37181 DISCRETE MATHEMATICS

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Lecture 2: more on logic

PLAN

- quantified statements
- negation of quantified statements
- SAT and P=?NP

VARIABLES

Statements can contain *variables*.

Eg:

- $P(x)$: “the number x is greater than or equal to 3”
- $Q(x)$: “x lives in Queensland”

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The *universe of discourse* is the set of objects over which the statement could be defined.

- for $P(x)$ the universe of discourse could be $\mathbb{R}$ or $\mathbb{Z}$ or $\mathbb{N}$ (we would need to be told)
- for $Q(x)$ the universe might be all people, or all students at QUT.

QUANTIFIERS

We have the symbols $\forall$ =“for all” and $\exists$ =“there exists”.

Eg: Let the universe of discourse be $\mathbb{Z}$.

- $\forall x, x^2 > x$ reads as “for all integers x, x^2 is greater than x”

  Is this true?

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  Is this true?

- $\exists x, x^2 \leqslant x$ reads as “there exists (there is) some integer x whose square is smaller than or equal to itself”

  Is this true?

Rather than say " Let the universe of discourse be" we often hide this (make it *implicit*), or write

- $\forall x \in \mathbb{Z}, x^2 > x$

PRACTICE

Let $B(x)$ be the statement "x lives in Bondi".

Let $C(x)$ be the statement "x lives in Cabramatta".

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Write these in symbols:

- "All UTS students live in Bondi"
- "All UTS students either live in Cabramatta or do not live in Bondi"

NEGATION OF QUANTIFIED STATEMENTS

Can you work out, *intuitively* what the meaning of

- $\neg(\forall x, B(x))$

is?

NEGATION OF QUANTIFIED STATEMENTS

Formally, to negate a quantified statement you switch $\forall$ and $\exists$ at the front, then negate the proposition.

$$\neg\left(\forall x P(x)\right) = \exists x \neg P(x)$$

$$\neg\left(\exists x P(x)\right) = \forall x \neg P(x)$$

PRACTICE

Check the course notes, and Week 1 homework sheet, to practice turning English sentences into symbolic statements, and backwards, and negating them.

SAT

3-SAT is the following problem: on input an expression of the form

$$(x_1 \vee y_1 \vee z_1) \wedge (x_2 \vee y_2 \vee z_2) \wedge \ldots (x_n \vee y_n \vee z_n)$$

where x_i, y_i, z_i are propositions p or $\neg p$, answer yes or no: there is some assignment of truth values to the variables which makes the whole statement true.

For example

$$(p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r)$$

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$$(p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r)$$

If I tell you a particular truth assignment, like $p = 0, q = 1, r = 0$ etc, you can easily compute (in a number of steps polynomial in n) the truth value of the statement.

SAT

If an instance of a solution can be *verified* in polynomial time (number of steps), we say a problem is in NP.

If a solution can be found in polynomial time (number of steps), we say the problem is in *P*.

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No-one knows if you can always find a truth assignment, or show there is none, making a general 3-SAT expression true, in polynomially many steps. If you can, you will get $1M

3-SAT is an important problem, even though it may seem abstract and useless, because Cook and Levin showed that every other candidate to solve the P=NP problem is related to this one. (Keyword: NP-complete)

COMING UP

In your tutorial class Wednesday/Thursday/Friday, lots of practice to fully understand the content presented in lectures 1-2.

After the workshop, do the homework sheet to consolidate your learning, and be ready for the uncoming quizzes.

Next lecture:

- proof methods: direct, contrapositive, contradiction