The sandwich theorem Let a_n, b_n, c_n be real valued sequences such that

$$a_n \leq b_n \leq c_n.$$

Suppose $a_n \to l$ and $c_n \to l$ as $n \to \infty$.
Then $b_n \to l$.

Proof Let $\varepsilon > 0$. Choose N_1 such that $n \geq N_1 \Rightarrow |a_n - l| < \varepsilon$. Choose N_2 such that $n \geq N_2 \Rightarrow |c_n - l| < \varepsilon$. Let $N = \max\{N_1, N_2\}$.

So for $n \geq N$, $a_n \leq b_n \leq c_n$ and $|a_n - l| < \varepsilon$ and $|c_n - l| < \varepsilon$.

Hence $a_n > l - \varepsilon$ and $c_n < \varepsilon + l$.

So

$$l - \varepsilon < a_n \leq b_n \leq c_n < \varepsilon + l.$$

Thus $b_n \in (l - \varepsilon, l + \varepsilon)$ for all $n \geq N$.

So $b_n \to l$.