	q	$(p \to q)$	$\leftrightarrow$	$(\neg q$	$\to$	$\neg p)$
1	1	1	1	0	1	0
1	0	0	1	1	0	0
0	1	1	1	0	1	1
0	0	1	1	1	1	1

This is a tautology. It is called *contrapositive*.

(c)

p	s	$s \quad \leftrightarrow \quad (p \to ((\neg p) \vee s))$
1	1	1
1	0	1
0	1	1
0	0	0

(d)

p	r	s	$(p \to (r \to s)) \quad \leftrightarrow \quad ((p \to r) \to s)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(e) (a) and (b) are tautologies.

2 C

3 B. For all real numbers x, y either one of them is zero or if not then $xy > 0$. This is not true, for example what if $x = -2andy = 1$?

4 (a) A (b) 3 1 0

5 (a) To write this in symbols, let the universe of discourse be the set of all people. Let $W(x)$ be the statement "x is white", and $S(x)$ be the statement "x is a senior leader at UTS". Then the statement is

$$\forall x \ \ [S(x) \to W(x)]$$

The negation is

$$\exists x \ \neg [S(x) \to W(x)]$$

which becomes

$$\exists x \ \neg [\neg S(x) \vee W(x)]$$

which becomes

$$\exists x \ \ [S(x) \wedge \neg W(x)]$$

There is (at least) one senior leader who is not white.

(b) Is it true? https://www.uts.edu.au/about/university/senior-executive