Real Analysis 35007
Friday Class Test One 2023

Answer All Questions. Time allowed. One Hour.
Each Question is worth five marks

(1) Let a sequence have nth term $a_n = \dfrac{4n^2+6}{5n^2+1}$. Use the definition of a limit to prove that the sequence converges to $\frac{4}{5}$.

(2) Use the ratio test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^{3n}}{(3n)!}$ is convergent or divergent.

(3) Let $f(x) = x^4 + 6x^2$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 1]$. Notice that
$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2).$$
Then factorise $(x^4 - y^4)$.

(4) Prove that $|\tan(2x) - \tan(2y)| \leq 4|x - y|$ on the interval $[0, \frac{\pi}{8})$. (Hint: Mean Value Theorem)

(5) Determine a Taylor series expansion for $f(x) = x\ln(1 + 2x)$ around the point $a = 0$.

Taylor's Formula

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$

Real Analysis 35007
Friday Class Test Two 2023
Answer All Questions. Time allowed. One Hour.
Each Question is worth five marks

(1) Let a sequence have nth term $a_n = \dfrac{3n+6}{7n+11}$. Use the definition of a limit to prove that the sequence converges to $\frac{3}{7}$.

(2) Use the ratio test to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(3n)!}$ is convergent or divergent.

(3) Let $f(x) = x^3 + 4x^2$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 2]$.

(4) Prove that $|\cos(x^2) - \cos(y^2)| \leq \sqrt{2\pi}|x - y|$ on the interval $[0, \sqrt{\frac{\pi}{2}}]$. (Hint: Mean Value Theorem)

(5) Determine a Taylor series expansion for $f(x) = x^3 \cos(2x)$ around the point $a = 0$.

Taylor's Formula

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$

Real Analysis 35007
Wednesday Class Test 2023
Answer All Questions. Time allowed: One Hour.
Each Question is worth five marks

(1) Define a sequence by setting $a_n = \dfrac{n^2 - 3}{4n^2 + 1}$. Use the definition of a limit to prove that the sequence converges to $\frac{1}{4}$.

(2) Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 2}$ is divergent using the comparison test. You may assume that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent for $p \leq 1$.

(3) Let $f(x) = x^4 + 3x$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 2]$.

(4) Prove that $|e^{\sin x} - e^{\sin y}| \leq e|x - y|$ on the interval $[0, \frac{\pi}{2}]$. (Hint: Mean Value Theorem) .

(5) Determine a Taylor series expansion for $f(x) = x \sinh(3x)$ around the point $a = 0$. Recall that $\sinh z = \frac{1}{2}(e^z - e^{-z})$. Hint: You can just find the series for $\sinh(3x)$ and the answer follows from that.

Taylor's Formula

$$f(x) = f(a) + f'(a)(x - a) + \frac{1}{2!} f''(a)(x - a)^2 + \frac{1}{3!} f'''(a)(x - a)^3 + \cdots$$

Real Analysis 35007.
Thursday Class Test 2023.
Answer All Questions. Time allowed. One Hour.
Each Question is worth five marks

(1) Let a sequence have nth term $a_n = \dfrac{2n^2}{8n^2+1}$. Use the definition of a limit to prove that the sequence converges to $\frac{1}{4}$.

(2) Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+n^3}$ is convergent using the comparison test. You may assume that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$.

(3) Let $f(x) = 4x^3 + 3x$. Use the $\epsilon - \delta$ definition of continuity to prove that f is continuous on the interval $[0, 3]$.

(4) Prove that $|\sin(2x) - \sin(2y)| \leq 2|x - y|$ on the interval $[0, \infty)$. (Hint: Mean Value Theorem)

(5) Determine a Taylor series expansion for $f(x) = x^3 \cosh(2x)$ around the point $a = 0$. Recall that $\cosh z = \frac{1}{2}(e^z + e^{-z})$. Hint: You can just find the series for $\cosh(2x)$ and the answer follows from that.

Taylor's Formula

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f'''(a)(x-a)^3 + \cdots$$