37181 DISCRETE MATHEMATICS

Lecture 12: pigeonhole principle

PLAN

- pigeonhole principle

Recall:

Lemma

Let A, B be finite sets. If $f : A \to B$ is

- *1-1 then $|A| \leqslant |B|$.*
- *onto then $|B| \leqslant |A|$.*

Proof: ?

To prove the 1-1 rigorously, we need another **Axiom**

Axiom (Pigeonhole principle)

If m pigeons occupy n pigeonholes and $m > n$ then some pigeonhole has at least two pigeons in it.

Remember, this is an *axiom* like well ordering (and induction) – they seem obvious, but we can't derive them from other things.

Axiom (Pigeonhole principle)

If m pigeons occupy n pigeonholes and $m > n$ then some pigeonhole has at least two pigeons in it.

Out of 13 people, what is the chance two of them have the same Western Zodiac sign?

Axiom (Pigeonhole principle)

If m pigeons occupy n pigeonholes and $m > n$ then some pigeonhole has at least two pigeons in it.

Out of 13 people, what is the chance two of them have the same Western Zodiac sign?

Out of 367 people, what is the chance two of them have the same birthday?

Axiom (Pigeonhole principle)

If m pigeons occupy n pigeonholes and $m > n$ then some pigeonhole has at least two pigeons in it.

Out of 145 people, what is the chance two of them have the same Western Zodiac sign AND Chinese Zodiac?

EX

Let $A \subseteq \mathbb{N}_+$ with $|A| = 28$. Then A contains at least two elements with the same remainder $\mod 27$.

Proof: the pigeons are ...

the pigeonholes (boxes) are ...

and the rule for placing the pigeons into the boxes is:

EX

If 11 integers are chosen from $\{1, 2, 3, \ldots 100\}$ then at least two, say x and y, are such that

$$|x - y| \leqslant 9$$

Proof: the pigeons are ...

the pigeonholes are ...

and the rule for placing the pigeons into the boxes is:

If 11 integers are chosen from $\{1, 2, 3, \ldots 100\}$ then at least two, say x and y, are such that

$$|x - y| \leqslant 9$$

Proof: the pigeons are ...

the pigeonholes are ...

and the rule for placing the pigeons into the boxes is:

If 11 integers are chosen from $\{1, 2, 3, \ldots 100\}$ then at least two, say x and y, are such that

$$|\sqrt{x} - \sqrt{y}| < 1$$

Proof: the pigeons are ...

the pigeonholes are ...

and the rule for placing the pigeons into the boxes is:

Recall:

Lemma

Let A, B be finite sets. If $f : A \to B$ is

- *1-1 then $|A| \leqslant |B|$.*
- *onto then $|B| \leqslant |A|$.*

Proof:

Axiom (Pigeonhole principle)

If m pigeons occupy n pigeonholes and $m > kn$ then some pigeonhole has more than k pigeons in it.

Ex: Use PHP to show that if $S \subseteq \mathbb{N}_+$ and

1. $|S| \geqslant 3$ then S contains two distinct elements x, y such that $x + y$ is even.

Ex: Use PHP to show that if $S \subseteq \mathbb{N}_+$ and

2. $|S| > 6$ then S contains three distinct elements x, y, z such that $x + y + z$ is a multiple of 3.

NEXT

- how to count
- inclusion-exclusion
- permutations and combinations