Real Analysis. Sample Test Questions.

1) (a) Use the definition of convergence to show that the sequence $a_n = \frac{n^2}{2n^2+1} \to \frac{1}{2}$

(b) If $a_n = \frac{n}{n+1}$, prove $a_n \to 1$.

2) If $a_n = \frac{e^{n}}{(n!)^n}$, determine whether or not $\sum_{n=1}^{\infty} a_n$ converges.

(3) Find the limit of the sequences

(i) $a_n = n\left(\sqrt{n^2+n+2} - \sqrt{n^2+n}\right)$

(ii) $a_n = \frac{2^n + n^3}{3^n + n^2}$

(4) Show that if $\{S_n\}$ is bounded and $t_n \to 0$, then $S_n t_n \to 0$.

(5) Prove that $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + 3\sqrt{n+1}}$ is divergent

(6) Use the definition of continuity (any of the three) to prove that $f(x) = x^3$ is continuous on $[0,5]$

(7) If f is continuous on $\mathbb{R}$ and periodic, prove that it is uniformly continuous

(8) Let f be three times differentiable and suppose that $f'(a) = 0$. Let f''' be continuous. Show that if $f''(a) < 0$ then f has a local maximum at $x = a$.
Hint: Taylor's Theorem

(9) Let $f(x) = x^3 + 3x + 2$. Let f^{-1} be the inverse of

f. Find $(f^{-1})'(2)$, where the $'$ denotes differentiation

10) Let $f(x) = \cosh x = \left(\frac{e^{x}+e^{-x}}{2}\right)$.

Calcalate the Taylor series expansion of f about $a=0$.