DISCRETE MATH 37181 HOMEWORK SHEET 3

Instructions. Try these sometime after your tutorial and before the next lecture. Set aside some time each week to keep up with the homework. Partial solutions at the end of the PDF.

1. Let $A, B \in \mathscr{U}$ be sets. The *complement* of the set
$$\emptyset \cup \overline{A \cap (\overline{B} \cup A)}$$
is equal to

   A. $\emptyset$
   B. $A \cup (\overline{A} \cap B)$
   C. $\overline{B} \cap A$
   D. A
   E. $A \setminus B$ [1]
   F. none of A–E.

2. Compute $\gcd(32, 124)$ using the Euclidean algorithm.

3. To show that the set theory statement
$$\overline{A} \cap (B \cup C) = (B \cap \overline{A}) \cup C$$
is incorrect, we could use the following example:

   A. $\mathscr{U} = \{1,2,3,4,5\}, A = \{1,2\}, B = \{3,4\}, C = \{5\}$

   B. $\mathscr{U} = \{1,2,3,4\}, A = \{1,2\}, B = \{2,3\}, C = \{4\}$

   C. $\mathscr{U} = \{1,2\}, A = \emptyset, B = \{1\}, C = \{2\}$

   D. $\mathscr{U} = \{1,2,3,4\}, A = \{2,4\}, B = \{1,3\}, C = \{4\}$

   E. $\mathscr{U} = \{1,2,3\}, A = \{1\}, B = \{2\}, C = \{3\}$

   F. none of (A)–(E).

Date: Week 4.

[1] $\setminus$ means “set minus”

4. Prove or disprove: [2]

   (a) If $A \subseteq B$ and $C \subseteq D$ then $A \cap C \subseteq B \cap D$. [3]

   (b) $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$. [4]

   (c) $\mathscr{P}(A \cup B) = \mathscr{P}(A) \cup \mathscr{P}(B)$.

   (d) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

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[2] remember a Venn diagram is not a proof. It may help you find a counterexample though if its false

[3] Hint: start with, let $x \in A \cap C$. Then ...

[4] if and only if means you have two proofs to do – one for each direction

Brief solutions:

1 **D**. A. Proof:

$$\emptyset \cup \overline{A \cap (\overline{B} \cup A)} = \overline{A \cap (\overline{B} \cup A)}$$

so its complement is

$$\begin{array}{lll} & A \cap (\overline{B} \cup A) & \\ = & (A \cap \overline{B}) \cup (A \cap A) & \text{distributive} \\ = & (A \cap \overline{B}) \cup A & \text{idempotent} \\ = & A & \text{adsorption} \end{array}$$

2.

$$\begin{aligned} 124 &= 3 \cdot 32 + 28 \\ 32 &= 1 \cdot 28 + 4 \\ 28 &= 7 \cdot 4 + 0 \end{aligned}$$

so $\gcd(32, 124) = 4$.

4. (a) *Proof.* Let $x \in A \cap C$. Then $x \in A$ and $x \in C$. Since $A \subseteq B$, we have $x \in B$, and since $C \subseteq D$, we have $x \in D$, so $x \in C \cap D$. □

(b) *Proof.* Suppose $A \cap \overline{B} \neq \emptyset$, so $\exists x \in A \cap \overline{B}$. Then $x \in A$ and $x \notin B$ which means $A \not\subseteq B$. This proves (contrapositive) the direction $A \subseteq B$ implies $A \cap \overline{B} = \emptyset$.

Now suppose $A \not\subseteq B$. This means there is some element of A that is not also in B, so this element (call is $a \in A$) lives in $\overline{B}$. Thus $a \in A \cap \overline{B}$ so this set is not empty. This proves (contrapositive) the direction $A \subseteq B$ implies $A \cap \overline{B} = \emptyset$. □

(c) This is false. Say $A = \{1\}$ and $B = \{2\}$, then $\{1, 2\} \in \mathscr{P}(A \cup B)$ but $\mathscr{P}(A) \cup \mathscr{P}(B) = \{\emptyset, \{1\}, \{2\}\}$ only.

(d) This is false. Let $\mathscr{U} = \{1, 2\}$ and $A = \{1\}$, $B = \{2\}$. Then $A \cap B = \emptyset$ so $\overline{A \cap B} = \{1, 2\}$ but $\overline{A} = \{2\}, \overline{B} = \{1\}$ so $\overline{A} \cap \overline{B} = \emptyset$.