Autumn 2015 – **Main Exam**

**STUDENT NUMBER:** | | | | | | | | |

**SURNAME:**
(FAMILY NAME) .....

**OTHER NAMES:** .....

**This paper and all materials issued must be returned at the end of the examination. They are not to be removed from the exam centre.**

**Examination Conditions:**

It is your responsibility to fill out and complete your details in the space provided on all the examination material provided to you. Use the time before your examination to do so as you will not be allowed any extra time once the exam has ended.

You are **not** permitted to have on your desk or on your person any **unauthorised material**. This includes but not limited to:

- Mobile phones
- Smart watches
- Electronic devices
- Draft paper (unless provided)
- Textbooks (unless specified)
- Notes (unless specified)

You are **not** permitted to obtain assistance by improper means or ask for help from or give help to any other person.

You are **not** permitted to leave your seat (including to use the toilet):

- Until 90 mins has elapsed
- During the final 15 mins

During the examination **you must first seek permission** (by raising your hand) from a supervisor before:

- Leaving early (after 90 mins)
- Using the toilet
- Accessing your bag

Disciplinary action will be taken against you if you infringe university rules.

# 33130 Mathematical Modelling 1

Tuesday 16th June. 2:00pm – 4:10pm

## Time Allowed: 2 hours and 10 mins

Includes 10 minutes of reading time.

Reading time is for reading only. You are not permitted to write, calculate or mark your paper in any way during reading time.

## This is a Restricted Open Book exam

Unauthorised materials as specified in the examination conditions are not allowed.

**Permitted materials for this exam:**

- Calculators (non-programmable only)
- One sheet of A4 paper, with notes on both sides

**Materials provided for this exam:**

- This examination paper
- Four (4) answer booklets (8 pages)

**Students please note:**

- Each question is to be attempted in a separate booklet
- You MUST indicate the question number on the cover of each booklet
- You MUST hand in your sheet of A4 paper

Examiner: Dr M. Coupland

Assessor: Associate Professor C. Poulton

**Do not turn over or open your exam paper until instructed.**

## PLEASE BEGIN A NEW BOOKLET
## WRITE THE QUESTION NUMBER ON THE COVER

**Question 1** ( 3 + 3 + 3 + 8 = 17 marks)

(a) By making a suitable substitution (or otherwise), evaluate the integral $\int 3x\sqrt{x-2}\,dx$ .

(b) A student is preparing to use Newton's Method with a starting point of $x = x_1$ to find the root of the function $f(x) = e^x + x - 7$ . Briefly describe how they might find a suitable value for $x_1$.

(c) (i) Sketch the graph of the function : $f(x) = 1 + 4\cosh^{-1}(5x)$,

(ii) State the domain of the function : $f(x) = 1 + 4\cosh^{-1}(5x)$.

(d) (i) Use the expansion $\cos(A + B) = \cos A\cos B - \sin A\sin B$ to show that $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$, and hence evaluate $\int \sin^2 x dx$.

(ii) A set of functions $f_1(x), f_2(x)$, ... is said to be *orthonormal* on an interval $a \le x \le b$ if

$$\int_a^b f_m(x) f_n(x) dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$$

Show that the set of functions $f_1(x) = \frac{1}{\sqrt{2\pi}}$, $f_2(x) = \frac{1}{\sqrt{\pi}}\sin x$, is orthonormal on the interval $0 \le x \le 2\pi$.

Remember to start the next question in a separate booklet, and write the question number on the cover.

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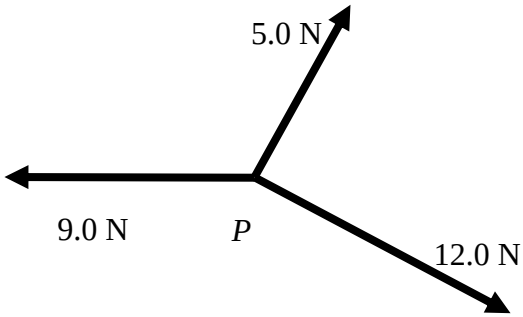
## PLEASE BEGIN A NEW BOOKLET
## WRITE THE QUESTION NUMBER ON THE COVER

**Question 2** ( 5 + 6 + 6 = 17 marks)

(a) In this diagram, the 9.0 N force acts horizontally to the left, and the 12.0 N force makes an angle of 30° with the horizontal. The 5.0 N force is perpendicular to the 12.0 N force. Find the magnitude and direction of the total force acting on point *P*.

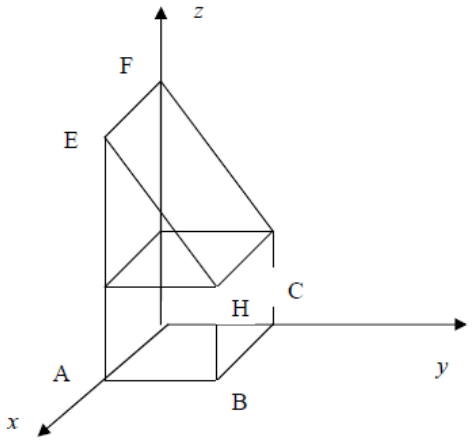



NOT TO SCALE

(b)

The structure consists of a cube of side 10 metres and a triangular prism. The point *A* is (10, 0, 0), measuring in metres. The point *F* is located at (0, 0, 22) and point *E* is vertically above point *A*. The distance *EA* is the same as the distance *FO*.

(i) Find the normal to the plane *EHCF*.
(ii) Find the Cartesian form of the equation of the line which passes through points *F* and *H*.
(iii) If the line *FH* is extended, where does it intersect the *xy* plane?




(c) Solve the differential equation

$$xy' - 4y = x^6 e^x$$

You may need the integrating factor method:

Write the DE in the form $y' + p(x)y = q(x)$, that is, $y' + py = q$.
Find the integrating factor, $I = e^{\int p\,dx}$. Then find $y$ from $Iy = \int Iq\,dx$,
or by multiplying both sides of $y' + py = q$ by the integrating factor.

Remember to start the next question in a separate booklet, and write the question number on the cover.

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## PLEASE BEGIN A NEW BOOKLET
## WRITE THE QUESTION NUMBER ON THE COVER

**Question 3** ( 6 + 2 + 6 + 3 = 17 marks)

(a) The motion of a vibrating system is given by the differential equation $y''+25y=3\sin(2t)$ .

(i) Find the general solution for this differential equation.

(ii) Find the particular solution if the initial conditions are $y(0)=0, y'(0)=1$.

(b) Find the real and imaginary parts of the admittance, $Y=\frac{1}{Z}$, in a particular circuit given that $Z=65-j20$ ohms. Use $j^2=-1$.

(c) A cylindrical tank of water is being drained through a hole in the base of the tank. If the height (in metres) of the water in the tank at time $t$ (in hours) is given by $h(t)$ then Torricelli's Law predicts that $\frac{dh}{dt}=-k\sqrt{h}$.

(i) Solve this separable differential equation.

(ii) For the case when $k$ = 0.3 (in appropriate units) and the initial depth of the water in the tank is 1.6 metres, find the time taken for all the water to drain from the tank.

(d) Describe in words, mathematical expressions, and also diagrams if necessary, the output of this *Mathematica* command:

```
Clear[f];
Clear[t];
f[t_]:= Sin[4*π*t]
Plot[2-f[t],{t,0,1}]
f''[t]
```

Remember to start the next question in a separate booklet, and write the question number on the cover.

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## PLEASE BEGIN A NEW BOOKLET
## WRITE THE QUESTION NUMBER ON THE COVER

**Question 4** ( 3 + 4 + 4 + 6 = 17 marks)

(a) What are the partial sums of a series, and how are they used to determine whether or not the series converges?

(b) Find the open interval of convergence for the series below. As you are finding the open interval, you may ignore the problem of convergence at the end points.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$$

(c) For the series $\sum_{n=1}^{\infty} \frac{n+3}{2^n(n+1)!}$, use the ratio test to test the series for convergence.

(d) The function $g(x) = \sin x$ can be expressed as $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

(i) Express this result using summation notation (Sigma notation).

(ii) Find the interval of convergence for this series.

(iii) Use the first three non-zero terms of this series to estimate sin(0.1). Comment on the accuracy of your result, using the alternating series estimation theorem if it applies here.