

A.5: Table of Taylor Expansions

Let $n \geq$ be an integer. Then if the function f has $n + 1$ derivatives on an interval that contains both x_0 and x , we have the Taylor expansion

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f''(x_0)(x - x_0)^2 + \cdots + \frac{1}{n!}f^{(n)}(x_0)(x - x_0)^n \\ + \frac{1}{(n+1)!}f^{(n+1)}(c)(x - x_0)^{n+1} \quad \text{for some } c \text{ between } x_0 \text{ and } x$$

The limit as $n \rightarrow \infty$ gives the Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

for f . When $x_0 = 0$ this is also called the Maclaurin series for f . Here are Taylor series expansions of some important functions.

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{1}{n!}x^n && \text{for } -\infty < x < \infty \\ &= 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots \\ \sin x &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} && \text{for } -\infty < x < \infty \\ &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \cdots \\ \cos x &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} && \text{for } -\infty < x < \infty \\ &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + \cdots \\ \frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n && \text{for } -1 \leq x < 1 \\ &= 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \\ \frac{1}{1+x} &= \sum_{n=0}^{\infty} (-1)^n x^n && \text{for } -1 < x \leq 1 \\ &= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots \\ \ln(1-x) &= -\sum_{n=1}^{\infty} \frac{1}{n}x^n && \text{for } -1 \leq x < 1 \\ &= -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \cdots - \frac{1}{n}x^n - \cdots \\ \ln(1+x) &= -\sum_{n=1}^{\infty} \frac{(-1)^n}{n}x^n && \text{for } -1 < x \leq 1 \\ &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots - \frac{(-1)^n}{n}x^n - \cdots \\ (1+x)^p &= 1 + px + \frac{p(p-1)}{2}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots \\ &\quad + \frac{p(p-1)(p-2) \cdots (p-n+1)}{n!}x^n + \cdots \end{aligned}$$

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