Real Analysis 35007.
Class Test. Monday
Answer All Questions. Time allowed. One Hour.
Each Question is worth five marks

(1) Let a sequence have nth term $a_n = \dfrac{2n-3}{4n+1}$. Use the definition of a limit to prove that the sequence converges to $\frac{1}{2}$.

(2) Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+2\sqrt{n}}$ is divergent using the comparison test. You may assume that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent for $p \leq 1$.

(3) Let $f(x) = 2x^4 + 3$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 3]$.

(4) Prove that $|e^{2x} - e^{2y}| \leq 8|x - y|$ on the interval $[0, \ln 2]$.

(5) Determine a Taylor series expansion for $f(x) = \sinh(2x)$ around the point $a = 0$. Recall that $\sinh z = \frac{1}{2}(e^z + e^{-z})$.

Real Analysis 35007.
Class Test. Tuesday
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(1) Let a sequence have nth term $a_n = \dfrac{n^2}{4n^2+1}$. Use the definition of a limit to prove that the sequence converges to $\frac{1}{4}$.

(2) Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+n^2}$ is convergent using the comparison test. You may assume that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$.

(3) Let $f(x) = 2x^3 + 3$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 3]$.

(4) Prove that $|\cos(2x) - \cos(2y)| \leq 2|x - y|$ on the interval $[0, \infty)$.

(5) Determine a Taylor series expansion for $f(x) = x \sinh(2x)$ around the point $a = 0$. Recall that $\sinh z = \frac{1}{2}(e^z + e^{-z})$.

Real Analysis 35007.
Class Test. Wednesday
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(1) Let a sequence have nth term $a_n = \dfrac{2n^2+3}{4n^2+1}$. Use the definition of a limit to prove that the sequence converges to $\frac{1}{2}$.

(2) Use the ratio test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$ is convergent or divergent.

(3) Let $f(x) = x^6$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 1]$. Notice that

$$x^6 - y^6 = (x^3 - y^3)(x^3 + y^3).$$

Then factorise $(x^3 - y^3)$.

(4) Prove that $|\sin(2x) - \sin(2y)| \le 2|x - y|$ on the interval $[0, \infty)$.

(5) Determine a Taylor series expansion for $f(x) = \dfrac{x^2}{(1+x)^2}$ around the point $a = 0$.

Real Analysis 35007.
Class Test. Thursday
Answer All Questions. Time allowed. One Hour.
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(1) Let a sequence have nth term $a_n = \dfrac{2n^2+3}{5n^2+9}$. Use the definition of a limit to prove that the sequence converges to $\frac{2}{5}$.

(2) Use the ratio test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^n}$ is convergent or divergent.

(3) Let $f(x) = x^3 + 4x$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 2]$.

(4) Prove that $|\sin(4x) - \sin(4y)| \leq 4|x - y|$ on the interval $[0, \infty)$.

(5) Determine a Taylor series expansion for $f(x) = \dfrac{x}{(1+x^2)}$ around the point $a = 0$.