37181 DISCRETE MATHEMATICS

LPC

3pm - 8pm

6 - 7:30

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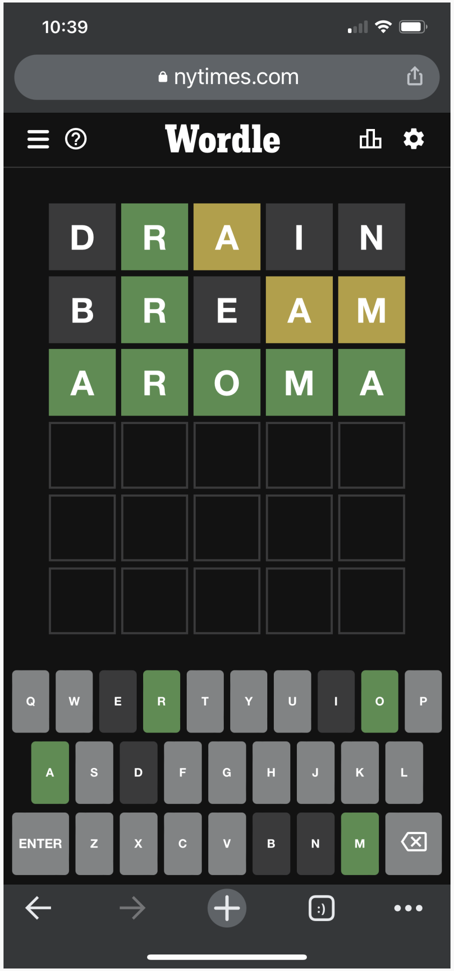
Lecture 1: propositional logic

PLAN

- introduction, subject outline
- truth tables
- logical equivalence
- tautology
- logic circuits

- please read the Subject Outline
- two lectures each week:
  - one live zoom lecture Monday 1pm-2:20pm;
  - second recorded lecture to watch anytime Tuesday.
- tutorial - online for overseas-based students; on-campus for Sydney-based students. Both running as "whiteboard workshops". Marks for "active participation" each week. Starts **this week**
- Learning Progress Checks (quizzes) (LPCs) online Tuesdays 3pm-8pm starting Week 2. Dates in Subject Outline

WHITEBOARD WORKSHOPS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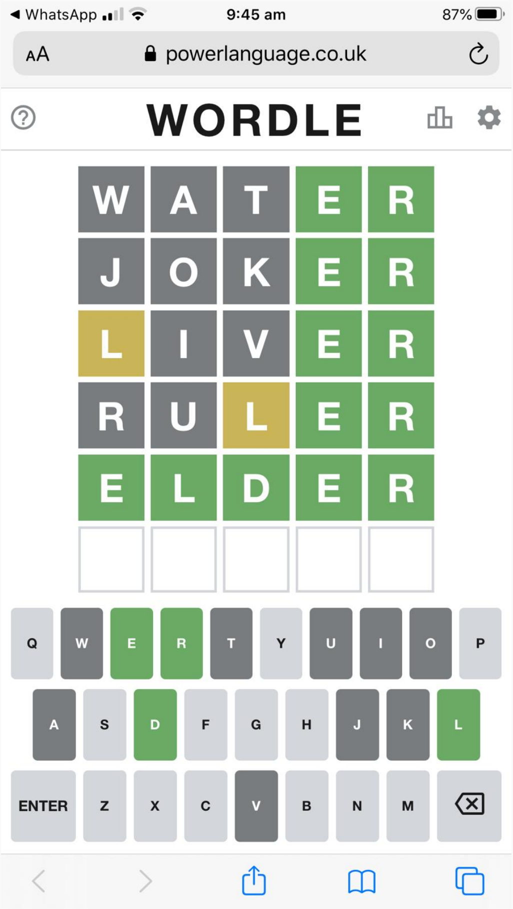
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Definition

A *statement* is a sentence that can (theoretically) be assigned a value of *true* or *false*.

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Definition

A *statement* is a sentence that can (theoretically) be assigned a value of *true* or *false*.

Eg:

1. Um, like, whatever NO
2. All positive integers are prime YES (False!)
3. In 2019, all live lectures were recorded at UTS YES False
4. In the year 4000BC, at the location of UTS Building 1, it was raining on the 5th of March at 10am YES
5. When will this lecture end? NO.

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6. 

YES

LOGICAL CONNECTIVES

We can build up more complicated statements out of simpler ones using *logical connectives* like *and* and *or*.

Eg:

1. Murray has neat handwriting and Murray has long hair.
2. Murray has neat handwriting or Murray has long hair.
3. Murray does not have neat handwriting and Murray has short hair.

not (Murray has long hair)

PRECISE MEANING: TRUTH TABLE

English (or any natural human language) can be imprecise, so instead of using our *"intuitition"* we **define** what *"and"* and *"or"* and *"not"* mean using ***truth tables***.

PRECISE MEANING: TRUTH TABLE

LaTeX

\wedge \vee \neg

English (or any natural human language) can be imprecise, so instead of using our *"intuitition"* we **define** what *"and"* and *"or"* and *"not"* mean using ***truth tables***.

and $\wedge$ or $\vee$ not $\neg$

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

p	$\neg p$
1	0
0	1

Define

1 = True
0 = False

English (or any natural human language) can be imprecise, so instead of using our *"intuitition"* we **define** what *"and"* and *"or"* and *"not"* mean using ***truth tables***.

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

p	$\neg p$
1	0
0	1

Teenager speech is more precise: Eg: "Maths is awesome — NOT"

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TRUTH TABLES FOR COMPOUND STATEMENTS

We can use truth tables to decide the truth values of more complicated statements, like $\neg p \vee q$:

$\cancel{\neg(p \vee q)}$

p	q	$(\neg p)$	$\vee$	q
1	1	0	1	
1	0	0	0	
0	1	1	1	
0	0	1	1	

1 2

Final true values.

We can use truth tables to decide the truth values of more complicated statements, like $\neg p \vee q$:

p	q	$\neg p$	$\vee$	q
1	1		1	
1	0		0	
0	1		1	
0	0		1	

p	q	$\neg(p$	$\vee$	$q)$
1	1	0	1	
1	0	0	1	
0	1	0	1	
0	0	1	0	
		2	1	

Note that this is different to saying $\neg(p \vee q)$, since the truth values are not the same

YOUR TURN

Complete the truth tables for these statements:

p	q	$\neg$	$(p \wedge q)$
1	1	0	1
1	0	1	0
0	1	1	0
0	0	1	0
		2	1

p	q	$\neg p$	$\vee$	$\neg q$
1	1	0	0	0
1	0	0	1	1
0	1	1	1	0
0	0	1	1	1
		1	2	1

When two (compound) statements have the same truth values we say they are *logically equivalent*.

$\neg(p \land q)$ IS LOG EQUIV. TO $\neg p \lor \neg q$

$\neg q \lor \neg(\neg p)$

When two (compound) statements have the same truth values we say they are *logically equivalent*.

Eg:

p	q	$p \lor \neg q$	$\neg(q \land \neg p)$
1	1	1 0	1 0 0
1	0	1 1	1 0 0
0	1	0 0	0 1 1
0	0	1 1	1 0 1
		2 1	3 2 1

p	$\neg(\neg p)$
1	1 0
0	0 1

p LOG EQUIV $\neg\neg p$

IMPLIES

In mathematics and logic we have a very specific meaning for "*p implies q*", or "*if p then q*", notation $p \to q$.

We define it using the following table:

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

In mathematics and logic we have a very specific meaning for "*p implies q*", or "*if p then q*", notation $p \to q$.

We define it using the following table:

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

p q

You may think that in English, "*if it is raining then I get wet*" means that the rain *caused* me to get wet. But in mathematics *if-then* has the meaning defined above: if "*I am wet*" is true and "*it is raining*" is false, the implication is still true. → Canvas - link.

Show that $p \to q$ is logically equivalent to $\neg p \vee q$.

p	q	$p \to q$	$\neg p$	$\vee q$
1	1	1	0	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1
			1	2

TAUTOLOGY

A statement that is true for all truth value assignments is called a *tautology*.

A statement that is true for all truth value assignments is called a *tautology*.

Eg:

p	q	$((p \to q) \land p) \to q$
1	1	1 1 1
1	0	0 0 1
0	1	1 0 1
0	0	1 0 1
		1 2 3

Tautology.

Eg:

p	q	r	$[(p \to q) \land (q \to r)] \to (p \to r)$
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

2^3

$(p \to q)$
$(q \to r)$

$(p \to r)$

YOUR TURN

Decide which of these are tautologies:

1. $((p \to q) \land \neg q) \to \neg p$

   modus tollens

2. $((p \to q) \land \neg\neg p) \to \neg q$

DRAW TABLE!!

p	q	$((p$	$\to$	$q)$	$\wedge$	$\neg q)$	$\to$	$\neg p$
1	1		1		0	0	1	0
1	0		0		0	1	1	0
0	1		1		0	0	1	1
0	0		1		1	1	1	1
			1		3	2	5	4

Final Column

Tautology – all 1's. ✓

p	q	$((p$	$\to$	$q)$	$\wedge$	$\neg$	$\neg p)$	$\to$	$\neg p$
1	1		1		1	1	0	0	0
1	0		0		0	1	0	1	1
0	1		1		0	0	1	1	0
0	0		1		0	0	1	1	1
			1		4	3	2	6	5

same as p

Final column

NOT A TAUTOLOGY.

\leftrightarrow.

Define $\leftrightarrow$ ("if and only if") by the truth table:

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

Definition

ANOTHER LOGICAL CONNECTIVE

Define $\leftrightarrow$ ("if and only if") by the truth table:

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

$(p \to q) \wedge (q \to p)$
1
0
0
1

Ex: show that $p \leftrightarrow q$ is logically equivalent to $(p \to q) \wedge (q \to p)$

YOUR TURN

Decide which of these are tautologies:

3. $(p \to q) \leftrightarrow (q \to p)$
4. $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	q	$(p \to q) \leftrightarrow \neg q \to \neg p$
		1
		1
		1
		1

YES TAUTOLOGY.

p	q	$(p \to q)$	$\leftrightarrow$	$(q \to p)$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	0
0	0	1	1	1

not a tautology

In Humanities/Law you might see tautological statements written in this form. Some rules have names.

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

$\therefore$

(Modus ponens)

$$((p \to q) \wedge p) \to q$$

In Humanities/Law you might see tautological statements written in this form. Some rules have names.

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

(Modus ponens)

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array}$$

$$\big((p \to q) \wedge \neg q \big) \to \neg p$$

(Modus tollens)

$M \to A$

If $\underbrace{\text{I am an axe murderer}}$, then $\underbrace{\text{I can use an axe}}$.
I cannot use an axe.
Therefore, I am not an axe murderer.

Which style of argument is this? (Write it in symbols).

$$\big((M \to A) \wedge \neg A \big) \to \neg M$$

$\boxed{\text{modus tollens}}$

OR, IF-THEN

Metropolitan Sydney

If you live or work in the City of Sydney, Waverley, Randwick, Canada Bay, Inner West, Bayside, and Woollahra local government areas, you cannot travel outside metropolitan Sydney.

If you live or work in metropolitan Sydney, other than those local government areas, you may travel anywhere in NSW.

Let F be a statement that is always false (has truth table 0, for example, $F = q \wedge \neg q$).

Then the statement

$$(\neg p \to F) \to p$$

is a tautology. Check it:

p	F	$\neg p$	$\neg p \to F$	$(\neg p \to F) \to p$
1	0	0	1	1
0	0	1	0	1
		1	2	3

Tautology.

$p \wedge \neg p$

CONTRADICTION: PREVIEW

Let F be a statement that is always false (has truth table 0, for example, $F = q \wedge \neg q$).

Then the statement

$$(\neg p \to F) \to p$$

is a tautology. Check it:

p	F	
1		
0		

It says, if not p implies something that is false, then it must be p (is true). This argument form is known as *proof by contradiction*. We will study this more when we start *proofs*

Last Section of Lecture 1

A *boolean function* is a function from $\{0,1\}^n$ to $\{0,1\}$. We will learn more about formal notation for functions in Week 3.

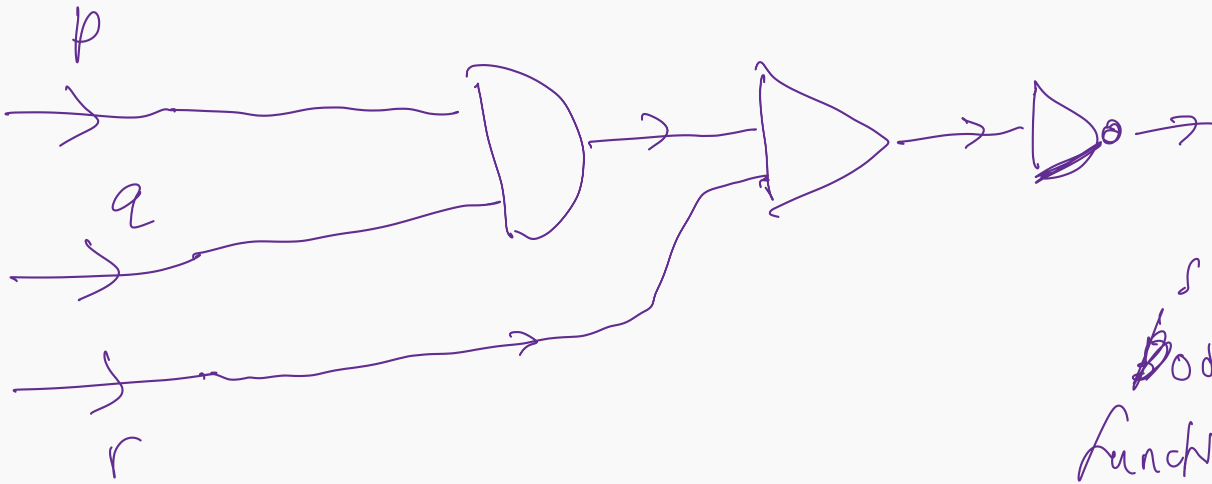
For example, the function f which sends (p,q) to $\neg(p \wedge q)$ is a boolean function.

0, 1 0, 1

$(0,0) \to 1$
$(0,1) \to 1$
$(1,0) \to 1$
$(1,1) \to 0$

We can represent boolean functions as *logic circuits* which are theoretical models of a computer:

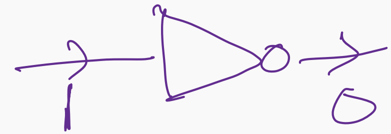
- input wires labeled by $p, q, r, \ldots$
- gates AND, OR, NOT ...
- single output wire, labeled by $f(p, q, r, \ldots)$. function



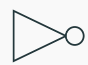
single Boolean function.

We can represent boolean functions as *logic circuits* which are theoretical models of a computer:

- input wires labeled by $p, q, r, \ldots$
- gates
- single output wire, labeled by $f(p, q, r, \ldots)$.



Here are three kinds of gates used:

and :  or:  not: 

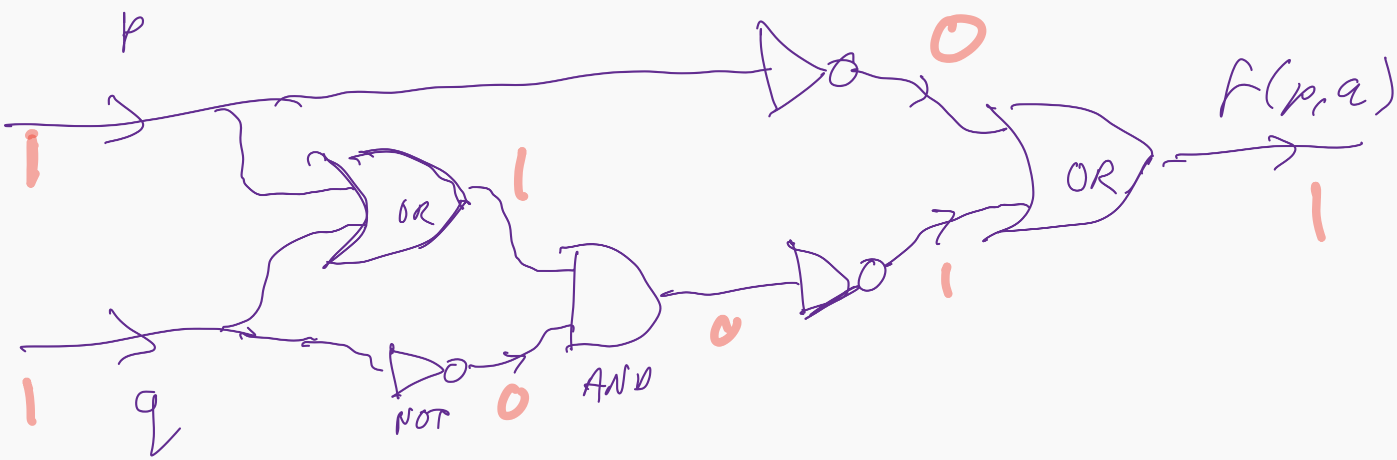
p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

p	$\neg p$
1	0
0	1

Draw a logic circuit for the boolean function

$$f(p,q) = \neg((p \vee q) \wedge \neg q) \vee \neg p$$



COMING UP

Next lecture (recorded, uploaded to Canvas by Tuesday 9am)

- quantified statements
- negation of quantified statements
- SAT and P=?NP

Please go through Lecture 2 recording with the blank slides, pause and do the problems as you watch.

Make sure you have seen all the content in lectures 1 and 2 before your tutorial class.

Note you are *not* expected to look at the Tutorial worksheet before your class, they are designed for you to work on together on-the-spot with your teammates during the class.

After your tutorial, do the Homework sheet by yourself (or with classmates if you can organise that).