DISCRETE MATH 37181 TUTORIAL WORKSHEET 12

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Instructions. Complete these problems in groups of 3-4 at the whiteboard. Partial solutions at the end of the PDF. Last one!

1. (a) Draw a rooted tree which encodes this arithmetic expression:

$$a(b + cd) + 4$$

   (b) Give the prefix, postfix and infix traversals of the tree in part (a).

2. The string

$$+ \ \div \ c \ + \ b \ \times \ 3 \ a \ 6$$

is the preorder traversal encoding of the arithmetic expression

A. $\left(\frac{b+3a}{c}\right) + 6$

B. $6 + \left(\frac{3}{b+ac}\right)$

C. $\frac{3+a+b}{6c}$

D. $\left(\frac{c}{b+3a}\right) + 6$

E. $\frac{c}{3b+a} + 6$

F. none of (A)–(E).

3. (a) Give the tree corresponding to the arithmetic expression $\frac{2}{f+gh} \times \left(b + \frac{3}{a}\right)$

   (b) Give the pre-order traversal of the tree in part (a)

4. Show that $a(b + c)$ when written as infix traversal (without using brackets) is ambiguous.

5. I have 8 shirts, 4 pairs of pants and 2 pairs of shoes. Use a rooted tree to count how many different outfits are possible.

6. A *binary rooted tree* is a rooted tree where every node has either 0 or 2 children.

   (a) Draw all binary rooted trees with $n + 1$ leaves for $n = 0, 1, 2, 3, 4$.

   (b) Prove that a binary rooted tree with $n + 1$ leaves has n nodes of outdegree 2. [1]

Date: Week 12 workshop (Wednesday 18, Thursday 19, Friday 20 May).

[1] We consider rooted trees to be directed (downwards) so we talk of indegree and outdegree. Obviously this proof is induction. Check cases $n = 0$ and $n = 1$ (need two initial cases since I was to assume $k \geqslant 1$ in my induction step). Assume true for k then find a piece $\wedge$ and remove it.

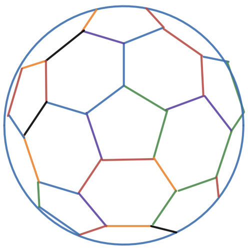
(c) If c_n is the number of binary rooted trees with $n+1$ leaves, show that

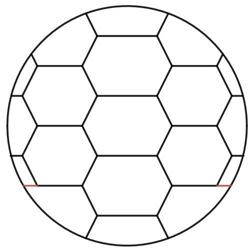
$$c_{n+1} = \sum_{i=0}^{n} c_i c_{n-i}$$

[2]

(d) What is this sequence called?

7. A soccer ball has a pattern on it made up of pentagons (5-sided shape) and hexagons (6-sided shape). Three faces (pentagons or hexagons) meet at each vertex.



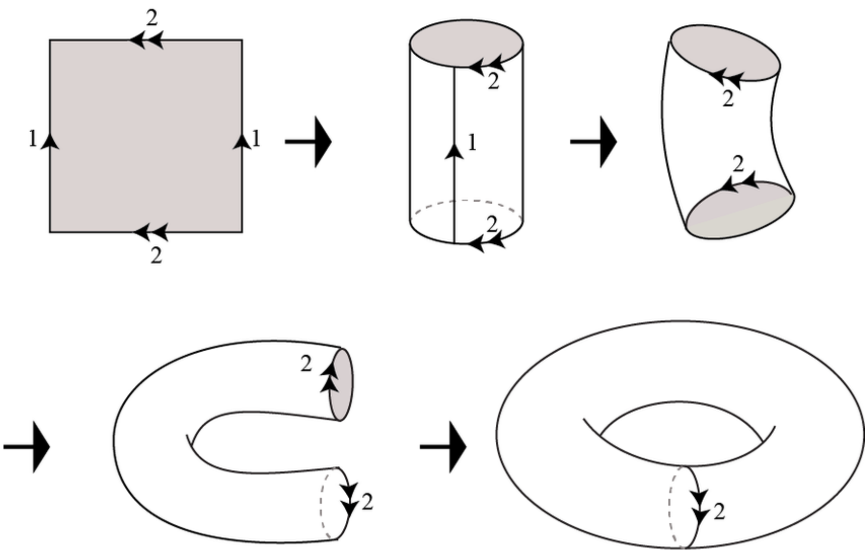


How many pentagons can a soccer ball have? Show all working. [3]

8. The formula $V - E + F$ can be used for graphs drawn on different surfaces (not just spheres). When we generalise it, we must make sure that F is counting actual flat regions (polygons), not different shapes (like cylinders, etc).

Draw (without any edges crossing) a graph on the surface of a donut (torus) so that each region in between the edges and vertices drawn is a flat polygonal shape, then count $V - E + F$. Does it equal 2 or something different. Compare with other groups in the class.

Hint: to draw on the surface of a donut, imagine the surface is a square TV screen, and if you exit the top (like an old-school video game) you enter at the bottom, and if you exit the left side you return on the right side, and vice versa.



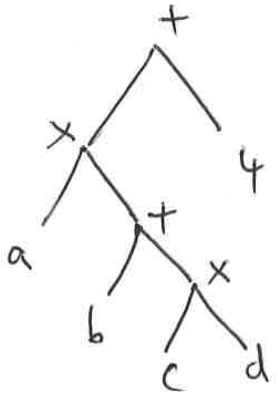
END OF TUTORIAL WORKSHEET 12

[2]Hint: consider the root in a tree with $n+1$ leaves. Its left child is a tree with __ leaves and its right child is a tree with __ leaves. Think of all the possible numbers you can put for __ to get distinct trees.

[3]Hint: sum of degrees of a graph, Euler's formula, etc

Brief solutions:

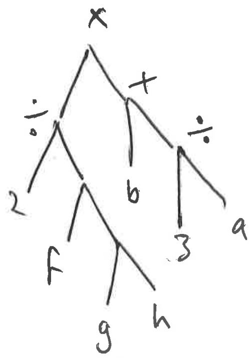
1. (a)



(b) pre: parent, left, right $+ \times a + b \times cd4$
in: left, parent, right $a \times b + c \times d + 4$
post: left, right, parent $abcd \times + \times 4+$

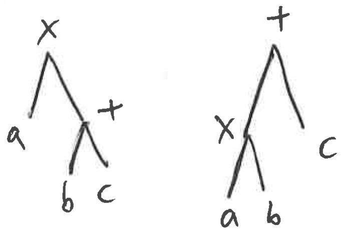
2. D.

3. (a)



(b) pre: parent, left, right $\times \div 2 + f \times gh + b \div 3a$

4. $a \times b + c$ is the inorder traversal for both these trees:



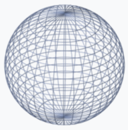
But the left tree encodes $a(b+c) = ab + ac$ while the right tree encodes $(a+b)c = ac + bc$ which is not equal (eg $a = 1, b = 2, c = 3$ gives 5 versus 9).

5. $8.4.2 = 64$. That is, assuming I like to have matching shoes. I also have the option of not wearing a shirt, or pants (lol).

6. (d) the Catalan numbers.

7. Soccer ball: always 12. This is true for nanocarbon molecules (buckyball, etc) as well.

8. The answer for everyone should have been $V - E + F = 0$ for a graph drawn (without any edges crossing) on the surface of a donut. If the donut has two or more holes, you get $-2, -4$ etc. See https://en.wikipedia.org/wiki/Euler_characteristic

Sphere		**2**
Torus *(Product of two circles)*		**0**
Double torus		**−2**
Triple torus		**−4**