Mathematics 1 and 33130
Assessment Task 2 – Take Home Examination
Spring 2022

Instructions

This examination is made available online at **9.30am** on **Tuesday 8th November 2022**.

Your completed answer file is due at **12.30pm** on **Tuesday 8th November 2022**, and must be submitted online via the https://canvas.uts.edu.au/courses/24606/assignments/115877 Assignments folder on Canvas.

There are **4 questions**. Your answer to each question attempted should commence on a new page and be appropriately numbered. Answers must be hand written.

The examination is worth **50%** of the marks available in this subject. Each question contributes equally to the total examination mark.

This examination is an open book examination.

This examination is expected to take approximately **2 hours** of working time. You are advised to allocate your time accordingly. Your answer file may be submitted at any time before the due time. Please allow time to complete the submission process.

Please submit your file in PDF format. Please name your file as follows:

EXAM_subject number_student number   e.g. EXAM_54000_12345678

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Important Notice – Exam Conditions and Academic Integrity

In attempting this examination and submitting an answer file, candidates are undertaking that the work they submit is a result of their own unaided efforts and that they have not discussed the questions or possible answers with other persons during the examination period. Candidates who are found to have participated in any form of cooperation or collusion or any activity which could amount to academic misconduct in the answering of this examination will have their marks withdrawn and disciplinary action will be initiated on a complaint from the Examiner.

Exam answers must be submitted via Canvas. Exam answers will not be accepted by email. Vivas or other invigilated tasks may be used to verify student achievement of learning outcomes to ensure they have completed the work on their own and to assess their knowledge of the answers they have submitted.

Students must not post any requests for clarification on the Discussion Boards on Blackboard, Canvas or Microsoft Teams. Any requests for clarification should be directed by email to Dr Mary Coupland on mary.coupland@uts.edu.au Where clarification is required it will be broadcast by email to all students in the exam group. **Dr Mary Coupland** will be available throughout the examination time.

Students with technical difficulties during the exam should ring the UTS Exam Hotline on +61 2 9514 3222 .

AS THIS IS A TAKE HOME EXAM AND NOT SUPERVISED, YOU MAY USE ONLINE RESOURCES TO CHECK YOUR WORK. HOWEVER FULL WORKING FOR SOLUTIONS IS EXPECTED.

NO MARKS WILL BE AWARDED FOR ANSWERS WITHOUT SUFFICIENT REASONS IN THE FORM OF HAND WRITTEN WORKING.

Question 1

1(a) Given the matrices $A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $C = [1 \quad 0 \quad 6]$, evaluate where possible:

(i) $A + A^T$

(ii) $C + B$

(iii) BC

(iv) The inverse of A

(v) CB.

1(b) Given the vectors **p** = < 3, 2, —1 > and **q** = < 1, 0, 2 > find

(i) The cross product of **p** and **q**

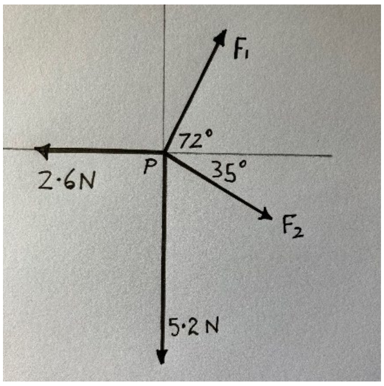
(ii) A unit vector in the direction of **q**

(iii) The parametric equation of a line in the direction of **p** and passing through the point (5, 6, 10)

(iv) The angle between **p** and **q.** (To the nearest degree).

1(c) There are four forces acting at P. Find the resultant force **as a vector** if F_1 is 3.2 N and F_2 is 4.8 N.

(Note that the diagram is not drawn to scale.)



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Question 2

2(a) Solve the following differential equations, subject to initial conditions if they are given.

Assume y is a function of t.

(i) $y'' + 10y' + 25y = 0$

(ii) $y'' = -64y, \quad y(0) = 30, \quad y'(0) = 6.$

(iii) $y'' + 8y' - 20y = 0,$

(iv) $y'' + 6y' + 13y = 0$

2(b) Linear, first order DE s may be solved by the integrating factor method:

Write the DE in the form $y' + p(x)y = q(x)$, that is, $y' + py = q$.
Find the integrating factor, $I = e^{\int p(x)dx}$.
Then find y from $Iy = \int Iq\, dx$,
or by multiplying both sides of $y' + py = q$ by the integrating factor.

Solve the DE $2\frac{dy}{dx} + 8y = 5.$

2(c) (i) In the identity $\frac{1}{(2x-1)(x+2)} \equiv \frac{A}{2x-1} + \frac{B}{x+2}$, find the values of A and B.

(ii) Under certain conditions, the time t (in minutes) required to form x grams of a particular chemical substance in a chemical reaction can be modelled by $\frac{dx}{dt} = k(1-x)(2-x) - 3kx^2$ where $x = 0$ grams when $t = 0$ minutes. Show that the differential equation can be written in the form

$$\int \frac{dx}{(2x-1)(x+2)} = -\int k dt$$

and then solve the initial value problem to find a relation between t and x. Your final answer should be in the form of $\frac{2x-1}{x+2} = f(t)$, where $f(t)$ is a function of t.

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Question 3

(a) Evaluate the following integrals:

(i) $\int x^2 e^{5x} dx$ (ii) $\int \frac{dx}{\sqrt{9+4x^2}}$ (iii) $\int e^x \sinh\left(\frac{x}{2}\right) dx$

(b) (i) Find the value of $\int_0^{2.1} sinx \; dx$, remembering to use radian mode on your calculator. Answer to two decimal places.

(ii) Use the trapezoidal rule with three equal sub-divisions to estimate the integral in (i). Answer to two decimal places.

(iii) Explain any differences in your answers to (i) and (ii). Your answer should indicate that you know how the trapezoidal rule works, and why it may underestimate or overestimate in particular cases. A sketch is expected.

(c) Consider the function $f(x) = x^2 + 5$, on its natural domain, the set of all real numbers.

(i) Explain why we need to restrict its domain before constructing its inverse function, $f^{-1}(x)$.

(ii) Show both these functions, $f(x)$ and $f^{-1}(x)$ on a sketch.

(iii) What is a formula for $f^{-1}(x)$?

(iv) What is the domain and range of $f^{-1}(x)$?

(d) Find $\frac{dy}{dx}$ if $y = 2x^{3x}$.

Question 4

(a) For the complex numbers $z = 5 - 5i$ and $w = 3 + 3\sqrt{3}i$,

(i) Find the real and imaginary parts of $2z + w$.

(ii) Find the exponential form of z, i.e. the form $re^{i\theta}$ where both r and θ are real, and $-\pi < \theta \leq \pi$.

(iii) Use your answer to (ii) to find a, b and c if $z^{20} = -5^a 2^b + ci$ and a, b, and c are real.

(b) The input impedance Z of a particular network is related to the terminating impedance z by the equation $Z = \frac{(1+j)z-2+j4}{z+2+j}$ where $j^2 = -1$. Find Z when $z = 1$. Answer in cartesian form.

(c) For the series $\sum_{n=1}^{\infty} \frac{(n+1)!}{2^n}$, (i) write out the first three terms; (ii) write out the first three partial sums; (iii) use the ratio test to test the series for convergence.

(d) It can be shown that the sine function can be represented as a series:

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

Use the first three terms of this series to estimate the sine of 3 degrees. Note that $3° = \frac{\pi}{60}$ in radians, which is the value you should use for x. Answer to three decimal places.

TABLE OF INTEGRALS

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln\left(x + \sqrt{x^2 - a^2}\right) + C_2$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C_1 = \ln\left(x + \sqrt{x^2 + a^2}\right) + C_2$$

$$\int \frac{dx}{1 - x^2} = \tanh^{-1} x + C_1 = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C_2$$

$$\int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| < a. \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C & \text{if } |x| > a. \end{cases}$$

$$\int \cosh x \; dx = \sinh x + C$$

$$\int \sinh x \; dx = \cosh x + C$$

$$\int \tanh x \; dx = \ln \cosh x + C$$