DISCRETE MATH 37181 TUTORIAL WORKSHEET 1

Instructions. Complete these problems in groups of 3-4 on the whiteboard (real or virtual). If online your tutor will email you the relevant software link. Partial solutions at the end of the PDF. Your tutor will come around to each group throughout the tutorial and check your progress/ask questions as you work together on the worksheet.

1. Each group member: write your name and degree/major on the top of the whiteboard, and say hi to your teammates. Eg: Murray — CompSci (AI major/undecided).

2. Draw truth tables for the following statements. (Take it in turns to draw the table for each part.) If you are unsure of the format taught in the first lecture, take a quick look at the solutions to see. Don't erase until you've answered part (g).

   (a) $((p \to q) \wedge p) \to \neg q$

   (b) $((p \to q) \wedge \neg q) \to \neg p$

   (c) $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

   (d) $((p \to q) \wedge (q \to r)) \to (p \to r)$

   (e) $(p \to q) \leftrightarrow (\neg q \to p)$

   (f) $s \leftrightarrow (p \to ((\neg p) \vee s))$

   (g) Which of (a)–(f) are *tautologies*?

3. For each of the following quantified statements, write a quantified statement which is logically equivalent to the *negation* of the given statement and only uses the symbol $\neg$ *after* the quantifiers.

   For example, $\neg(\forall x \forall y (x > y))$ is rewritten as $\exists x \exists y (x \leqslant y)$.

   For (b),(e),(f), translate the statement into logical symbols, find the negation, then put your answer back in English.

   (a) $\forall x \exists y\, (x^2 > y \vee x < 2y)$

   (b) Every person has someone who loves them [1]

   (c) $\forall x \forall y\, (x < y \to \exists z (x < y < z))$

   (d) $\exists x \forall y \exists z\, (z > y \to z < x^2)$

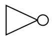
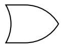
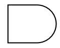
   (e) Every person loves at least two people

   (f) Every person loves at least two people who do not love each other.

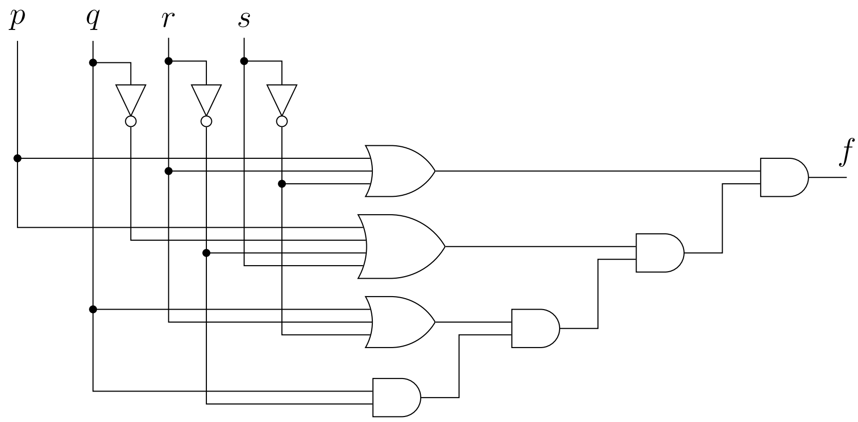
Date: Week 1 workshop (Wednesday 23, Thursday 24, Friday 25 February 2022).

[1] define $L(x, y)$ to mean "x loves y". Note $L(x, y)$ does not always have the same truth value as $L(y, x)$ right?

4. Recall that a *logic circuit* has input wires labeled $p, q, r, s, \ldots$, *logic gates*:

not:  or:  and : 

and an output wire labeled f. An example is given here:



(a) On input $p = 0, q = 1, r = 0, s = 1$, what is the output at f?

(b) Write down the logical expression corresponding to the circuit diagram above.

(c) Draw a logic circuit representing this formula: [2]

$$(\neg(p \wedge q) \rightarrow \neg p) \vee q$$

5. Is the following formula *satisfiable*?

$$(p \vee q \vee \neg r) \wedge (r \vee w \vee \neg q) \wedge (\neg w \vee \neg p \vee \neg r)$$

6. Use propositional logic (*i.e.* convert to symbols) to determine which of the following are valid arguments:

(a) If I do not work hard, I will sleep. If I am worried, I will not sleep. Therefore, if I am worried, I will work hard.

(b) If it is raining, I am wet. I am dry. Therefore it is not raining. [3]

(c) If it is raining, I am wet. It is not raining. Therefore I am dry.

7. Express the following sentence using the ideas learned this week:

"Nothing in this world is not non-binary".

[4]

Then find the negation and convert the negation back into English.

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[2] hint: $p \rightarrow q$ is the same as $\neg p \vee q$

[3] "I am dry" is the same as "I am not wet"

[4] From https://wackieju.com/About

Brief solutions:

2. (a)

p	q	$((p \to q)$	$\wedge$	$p)$	$\to$	$\neg q$
1	1	1	1		0	0
1	0	0	0		1	1
0	1	1	0		1	0
0	0	1	0		1	1

(b)

p	q	$((p \to q) \wedge \neg q)$	$\to$	$\neg p$
1	1		1	
1	0		1	
0	1		1	
0	0		1	

(c)

p	q	$\neg(p \wedge q)$	$\leftrightarrow$	$(\neg p \vee \neg q)$
1	1		1	
1	0		1	
0	1		1	
0	0		1	

(d)

p	q	r	$((p \to q) \wedge (q \to r))$	$\to$	$(p \to r)$
1	1	1		1	
1	1	0		1	
1	0	1		1	
1	0	0		1	
0	1	1		1	
0	1	0		1	
0	0	1		1	
0	0	0		1	

(g) b,c,d only.

3. (a)

$$\neg \forall x \exists y \left(x^2 > y \vee x < 2y\right)$$

$$\exists x \forall y \neg \left(x^2 > y \vee x < 2y\right)$$

$$\exists x \forall y \left(\neg(x^2 > y) \wedge \neg(x < 2y)\right)$$

$$\exists x \forall y \left((x^2 \leqslant y) \wedge (x \geqslant 2y)\right)$$

(b) Let $L(x, y)$ be the statement "x loves y", and the universe of discourse be the set of all people.

$$\forall x \exists y L(y, x)$$

Negation:

$$\exists x \forall y \neg L(y, x)$$

There is a person that nobody loves. (Note: including themself).

(c) This is equivalent to

$$\forall x \forall y\, (\neg(x < y) \vee \exists z(x < y < z))$$

Negation:

$$\exists x \exists y \neg\, (\neg(x < y) \vee \exists z(x < y < z))$$
$$\exists x \exists y\, ((x < y) \wedge \neg \exists z(x < y < z))$$
$$\exists x \exists y\, ((x < y) \wedge \forall z \neg(x < y < z))$$
$$\exists x \exists y\, ((x < y) \wedge \forall z \neg((x < y) \wedge (y < z)))$$
$$\exists x \exists y\, ((x < y) \wedge \forall z((x \geqslant y) \vee (y \geqslant z)))$$

There are two numbers x, y so that $x < y$ and either $x \geqslant y$ (which cannot be true so we just have) or y is bigger than or equal to all numbers z.
There are two numbers x, y so that $x < y$ and y is bigger than or equal to all numbers z. If we assume the universe of discourse is $\mathbb{R}$ or $\mathbb{Z}$ then this is false (∞ is not included in $\mathbb{R}, \mathbb{Z}$). and the original statement was true: for all pairs of numbers (x, y), if $x < y$ then you can find some $z = y + 1$ say so that $x < y < z$ is true.

(d)

$$\forall x \exists y \forall z\, \big(z > y) \wedge (z \geqslant x^2)\big)$$

(e)

$$\forall x \exists y \exists z\, [L(x, y) \wedge L(x, z) \wedge (y \neq z)]$$

(This includes the possibility that $x = z$ or $x = y$, that is, one of the people x loves is themself.)
Negation:

$$\neg \forall x \exists y \exists z\, [L(x, y) \wedge L(x, z) \wedge (y \neq z)]$$
$$\exists x \forall y \forall z \neg\, [L(x, y) \wedge L(x, z) \wedge (y \neq z)]$$
$$\exists x \forall y \forall z \neg L(x, y) \vee \neg L(x, z) \vee (y = z)$$

There is a person that either does not love anyone, or loves only one person. That is, $\forall y \forall z$, as you consider all pairs of people (y, z), if x loves both of them, then $y = z$.

(f)

$$\forall x \exists y \exists z\, [L(x, y) \wedge L(x, z) \wedge (y \neq z) \wedge \neg L(y, z) \wedge \neg L(z, y)]$$

Negation:

$$\exists x \forall y \forall z\, [\neg L(x, y) \vee \neg L(x, z) \vee (y = z) \vee L(y, z) \vee L(z, y)]$$

There is a person that either loves at most one person, or if they more than one person, then in each pair of people that x loves, one of the people loves the other one.

4. (a) 0. From the top *or* gate, all inputs are 0, and this feeds into the top *and* gate.

(b)

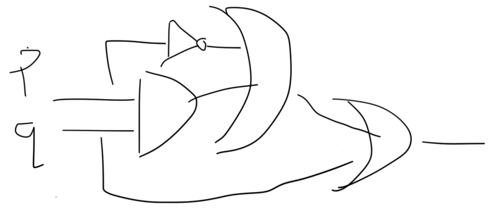
$$(p \vee r \vee \neg s) \wedge [(p \vee \neg q \vee \neg r \vee s) \wedge [(q \vee r \vee \neg s) \wedge (q \wedge \neg r))]$$

(c)

$$(\neg(p \wedge q) \to \neg p) \vee q$$

is equivalent to

$$((p \wedge q) \vee \neg p) \vee q$$



5. Try some values: $p = 1, r = 1, w = 0$ makes the entire formula true (for any q).

6. (a) Let $h =$ work hard, $s =$ sleep, $w =$ worried.

$$((\neg h \to s) \wedge (w \to \neg s)) \to (w \to h)$$

h	s	w	$((\neg h \to s)$	$\wedge$	$(w \to \neg s))$	$\to$	$(w \to h)$
1	1	1				1	1
1	1	0				1	1
1	0	1				1	1
1	0	0				1	1
0	1	1				1	0
0	1	0				1	1
0	0	1				1	0
0	0	0				1	1

A quick way to show this is to argue as follows: how could this statement be false? Only if $w \to h$ is false (and $(\neg h \to s) \wedge (w \to \neg s)$ is true).
So only need to check two rows of the truth table.

Alternatively, $\neg h \to s$ is logically equivalent to $\neg s \to h$ (contrapositive), so $((w \to \neg s) \wedge (\neg s \to h)) \to (w \to h)$ is syllogism.

(b) Let $r =$ it is raining, $w =$ I am wet.

$$((r \to w) \wedge \neg w) \to (\neg r)$$

Valid – this is modus tollens.

7. To write this in symbols, let the universe of discourse be the set of all things in this world. Let $B(x)$ be the statement "x is binary".
Let me try: for all things in this world, there is not one thing that is not non-binary.
For all things in this world, there is not one thing that is binary. (not not)
For all things x in this world, x is not binary. (there is not one thing, means all things are not)
$\forall x[\neg B(x)]$

Negation: $\exists x[B(x)]$, there is something in this world that is binary.

Note that in a humanities-type context binary and non-binary refers to gender, not 0 and 1.