Real Analysis 35007.
Four Week Review

(i) Use the definition to prove that the sequence with nth term $a_n = \dfrac{4n+2}{3n+5}$ is convergent.

(ii) Explain why the sequence with nth term $a_n = \dfrac{(-1)^n n}{3n+2}$ has convergent subsequences. Find the lim inf and lim sup of the sequence.

(iii) Use properties of limits to determine $\lim_{n\to\infty} (3^n + 2^n)^{1/n}$.

(iv) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of positive terms and suppose $x_n \to l$ with $l > 0$. Prove that $\sqrt{x_n} \to \sqrt{l}$. Hint $|x_n - l|$ is a difference of two squares and $\frac{\sqrt{x_n}+\sqrt{l}}{\sqrt{x_n}+\sqrt{l}} = 1$. What happens if $l = 0$?

(v) From the definition, prove that $a_n = \dfrac{1}{n^2+4}$ is a Cauchy sequence.

(vi) Use the comparison test to prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+4}$ is convergent.