37181 DISCRETE MATHEMATICS

Lecture 13: Counting

PLAN

- multiplication
- addition
- inclusion-exclusion
- permutations
- permutations with repetition
- combinations

In this lecture, we will learn how to count.

Ex 1: let $|A| = 4, |B| = 7$. How many one-to-one functions are there from A to B?

Multiplication rule: if there are a ways to do task A, and b ways to do task B, then there are ab ways to do task A then B.

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Ex 2: I have 3 pairs of shorts, 2 pairs of jeans, 5 t-shirts, 4 shirts, and 3 hats.

How many different outfits can I wear?

Ex 3: I have 3 pairs of shorts, 2 pairs of jeans, 5 t-shirts and 4 shirts, and my outfit is always either shorts and t-shirt, or jeans and a shirt.

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Addition rule: if there are a ways to do task A, and b ways to do task B, and you can only do A or B (not both), then there are $a+b$ ways to do A or B.

Ex 4: Suppose a survey of 100 people asks if they have a cat or dog as a pet. The results are as follows:

55 answered yes for cat, 58 answered yes for dog and 20 people said yes for both cat and dog.

How many people have a cat or a dog?

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formula: $|A \cup B| = |A| + |B| - |A \cap B|$

Inclusion-exclusion principle: if $A, B, C, \ldots$ are sets (whose elements might be some events like some tasks which may or may not be able to be performed simultaneously, or people having a cat or a dog or both ...)

then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Ex 5: How many numbers between 1 and 100 (inclusive) are not divisible by either 2 or 3?

Ex 6: How many numbers between 1 and 100 (inclusive) are not divisible by either $2, 3$ or 7?

Ex 7: How many ways can you arrange the letters in the word *meat*?

Ex 8: How many ways can you arrange the letters in the word *meet*?

Ex 9: How many ways can you arrange the letters in the word *coonabarabran*?

Ex 10: How many ways can you choose k elements from a set of size n?

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Step 1: How many *ordered lists* of k elements from n? (overcount first)

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Step 2: deal with the overcount

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Formula for $\binom{n}{k}$:

Notation: a *binary string* of length $n \in \mathbb{N}_+$ is an expression of the form $d_1 d_2 \dots d_n$ where $d_i \in \{0,1\}$ for $1 \leqslant i \leqslant n$. For example, 11011 is a binary string of length 5. The set of all binary strings of length n is denoted $\{0,1\}^n$.

A finite length string is also called a *word*.

Whenever people talk about strings, it is helpful to include strings of length 0. For this subject, let us denote the (unique) string of length 0 by the symbol λ.

How many binary strings are there of length n?

A *square free word* is a word (a sequence of symbols) that does not contain any squares. A square is a word of the form XX, where X is not empty. `https://en.wikipedia.org/wiki/Square-free_word`

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How many binary strings of length 5 are square free?

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How many binary strings of length 5 are square free?

How many *ternary* strings ($d_i \in \{0, 1, 2\}$) of length 5 are square free?

NEXT TIME

- binomial theorem
- combinatorial proofs
- some famous counting sequences
- Catalan numbers

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(LPC6 tomorrow, then tutorials on counting, then StuVac next week)