UTS

SEAT NUMBER:

STUDENT NUMBER:

SURNAME:
(FAMILY NAME)

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33230_v4 Mathematical Modelling 2

Time Allowed: 120 minutes.

Reading time: 10 minutes.

Reading time is for reading only. You are not permitted to write, calculate or mark your paper in any way during reading time.

Closed Book

Permitted materials for this exam:

Drawing Instruments
Non-programmable Calculator

Materials provided for this exam:

4 x 5 Page Booklet

Students please note:

All questions are of equal value.
Answer all questions.
Answer each question in a separate booklet clearly indicating the question number on the front cover.
The necessary statistical tables and formula sheet are at the end of the paper.

Examination Conditions:

It is your responsibility to fill out and complete your details in the space provided on all the examination material provided to you. Use the time before your examination to do so as you will not be allowed any extra time once the exam has ended.

You are **not** permitted to have on your desk or on your person any unauthorised material. This includes but not limited to:

- Mobile phones
- Smart watches and bands
- Electronic devices
- Draft paper (unless provided)
- Textbooks (unless specified)
- Notes (unless specified)

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If you wish to **leave and be re-admitted** (including to use the toilet), you have to wait until **90 mins** has elapsed.

If you wish to **leave the exam room permanently**, you have to wait until **60 mins** has elapsed.

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During the examination **you must first seek permission** (by raising your hand) from a supervisor before:

- Leaving early
- Using the toilet
- Accessing your bag

Misconduct action will be taken against you if you breach university rules.

Declaration: I declare that I have read the advice above on examination conduct and listened to the examination supervisor's instructions for this exam. In addition, I am aware of the university's rules regarding misconduct during examinations. I am not in possession of, nor do I have access to, any unauthorised material during this examination. I agree to be bound by the university's rules, codes of conduct, and other policies relating to examinations.

Signature:

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Do not write your answers on this page.

********ANSWER IN A SEPARATE BOOK********

Question 1. (2 marks + 3 marks + 2 marks + 3 marks + 10 marks = 20 marks)

a) Find the determinant of the matrix

$$B = \begin{pmatrix} 5 & 6 & 4 \\ 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix}$$

b) Find the first partial derivatives of the function

$$f(x, y, z) = 6x \tan(x + 2y - z)$$

c) Find the directional derivative of the function at the given point in the direction of vector **v**.

$$\mathrm{g(x, y, z)} = (x + 2y + 7z)^{3/2},$$

at $(4, 7, 9)$, in the direction of **v** = **4j** - **k**.

d) Calculate the integral

$$\int_0^1 \int_1^3 12x^2 - 27x^3y^2 dydx.$$

e) The temperature at the point (x, y, z) is given by

$$T(x, y, z) = x^2yz \text{ °C}$$

Use the method of Lagrange multipliers to find the hottest and coldest points on the surface of the sphere

$$x^2 + y^2 + z^2 = 12.$$

What are the hottest and coldest temperatures on the surface of the sphere in degrees Celsius?

********ANSWER IN A SEPARATE BOOK********

Question 2. (6 marks + 4 marks + 2 marks + 3 marks + 5 marks = 20 marks)

a) Find **all** solutions of the system of linear equation $Ax = b$ where:

$$A = \begin{pmatrix} 3 & 2 & 0 & 12 \\ 2 & -3 & 1 & -5 \\ 5 & -2 & 4 & 4 \\ 1 & 1 & 1 & 5 \end{pmatrix} \quad b = \begin{pmatrix} 18 \\ 1 \\ 22 \\ 9 \end{pmatrix},$$

or if there are no solutions, explain why this is so.

b) Find the inverse of the matrix

$$A = \begin{pmatrix} 6 & 2 & 3 \\ 6 & 3 & 2 \\ 6 & -2 & -3 \end{pmatrix}.$$

c) Let $A = \begin{pmatrix} -13 & 8 & 6 \\ -12 & 9 & 4 \\ -24 & 12 & 13 \end{pmatrix}$

(i) Show that $\lambda_1 = 3$ is an eigenvalue of the matrix A.

(ii) Find the eigenvectors u_1 associated with λ_1.

olume of the solid that lie

above the plane

and below the paraboloid

********ANSWER IN A SEPARATE BOOK********

Question 3. (3 marks + 2 marks + 3 marks + 4 marks + 4 marks + 4 marks = 20 marks)

a) The average lifetime of a LCD television is 7 years. Suppose that the lifetime follows an exponential distribution.

(i) Find the probability that a particular LCD television lasts for fewer than 4 years.

(ii) What is the probability that an LCD television lasts between 5 and 10 years?

(iii) If a consumer immediately replaces an LCD television once it has ceased to work, what is the probability that they need to replace the television twice in a period of 5 years?
(Hint: use the relationship between the Exponential and Poisson distributions)

b) An engineer studying whether a downhill section of a highway needs to be widened measures the speed of motorists travelling along the highway. A sample of 90 cars gives a mean speed of 110.2 km/h and a standard deviation of 6km/h. Minitab Output 3.1 and 3.2 was obtained.

(i) Find the 95% Confidence interval for the mean speed.

(ii) Suppose that the current width is only appropriate for speeds of less than 112km/h. Perform a hypothesis test to test whether the road is the width is appropriate (i.e. mean speed is less than 112km/h). State your hypotheses, the test statistic, a p-value, the decision made and your conclusion.

(iii) Explain what making a Type I error would mean in the context of the test in Part (ii).

Minitab Output 3.1:

One-Sample T
Test of mu = 112 vs not = 112

N	Mean	StDev	SE Mean	95% CI	T	P
90	110.200	6.000	0.632	(108.943, 111.457)	-2.85	0.005

Minitab Output 3.2:

One-Sample T
Test of mu = 112 vs < 112

N	Mean	StDev	SE Mean	95% Upper Bound	T	P
90	110.200	6.000	0.632	111.251	-2.85	0.003

********ANSWER IN A SEPARATE BOOK********

Question 4. (8 marks + 3 marks + 3 marks + 6 marks = 20 marks)

a) A company identifies that they were having quality problems in one of their manufacturing processes, so they implemented a new set of procedures. They would now like to find out whether the new procedures have been effective. Before the change, a sample of 150 products were taken, and they found that 37 were defective. After the new procedures were implemented, 24 defective products were found in a sample of 150. Have the new procedures been effective?

State your hypotheses, a p-value, the decision made and your conclusion.

Minitab Output 4.1:

Test and CI for Two Proportions

Sample	X	N	Sample p
1	24	150	0.160000
2	37	150	0.246667

Difference = p (1) - p (2)
Estimate for difference: -0.0866667
95% upper bound for difference: -0.0106678
Test for difference = 0 (vs < 0): Z = -1.88 P-Value = 0.030

b) Civil engineers have developed a 3D model to predict the response of jointed concrete pavements to temperature variations. To check this model, they measured the change in traverse strain on six different occasions and compared this to the modelled values. The table below was obtained.

	Change in Strain	
Date	**Field Measurement**	**Modelled Measurement**
Jul 25	-58	-52
Aug 4	69	59
Aug 16	35	32
Sep 3	-32	-24
Oct 26	-40	-39
Nov 3	-83	-71

Continued on the next page.

Use the Minitab output below to answer the following questions.

(i) Explain why this data is paired.

(ii) Test whether there is a significant difference between the modelled measurement and the field measurement. State your hypotheses, a test statistic, a p-value, the decision made and your conclusion.

(iii) Test whether the differences between the measurements are normally distributed. State your hypotheses, a test statistic, a p-value, the decision made and your conclusion.

Minitab Output 4.2:

Paired T-Test and CI: Field, Model

Paired T for Field - Model

	N	Mean	StDev	SE Mean
Field	6	-18.2	58.1	23.7
Model	6	-15.8	50.7	20.7
Difference	6	-2.33	8.02	3.27

95% CI for mean difference: (-10.75, 6.08)

T-Test of mean difference = 0 (vs not = 0): T-Value = -0.71 P-Value = 0.508

Minitab Output 4.3:

Paired T-Test and CI: Field, Model

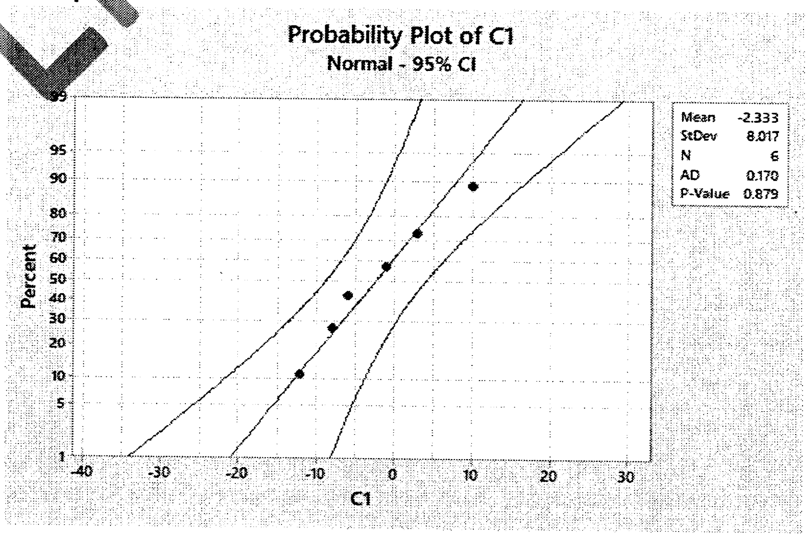
Paired T for Field - Model

	N	Mean	StDev	SE Mean
Field	6	-18.2	58.1	23.7
Model	6	-15.8	50.7	20.7
Difference	6	-2.33	8.02	3.27

95% upper bound for difference: 4.34

T-Test of mean difference = 0 (vs <): T-Value = -0.71 P-Value = 0.254

Minitab Output 4.4:



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FORMULA SHEET

Table of Integrals

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + K, n \neq -1 \qquad \int \frac{1}{x}\,dx = \log|x| + K$$

$$\int \sin x\,dx = -\cos x + K \qquad \int \cos x\,dx = \sin x + K$$

$$\int \sinh x\,dx = \cosh x + K \qquad \int \cosh x\,dx = \sinh x + K$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} + K \qquad \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + K$$

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + K \qquad \int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + K$$

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$$

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$$

Formulas for multiple integrals

1. Cylindrical coordinates: $x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ dV = r\,dz\,dr\,d\theta$

2. Spherical coordinates: $x = \rho\sin\varphi\cos\theta, y = \rho\sin\varphi\sin\theta, z = \rho\cos\varphi, dV = \rho^2\sin\varphi\ dp\,d\varphi\,d\theta$

3. Mass of a solid: $m = \iiint \rho(x,y,z)\,dV$, where $\rho(x,y,z)$ is the density

4. Centre of mass, $(\bar{x},\bar{y},\bar{z})$, of a solid:

$$\bar{x} = \frac{1}{m}\iiint x\,\rho(x,y,z)dV, \bar{y} = \frac{1}{m}\iiint y\,\rho(x,y,z)dV, \bar{z} = \frac{1}{m}\iiint z\,\rho(x,y,z)dV$$

5. Area = $\iint_R dA$ Volume = $\iint_R f(x,y)\,dA$ Volume = $\iiint_D dV$

6. The Jacobian of the transformation given by $x = x(u,v)$ and $y = y(u,v)$ is

$$J(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

Statistical Formulae

Basic Statistical Results

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} {x_i}^2 - n\bar{x}^2\right)$$

$$(Q_i) = \frac{i(n+1)}{4} \qquad IQR = Q_3 - Q_1 \qquad Z - score = \frac{x_i - \bar{x}}{s}$$

Probability distributions

Discrete Distributions

$$P(X \le x) = \sum_{x_i \le x} P(X = x_i) \qquad E(X) = \sum_{x_i} x_i P(X = x_i)$$

$$Var(X) = E(X^2) - \big(E(x)\big)^2 \qquad E(X^2) = \sum_{x_i} {x_i}^2 P(X = x_i)$$

Continuous Distributions

$$P(X \le x) = \int_{-\infty}^{x} f(t)dt \qquad E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$$

$$Var(X) = E(X^2) - \big(E(X)\big)^2 \qquad E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

Specific Distributions

Binomial Distribution (Discrete)

$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}\ x = 0,1,\dots,n$ $\qquad E(X) = np \qquad Var(X) = np(1-p)$

Poisson Distribution (Discrete)

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0,1,\dots \qquad E(X) = \lambda \qquad Var(X) = \lambda$

Exponential distribution (Continuous)

$f(x) = \lambda e^{-\lambda x} \qquad x > 0 \qquad E(X) = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$

Linear Combinations of Random Variables

If $Y = a_1X_1 + a_2X_2 + \cdots + a_nX_n$, then

$$E(Y) = a_1E(X_1) + a_2E(X_2) + \cdots + a_nE(X_n), \text{ and}$$

$$Var(Y) = a_1{}^2\,Var(X_1) + a_2{}^2\,Var(X_2) + \cdots + a_n{}^2\,Var(X_n)$$

Normal Approximation to the Binomial Distribution

$$P(a \le X \le b) = P\left(\frac{a - 0.5 - np}{\sqrt{np(1-p)}} \le Z \le \frac{b + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

Inference for a Single Mean

Population Variance Known

$$Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \qquad \bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2$$

Population Variance Unknown

$$T = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \qquad \bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

Inference for a Single Proportion

$$Z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} \qquad \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p})$$

Inference for a Contingency Table

$$E_i = \frac{Row\ Total \times Column\ Total}{Grand\ Total} \qquad X^2 = \sum_{i=1}^{R\times C}\frac{(O_i-E_i)^2}{E_i} \qquad df = (R-1)\times(C-1)$$

Simple Linear Regression

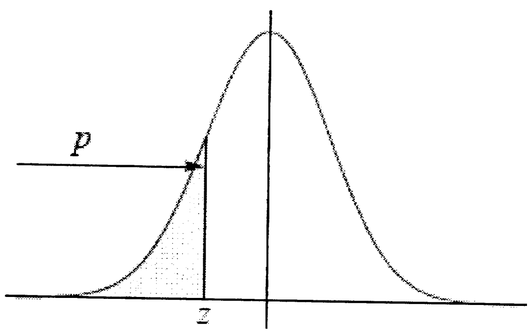
$$Y = \beta_0 + \beta_1X_i + \varepsilon_i \quad Y = \beta_0 + \beta_1X_i + \varepsilon_i \quad T = \frac{b_1}{se(b_1)} \quad b_1 \pm t_{\alpha/2,n-2} \times se(b_1)$$

Cumulative Standard Normal Distribution

z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00
-3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
-3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
-3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
-3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
-3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
-3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
-3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
-3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
-3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
-3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
-2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
-2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
-2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
-2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
-2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
-2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
-2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
-2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
-2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
-2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
-1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
-1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
-1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
-1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
-1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
-1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
-1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
-1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
-1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
-1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
-0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
-0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
-0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
-0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
-0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
-0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
-0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
-0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
-0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

The tabulated value, p, is such that

$$P(Z < z) = p$$

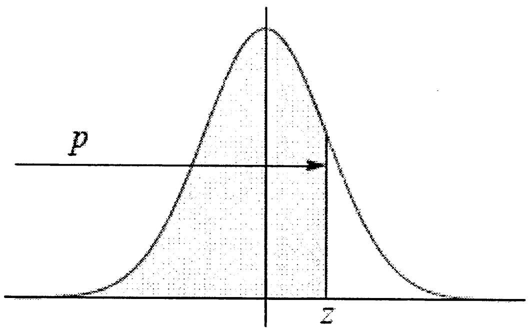


Cumulative Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523 922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

The tabulated value, p, is such that

$$P(Z < z) = p$$

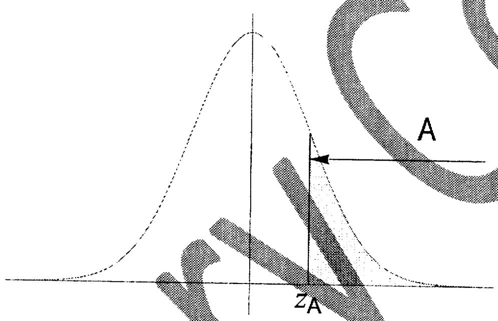


Percentage Points of the N(0,1) Distribution

α	z_α
0.100	1.2816
0.050	1.6449
0.025	1.9600
0.010	2.3263
0.005	2.5758
0.001	3.0902

The tabulated value, z_α, is such that

$$P(Z > z_\alpha) = \alpha$$



Cumulative Student t Distribution

α

1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

The tabulated value, $t_{\alpha,v}$, is such that

$$P(T > t_{\alpha,v}) = \alpha$$