35007 Real Analysis



31

Course area	UTS: Science
Delivery	Spring 2023; City
Credit points	6cp
Requisite(s)	((33130 Mathematics 1 OR 33190 Mathematical Modelling for Science OR 3713 Introduction to Linear Dynamical Systems) AND 37181 Discrete Mathematics) These requisites may not apply to students in certain courses. See access conditions.

Result type Grade and marks

Attendance: 2 hpw (lecture, on campus); 2 hpw (tutorial, on campus)

Subject coordinator

Dr Mark Craddock

07.05.037

95142244

Teaching staff

Dr Mark Craddock

Subject description

Real Analysis develops the underpinnings of calculus and its extensions. It begins with the structure of the real numbers and the least upper bound axiom. It then develops key ideas involving limits of sequences, continuous functions and the derivative. Applications of the derivative are developed and Taylor series are introduced. The subject then introduces Riemann sums and the Riemann integral. It develops properties of the Riemann integral. It introduces the notion of uniform convergence and study the problem of the convergence of a sequence of Riemann integrals. Throughout the treatment is entirely rigourous. One of the aims of the subject is to teach students how to construct and present a correct proof of a mathematical result.

Subject learning objectives (SLOs)

Upon successful completion of this subject students should be able to:

- 1. Develop skills in constructing rigourous mathematical arguments and understanding the principles of mathematical analysis.
- 2. Develop the ability to present mathematical arguments using clear and precise language.
- 3. Be able to understand the proofs of the major results in elementary mathematical analysis.
- 4. Develop the ability to prove elementary theorems and results that the student has not previously seen.
- 5. The student should be able to apply the skills learned to solve problems in areas where mathematical analysis is used.

Course intended learning outcomes (CILOs)

This subject also contributes specifically to the development of following course intended learning outcomes:

- Demonstrate theoretical and technical knowledge of mathematical sciences including calculus, discrete mathematics, linear algebra, probability, statistics and quantitative management. (1.1)
- Evaluate mathematical and statistical approaches to problem solving, analysis, application, and critical thinking to make mathematical arguments, and conduct experiments based on analytical, numerical, statistical, algorithms to solve new problems. (2.1)

- Work autonomously or in teams to demonstrate professional and responsible analysis of real-life problems that require application of mathematics and statistics. (3.1)
- Design creative solutions to contemporary mathematical sciences-related issues by incorporating innovative methods, reflective practices and self-directed learning. (4.1)
- Use succinct and accurate presentation of reasoning and conclusions to communicate mathematical solutions, and their implications, to a variety of audiences, using a variety of approaches. (5.1)

Contribution to the development of graduate attributes

This subject also contributes specifically to the development of the following Graduate Attributes to prepare students for professional practice:

Graduate Attribute 1 – Disciplinary knowledge

Activities in this subject develop practical skills to analyse problems which involve elementary mathematical analysis. We develop the foundations of analysis which underpin calculus and the many applications that have flowed from it.

Graduate Attribute 2 - Research. Inquiry and Critical Thinking. Students will have to develop the skills to prove and present elementary results in analysis which they have not seen before.

Graduate Attribute 3 - Professional, Ethical and Social Responsibility

Graduate Attribute 4 - Reflection, Innovation, Creativity

The assignments focus on the understanding of mathematical systems and their analysis in both formal mathematical language and in peer-to-peer (non-mathematical) language. Students should be able to make conjectures, then use the tools they have learned to prove their conjecture.

Graduate Attribute 5 - Communication

The assignments require students to write in clear English explaining their mathematical reasoning and its conclusions. The ability to communicate the ideas on which analysis is based and their extensions and interrelations is crucial.

All assessment tasks require the student to be able to demonstrate clear reasoning and correct conclusions in a language that can be understood by their peers. The students must be able to write their answers in English that is both grammatically and mathematically correct.

Teaching and learning strategies

Interactive workshops and tutorials on campus. The material will be developed with the students in interactive workshops. Students will have a set of problems which they will work through outside of class. In the tutorials students will be expected to work with their peers on particular questions from the tutorial sheet and may be asked to present them to the class on a whiteboard. A key skill in mathematics is the ability to communicate with your peers in clear mathematical language. Constructing a proof is the hardest part of mathematics. There are however various proof strategies and the students will be shown several of these. Students will see how mathematicians formulate theorems and write proofs, with each step laid out clearly and precisely. Each part of a proof must follow logically from the previous parts. Students will have a chance to write their own proofs in the various assessment taks. They will be required to set them out in a clear logical order with every part of the proof fully justified. This is the essence of communication in mathematics. They will receive feedback on their work in tasks 1 and 2, which are returned to the students after marking with helpful comments, as well as in the tutorials. This will help them to see how their work can be improved. They will also be given worked solutions to all problems so that they can compare their own work and privately reflect on how they can improve.

Content (topics)

The Fundamental principles of real analysis. The least upper bound axiom. Epsilon and delta proofs. Limits. The Bolzano-Weierstrass Theorem. Sequences and Series. Continuous functions and their properties. The derivative. Maxima and Minima. Taylor series. The Riemann integral. Limits of sequences of functions.

Program

Week/Session	Dates	Description
	Aug 8	Introduction to Real Analysis. Epsilon and delta proofs

Aug 15	The concept of a sequence. Limits and limit theorems.
Aug 22	Cauchy sequences and the Bolzano-Weierstrass Theorem
Aug 29	Continuous functions. Properties of continuos functions.
Sept 5	The derivative and its properties.
Sept 12	Applications of the derivative. The Mean Value Theorem.
Sept 19	Series. Convergence tests.
Sept 26	Stuvac. No classes
Oct 3	Taylor's Theorem and its application.
Oc 10	Introduction to the Riemann integral,
Oct 17	Properties of the Riemann integral.
Oct 24	Sequences of functions and convergence theorems for the Riemann integral. Uniform and pointwise convergence.
Oct 31	Methods of Integration and subject review.

Assessment

Assessment task 1: Assignment One

Intent: This assessment item addresses the following graduate attributes:

- 1. Disciplinary Knowledge
- 2. Research, Inquiry and Critical Thinking
- 3. Professional, Ethical and Social Responsibility
- 4. Reflection, Innovation, Creativity
- 5. Communication

Objective(s): This assessment task addresses subject learning objective(s):

Objective(s).	This assessment task addresses subject learning objective(s).
	1, 2, 3, 4 and 5
	This assessment task contributes to the development of course intended learning outcome(s):
	1.1, 2.1, 3.1, 4.1 and 5.1
Туре:	Report
Groupwork:	Individual
Weight:	25%
Task:	To assess the students ability to solve problems and prove elementary results in Real Analysis. This will also provide formative support to your learning.
	This is an assignment that requires the student to answer a set of ten questions using the methods developed over the course of the subject. It is intended to allow the student to develop their skills in analysis by attempting to solve harder problems than those that will be encountered in the tutorials.
Length:	There will be six questions.
Due:	Assignment is to be uploaded to Canvas by COB October 27.
Criteria:	Capacity to do calculations and construct valid mathematical arguments.
Assessment	task 2: Mid semester test
Intent:	This assessment item addresses the following graduate attributes:
	1. Disciplinary Knowledge
	2. Research, Inquiry and Critical Thinking
	3. Professional, Ethical and Social Responsibility
	4. Reflection, Innovation, Creativity
	5. Communication
Objective(s):	This assessment task addresses subject learning objective(s):
	1, 2, 3, 4 and 5
	This assessment task contributes to the development of course intended learning outcome(s):
	1.1, 2.1, 3.1, 4.1 and 5.1
Туре:	Quiz/test
Groupwork:	Individual

Weight: 25%

Task: To provide formative assessment of your progress in understanding real analysis.

The student will answer four questions. The student will be expected to give a correct proof of a mathematical result that they have not seen before. Answers should take no more than half a page of calculations. Students will also be asked to decide if a statement is true or false and give accurate reasons for their answer. The final question will require students to prove some basic facts, which must then be put together to prove a more substantial result. This reflects the way that research in mathematics is conducted, where simpler pieces are often put together to produce major results.

- **Length:** The test should take one hour to complete.
- **Due:** The test will be in your tutorial in the week beginning October 23.
- Criteria: Capacity to do calculations and construct valid mathematical arguments.

Assessment task 3: Final Exam

Intent: This assessment item addresses the following graduate attributes: 1. Disciplinary Knowledge 2. Research, Inquiry and Critical Thinking 3. Professional, Ethical and Social Responsibility 4. Reflection, Innovation, Creativity 5. Communication Objective(s): This assessment task addresses subject learning objective(s): 1, 2, 3, 4 and 5 This assessment task contributes to the development of course intended learning outcome(s): 1.1, 2.1, 3.1, 4.1 and 5.1 Type: Examination Groupwork: Individual 50% Weight: Task: To test your overall competency with the subject's content and concepts. This is the final exam. It will assess all aspects of the subject. It will consist of six questions. It is restricted access open book. The student will have to demonstrate their ability to construct elementary proofs and perform the standard types of calculations that arise in Real Analysis. Length: The exam will be two hours in length. Due: The test will be in the official university exam period. Criteria: Capacity to do calculations and construct valid mathematical arguments.

Minimum requirements

Students must obtain a mark of 50 or higher to pass the subject.

Required texts

A complete set of lecture notes will be provided.

Recommended texts

A Concise Approach to Mathematical Analysis

M.A. Robdera

Springer 2003

Academic liaison officer

Academic liaison officers

Download the complete list of the University's ALOs (PDF 52kB), including their contact details.

Support

UTS Library

The Library has a wide range of resources, facilities and services to support you throughout your studies including textbooks, subject readings, old exam papers, academic writing guides, scientific literature databases, workshops, a gaming room and bookable group study rooms. There is also a team of librarians to help you with all your questions.

w: lib.uts.edu.au facebook: utslibrary twitter: @utslibrary ph: 9514 3666

Mathematics & Science Study Centre

The Mathematics and Science Study Centre (MSSC) operates a Drop-in Room located on UTS City Campus, in Building 4, level 3, room 331 (CB04.03.331). Academic staff members are available for one-to-one assistance. For timetabling and other MSSC resources see:

w: https://tinyurl.com/UTS-maths-study-centre

Statement on copyright

Australian copyright law allows you as a student or researcher to copy and use limited amounts of other people's material in your study or research without their permission and free of charge.

This applies to any sort of published or unpublished work, and includes written material, tables and compilations, designs, drawings (including maps and plans), paintings, photographs, sculpture, craft work, films (such as feature films, television programs, commercials and computer video games), software (such as computer programs and databases), sound recordings, performances and broadcasts (including podcasts and vodcasts of these) and text, including books, journals, websites, emails and other electronic messages.

It is important to remember that you can only use a limited amount for your study or research purposes and that you need to correctly acknowledge the author and reference their material when you use it in your work.

Incorrect or improper use of copyright-protected material could result in breaking Australian copyright law, for which significant penalties apply. Incorrect or improper use of copyright-protected material at UTS would result in consideration under the UTS Student Misconduct rules.

UTS Rules and the UTS Student Charter require that students familiarise themselves and comply with UTS student policies and procedures. Student should also see the copyright information advising what you can copy and how much you can use.

Copyright notice concerning teaching materials

Please remember that teaching materials and resources provided to you at UTS are protected by copyright. You are not permitted to re-use those for any purposes (including commercial purposes, in kind benefit or gain) without permission of the copyright owner. Breaching copyright in relation to teaching materials and resources could lead to a legal action being brought against you.

Your presentation submitted as an Assessment may be used for future teaching activities at UTS without further notification.

Statement on plagiarism

The University and Faculty of Science encourage students to undertake their academic studies with the highest integrity and take seriously any instances of student misconduct.

Student misconduct as defined by Rule 16.2 can include cheating (examples of which may be in formal or informal examinations, copying work from another student for individual reports or assignments, altering data, submitting work which has been written by another person as your own, resubmitting work that has been submitted previously for academic credit; manipulating an assessment to avoid the UTS detection software; using third-party service(s) to bypass the UTS detection software; copying or reworking any material (e.g., text, images, music, video) from generative AI tools, and claiming this work as your own without declaring use of the relevant tool; using generative AI tools, unless permitted use is specified for that assessment) or plagiarism as defined in Rule 16.2.1(4).

Penalties for misconduct relating to a specific subject are outlined in Rule 16.3.1(9).

Students should be aware that any incident of misconduct is placed on record with the Registrar.

If you are uncertain as to what constitutes student misconduct or plagiarism, you are strongly advised to:

- 1. read Section 16 Student Misconduct and Appeals of the Student and Related Rules
- 2. consult the plagiarism help site
- 3. speak to the academic staff responsible for your subject/s.

Plagiarism detection software such as Turnitin or other methods to detect plagiarism may be used to check your work in any subject.

Statement on UTS email account

Email from the University to a student will only be sent to the student's UTS email address. Email sent from a student to the University must be sent from the student's UTS email address. University staff will not respond to email from any other email accounts for currently enrolled students.