NEW ZETA FORMULA SOLVED THE RIEMANN HYPOTHESIS AND UNIFIED THE QUANTUM AND THE CLASSICAL PHYSICS

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Abstract. Duality exists in nature and physics but does not clearly exist in mathematics. In this current study, we try to investigate the duality in mathematics. Firstly, we develop and derive a new formula for the Zeta function. We prove that this formula has duality characteristics representing a new kind of mathematics. Besides, we show that the Riemann hypothesis proof is directly related to this property. In the second part of this theoretical study, the behavior of the energy or particle in both quantum and classical physics theories is explained within the proposed mathematical model framework. The presented model can give answers to several important physics questions that were never answered, like, for instance, the reason for the collapsing of the wave function when being measured. In addition to explaining the Quantum entanglement phenomenon and its instantaneous communication, the double slits experiment and all its mysteries as well as the Stern-Gerlach experiment can be explained based on the model. Furthermore, regarding general relativity, the model can describe a mechanism of motion that achieves a uniform acceleration as a default particle motion in the universe, and that may explain its extension. Finally, this model presents a paradigm for gravity and provides a deeper explanation to understand its force and causes.

1. Introduction

What is the Riemann hypothesis ? and what is its importance?

$\mathbb{Z}$eta or Direichlet series form is,

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \qquad s \in \mathbb{C}, \tag{1.1}$$

It takes a function form in general as $\int_0^\infty x^{-s} \ \sum f(x,n) \ dx$ [3][14]. This function has been connected with many applied scientific fields and mathematics[7]. Its importance starts to appear in the nineteenth century when the mathematicians tried to find out the distribution of the prime numbers. Zeta series has the advantage

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that its product formula contains all the prime numbers.

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \left[\left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{5^s}\right)...\right]^{-1} = \prod_{p_{(prime)}} \left(1-\frac{1}{p^s}\right)^{-1} \tag{1.2}$$

This is called Euler's product formula. In 1859, Riemann, a German mathematician, could use this formula to derive a relationship that expresses the prime number distribution function which is represented by the $\pi(x)$ function,

$$\pi(x) \sim \int_2^x \frac{dt}{\ln t} \tag{1.3}$$

It determines the number of primes (less than or equal) $x \in \mathbb{R}$.
Riemann converted the Zeta series into a function, and concluded that Zeta is a polynomial function then calculated its roots (zeros). He conjuncted that all these zeros have the values $s = 1/2 + it$, they have different imaginary numbers, but they all have the same real value, which is $\Re(s) = 1/2$, he named them non-trivial zeros [1] . This is *The Riemann hypothesis: (All non-trivial zeros of the $\mathbb{Z}$eta function lie on* $\Re(s) = 1/2$*)* [see [3], [12]] Many mathematicians believe that this hypothesis may reveals the secrete of the prime numbers. The hypothesis gained importance with time by increasing the number of theories that depend on it as if it is true. This pushed some institutions such as the Clay Institute to allocate a big prize to prove (or disprove) it [6].

The zeros of the Zeta function appeared unexpectedly in the field of nuclear physics by Montgomery (1973) [8] when he linked it to the random matrices, and by Odlyzko who made a numerical study of the distribution of spacings between zeros of the Riemann zeta function and proved that the zeros of the zeta function behave like eigenvalues of random Hermitian matrices. Matrices of this type are used in modeling energy levels in physics, [10].
The applications of random matrices have begun to appear in many fields of applied physics, especially, with regard to atoms and molecules[11]. Thus the applied areas of the $\mathbb{Z}$eta function were expanded.

This research consists of two parts, Mathematics and Physics. As for mathematics part, it shall be started with deriving a new $\mathbb{Z}$eta formula, which is the main formula in the research. It is mainly defined in the strip region $0 < s < 1$ and can also extend to cover all s-domain. Follow that by testing it by deriving formulas for Zeta that are already known. After that proving the Riemann Hypothesis. Finally studying the Zeta function zeros.

The physics part shall concern Zeta function as a unification model for the Classic and Quantum physics.
The points that would be discussed:

[1]The Zeta function has two types of zeros according to the definition region, for the region $\Re(s) \notin]0,1[$ it equals zero for the values $s = -3, -5, -7, ...$ and can easily calculated. But for the definition region $0 < s < 1$, it has another zeros which are mentioned in the Riemann hypothesis and non-trivial means they are hard to find. What we will see later, these zeros (non-trivial) relate specifically to the Zeta as a function, while the other zeros (trivial one) relate to the Zeta as a series, they are defined into two independent domains.

- A new theory describes new *matter-space-time dynamic* picture.
- The proof of *Plank*'s *law* and the concept of *wave-particle duality.*
- Achieving the *Heisenbergs uncertainty principle.*
- Explaining the *superposition and states.*
- Explaining *The Entanglement phenomenon.*
- Explaining *The double slits experiment.*
- Explaining *The Stern-Gerlach experiment.*
- Achieving *The Lorentz invariant.*
- The acceleration frame in the model and Einsteins general relativity.
- New perspective for the Force of Gravity .

2. PART1: Mathematics part

In this section, the new $\mathbb{Z}$eta formula will be derived starting with the following theory:

Theorem 2.1. *For $s \in \mathbb{C}$, the Dirichlet series form of zeta-function*

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots + \frac{1}{k^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{2.1}$$

can be expressed as,

$$\zeta(s) = \frac{2^{s-1}}{s-1} + \int_{\frac{1}{2}}^{\infty} x^{-s}\, 2 \sum_{m=1}^{\infty} \cos 2\pi m x \, dx \tag{2.2}$$

Proof. The idea here is to start with zeta integral form for combination $\Gamma(s)\zeta(s)$, then manipulating this integral by extract $\Gamma(s)$ function from it in the right-hand-side then canceling $\Gamma(s)$ from both sides to get again $\zeta(s)$ in a new form. So we have Gamma function

$$\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} \, dt \tag{2.3}$$

By Combing Gamma function with Zeta we get

$$\Gamma(s) \sum_{n=1}^{\infty} n^{-s} = \int_0^{\infty} x^{s-1} \sum_{n=1}^{\infty} e^{-nx} \, dx \tag{2.4}$$

or

$$\Gamma(s)\zeta(s) = \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} \, dx \tag{2.5}$$

Now, we're going to extract Gamma function from the right-hand-side. We have

(2.6)

$$\begin{aligned}
\Gamma(s)\zeta(s) &= \int_0^\infty \frac{x^{s-1}}{e^x-1}\,dx \\
&= \int_0^\infty \frac{x^{s-1}e^{-\frac{x}{2}}}{e^{\frac{x}{2}}-e^{-\frac{x}{2}}}\,dx \\
&= \int_0^\infty \frac{x^{s-1}e^{-\frac{x}{2}}}{2\sinh\frac{x}{2}}\,dx \\
&= \int_0^\infty x^{s-1}e^{-\frac{x}{2}}\left(\frac{1}{x}+2\sum_{m=1}^\infty \frac{(-1)^m}{x^2+(2\pi m)^2}\right)\,dx \\
&= \int_0^\infty x^{s-1}e^{-\frac{x}{2}}\,\frac{dx}{x}+2\int_0^\infty x^{s-1}e^{-\frac{x}{2}}\left(\sum_{m=1}^\infty \frac{(-1)^m x}{x^2+(2\pi m)^2}\right)\,dx \\
&= 2^{s-1}\Gamma(s-1)+2\sum_{m=1}^\infty(-1)^m\int_0^\infty x^{s-1}e^{-\frac{x}{2}}\frac{x}{x^2+(2\pi m)^2}\,dx \\
&= 2^{s-1}\frac{\Gamma(s)}{s-1}+2\sum_{m=1}^\infty(-1)^m\int_0^\infty x^{s-1}e^{-\frac{x}{2}}\left(\int_0^\infty e^{-xt}\cos 2\pi mt\,dt\right)dx \\
&= 2^{s-1}\frac{\Gamma(s)}{s-1}+2\sum_{m=1}^\infty(-1)^m\int_0^\infty\int_0^\infty x^{s-1}e^{-x(t+\frac{1}{2})}\cos 2\pi mt\,dt\,dx
\end{aligned}$$

By taking the integration with respect to x, we get

$$\Gamma(s)\zeta(s) = 2^{s-1}\frac{\Gamma(s)}{s-1}+2\sum_{m=1}^\infty(-1)^m\Gamma(s)\int_0^\infty\left(t+\frac{1}{2}\right)^{-s}\cos 2\pi mt\,dt \tag{2.7}$$

on sitting $x=t+\frac{1}{2}$, we obtain

$$\Gamma(s)\zeta(s) = 2^{s-1}\frac{\Gamma(s)}{s-1}+2\Gamma(s)\int_{\frac{1}{2}}^\infty x^{-s}\sum_{m=1}^\infty(-1)^m\cos 2\pi m(x-\frac{1}{2})\,dx \tag{2.8}$$

since for $m=1,2,3,\ldots$

$$\begin{aligned}
\sum_{m=1}^\infty(-1)^m cos2\pi m(x-\frac{1}{2}) &= \sum_{m=1}^\infty(-1)^m\cos(2\pi mx-\pi m) \\
&= \sum_{m=1}^\infty \cos 2\pi mx
\end{aligned} \tag{2.9}$$

hence

$$\Gamma(s)\zeta(s) = 2^{s-1}\frac{\Gamma(s)}{s-1}+2\Gamma(s)\int_{\frac{1}{2}}^\infty x^{-s}\sum_{m=1}^\infty \cos 2\pi mx\,dx \tag{2.10}$$

Remove Gamma from both sides, we obtain

$$\zeta(s) = \frac{2^{s-1}}{s-1}+\int_{\frac{1}{2}}^\infty x^{-s}\,2\sum_{m=1}^\infty \cos 2\pi mx\,dx \tag{2.11}$$

□

Corollary 1. *For $s \in \mathbb{C}$ we have*

$$\zeta(s) = \int_0^\infty x^{-s}\, 2\sum_{m=1}^{\infty} \cos 2\pi mx\, dx \qquad 0 < \Re(s) < 1 \tag{2.12}$$

Proof. from the previous theorem 2.1 and by using Dirishlet kernel and Dirac delta function relation, where

$$D_m = \sum_{m=-\infty}^{\infty} e^{i2\pi mx} = \delta(x) = 1 + 2\sum_{m=1}^{\infty} \cos(2\pi mx) \tag{2.13}$$

or

$$2\sum_{m=1}^{\infty} \cos(2\pi mx) = \delta(x) - 1 \qquad -\frac{1}{2} < x < \frac{1}{2} \tag{2.14}$$

For a long positive x, we get

$$2\sum_{m=1}^{\infty} \cos(2\pi mx) = \sum_{n=1}^{\infty} \delta(x-n) - 1_n \qquad n - \frac{1}{2} < x < n + \frac{1}{2} \tag{2.15}$$

Since, n starting from 1, hence for $0 < x < 1/2$ the $2\sum \cos(2\pi mx) = -1$. Then the first term in 2.11 in the right-hand-side can be expressed as

$$\frac{2^{s-1}}{s-1} = \int_0^{\frac{1}{2}} x^{-s} \times (-1)\, dx = \int_0^{\frac{1}{2}} x^{-s}\, 2\sum_{m=1}^{\infty} \cos 2\pi mx\, dx \qquad \Re(s) < 1 \tag{2.16}$$

Hence

$$\zeta(s) = \int_0^\infty x^{-s}\, 2\sum_{m=1}^{\infty} \cos 2\pi mx\, dx \qquad 0 < \Re(s) < 1 \tag{2.17}$$

□

In fact, the first term in the right-hand-side in 2.11 can be expressed by two ways, one way as integral from $0 \to 1/2$, and that is the previous case, the other way as integral from $1/2- \to \infty$, and in this case, we get

$$\frac{2^{s-1}}{s-1} = \int_{1/2}^\infty x^{-s} \times (1)\, dx \qquad \Re(s) > 1 \tag{2.18}$$

So $\zeta(s)$ can be written as

$$\begin{aligned}
\zeta(s) &= \int_{1/2}^{\infty} x^{-s} \times (1)\, dx + \int_{1/2}^{\infty} x^{-s}\, 2\sum_{m=1}^{\infty} \cos 2\pi m x\, dx \\
&= \int_{1/2}^{\infty} x^{-s} \times (1)\, dx + \int_{1/2}^{\infty} x^{-s} \sum_{m=1}^{\infty} [\delta(x-n) - 1_n]\, dx \\
&= \int_{1/2}^{\infty} x^{-s} \times (1)\, dx + \int_{1/2}^{\infty} x^{-s} \sum_{m=1}^{\infty} \delta(x-n)\, dx - \int_{1/2}^{\infty} x^{-s} \times (1)\, dx \qquad (2.19) \\
&= \int_{1/2}^{\infty} x^{-s} \sum_{m=1}^{\infty} \delta(x-n)\, dx \\
&= \sum_{m=1}^{\infty} n^{-s} \qquad \Re(s) > 1
\end{aligned}$$

So, we get $\mathbb{Z}$eta series again, so this function has double-faced, one for $\Re(s) < 1$, or $0 < \Re(s) < 1$, and the other for $\Re(s) > 1$, but the form 2.17 is also, in general, boundedly convergent, term-by-term integration, so it can be taken over any finite range to converge for all *s-domain*.
In the next section, we will test the new form to ensure that its main convergence region is $0 < \Re(s) < 1$.

3. The new Zeta formula testing

First, we will rewrite 2.17 as

$$\zeta(s) = \sum_{n=1}^{\infty} \int_{0}^{\infty} x^{-s}\, \delta(x-n)\, dx - \int_{0}^{\infty} x^{-s} \times (1)\, dx \qquad (3.1)$$

The approach in this test is by deriving some well-known $\mathbb{Z}$eta function formulas which are already defined for $0 < \Re(s) < 1$

3.1. **First test.**

Deriving the equation,

$$\Gamma(s)\zeta(s) = \int_{0}^{\infty} x^{s-1} \left(\frac{1}{e^{x}-1} - \frac{1}{x} \right) dx \qquad 0 < \Re(s) < 1 \qquad (3.2)$$

This function is defined for $0 < \Re(s) < 1$ (see Titchmarsh [14] ch2).

Proof. By multiplying $\zeta(s)$, the new form, with $\Gamma(s)$, we get

(3.3)

$$\begin{aligned}
\Gamma(s)\zeta(s) &= \int_0^\infty x^{s-1}e^{-x}\Big(\sum_{n=1}^\infty \int_0^\infty t^{-s}\ \delta(t-n)\ dt - \int_0^\infty t^{-s}\times(1)\ dt\Big)\ dx \\
&= \int_0^\infty x^{s-1}\Big(\sum_{n=1}^\infty \int_0^\infty e^{-tx}\ \delta(t-n)\ dt - \int_0^\infty e^{-tx}\times(1)\ dt\Big)\ dx \qquad \text{(Laplace Transform } \mathcal{L}\{1\}) \\
&= \int_0^\infty x^{s-1}\left(\sum_{n=1}^\infty e^{-xn}\right)\ dx - \int_0^\infty x^{s-1}\frac{1}{x}\ dx \\
&= \int_0^\infty x^{s-1}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)\ dx \qquad 0<\Re(s)<1
\end{aligned}$$

□

3.2. **Second test.**

Deriving the Hardy's equation [2].

(3.4)

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \frac{2\xi(s)}{s(s-1)} = \int_0^\infty x^{s-1}\left(\sum_{n=-\infty}^\infty e^{-\pi n^2x^2}-1-\frac{1}{x}\right)\ dx \qquad 0<\Re(s)<1$$

Proof. First we need to define

(3.5)

$$\Gamma(\frac{s}{2}) = 2\pi^{\frac{s}{2}}\int_0^\infty x^{s-1}e^{-\pi x^2}\ dx \qquad \text{(by using } x\to\pi x^2 \text{ in Gamma integral)}$$

By multiplying the new form of $\zeta(s)$ with $\Gamma(\frac{s}{2})$, we get,

(3.6)

$$\begin{aligned}
\Gamma(\frac{s}{2})\zeta(s) &= 2\pi^{\frac{s}{2}}\sum_{n=0}^\infty\int_0^\infty x^{s-1}e^{-\pi x^2}\left(\sum_{n=1}^\infty\int_0^\infty t^{-s}\ \delta(t-n)\ dt - \int_0^\infty t^{-s}\times(1)\ dt\right)\ dx \\
&= 2\pi^{\frac{s}{2}}\sum_{n=0}^\infty\int_0^\infty x^{s-1}\left(\sum_{n=1}^\infty\int_0^\infty e^{-\pi t^2x^2}\ \delta(t-n)\ dt - \int_0^\infty e^{-\pi t^2x^2}\times(1)\ dt\right)\ dx \\
&= 2\pi^{\frac{s}{2}}\int_0^\infty x^{s-1}\left(\sum_{n=1}^\infty e^{-\pi n^2x^2}\right)\ dx - 2\int_0^\infty x^{s-1}\left(\int_0^\infty e^{-\pi t^2x^2}(1)\ dt\right)\ dx
\end{aligned}$$

By using the equality

$$2\sum_{n=1}^\infty e^{-\pi n^2x^2} = \sum_{n=-\infty}^\infty e^{-\pi n^2x^2}-1 \tag{3.7}$$

And by setting $xt=y$ in the second term in right-hand-side, hence

(3.8)

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \int_0^\infty x^{s-1}\left(\sum_{n=-\infty}^\infty e^{-\pi n^2x^2}-1\right)\ dx - 2\int_0^\infty x^{s-1}\left(\frac{1}{x}\int_0^\infty e^{-\pi y^2}\ dy\right)\ dx$$

[2]Hardy derived this formula when he was trying to prove the Riemann hypothesis, and proved that there are an infinite number of zeros for $\mathbb{Z}$eta on the vertical line $s=\frac{1}{2}+it$ [Edwards [3]]

Since the integral

$$\int_0^\infty e^{-\pi y^2}\ dy = \frac{1}{2} \tag{3.9}$$

Then

(3.10)

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \frac{2\xi(s)}{s(s-1)} = \int_0^\infty x^{s-1}\left(\sum_{n=-\infty}^{\infty} e^{-\pi n^2 x^2} - 1 - \frac{1}{x}\right)\ dx \qquad 0 < \Re(s) < 1$$

□

3.3. **Third test.** Calculating the (*Euler's constant*).

This formula of $\zeta(s)$ can be used to calculate (*Euler's constant* γ), since

$$\begin{aligned}
\zeta(s) &= \sum_{n=1}^{\infty}\int_0^\infty x^{-s}\ \delta(x-n)\ dx - \int_0^\infty x^{-s} \times (1)\ dx \\
&= \sum_{n=1}^{\infty} n^{-s} - \int_0^\infty x^{-s}\ dx \\
&= -\int_0^1 x^{-s}\ dx + \sum_{n=1}^{\infty} n^{-s} - \int_1^\infty x^{-s}\ dx \\
&= \frac{1}{s-1} + \lim_{N\to\infty}\left(\sum_{n=1}^{N} n^{-s} - \int_1^N x^{-s}\ dx\right)
\end{aligned}$$

hence, to calculate (*Euler's constant* γ), where $s = 1$, we obtain

$$\gamma = \lim_{s\to 1}\left(\zeta(s) - \frac{1}{s-1}\right) = \lim_{N\to\infty}\left(\sum_{n=1}^{N} n^{-1} - \int_1^N x^{-1}\ dx\right) \ = \lim_{N\to\infty}\left(\sum_{n=1}^{N} n^{-1} - \ln N\right)$$

This is one formula of many other formulas to calculate γ [3]

3.4. **Fourth test.**

The Zeta functional equation

$$\zeta(1-s) = 2(2\pi)^{-s}\cos(\frac{\pi}{2}s)\Gamma(s)\zeta(s) \tag{3.11}$$

It is a must to test the new $\zeta(s)$ formula by multiplying it with $2(2\pi)^{-s}\cos(\frac{\pi}{2}s)\Gamma(s)$ to get $\zeta(1-s)$, and make sure it has the same structure.
To restrict this study for only the strip region, we will define the first part as a Millen transform for $\cos 2\pi x$, so

$$2(2\pi)^{-s}\cos(\frac{\pi}{2}s)\Gamma(s) = 2\int_0^\infty x^{s-1}\ \cos 2\pi x\ dx \qquad 0 < \Re(s) < 1 \tag{3.12}$$

[3] https://en.wikipedia.org/wiki/Euler-Mascheroni_constant

Then, we have
(3.13)

$$\left\{2(2\pi)^{-s}\cos\left(\frac{\pi}{2}s\right)\Gamma(s)\right\}\zeta(s) = 2\left(\int_0^\infty x^{s-1}\ \cos 2\pi x\ dx\right)\left(\sum_{n=1}^\infty \int_0^\infty u^{-s}\ \delta(u-n)\ du - \int_0^\infty u^{-s}\times(1)\ du\right)$$

on setting x=uy, then,

$$\begin{aligned} 2(2\pi)^{-s}\cos\left(\frac{\pi}{2}s\right)\Gamma(s)\zeta(s) &= 2\sum_{n=1}^\infty \int_0^\infty y^{s-1}\left(\int_0^\infty \cos(2\pi uy)\ \delta(u-n)\ du\right)\ dy \\ &\qquad - 2\int_0^\infty y^{s-1}\left(\int_0^\infty \cos(2\pi uy)\times 1\ du\right)\ dy \\ &= \int_0^\infty y^{s-1} 2\sum_{n=1}^\infty \cos(2\pi ny)\ dy - \int_0^\infty y^{s-1}\int_{-\infty}^\infty e^{-i2\pi uy}\times 1\ du\ dy \quad \text{(F.T)} \\ &= \int_0^\infty y^{s-1}\ \left(2\sum_{n=1}^\infty \cos(2\pi ny)\right)\ dy - \int_0^\infty y^{s-1}\times\delta(y)\ dy \end{aligned}$$

This is an unexpected result because, it is supposed to be

$$\zeta(1-s) = \sum_{n=1}^\infty \int_0^\infty y^{s-1}\ \delta(y-n)\ dy - \int_0^\infty y^{s-1}\times(1)\ dy \tag{3.14}$$

In addition, the first integral in the right-hand-side has a kernel function similar to the Zeta function itself,so how can we understand this ? We need to re-study the main function 2.17 again by different way. In that way, we will consider the summation of $\sum_{n=1}^\infty \cos(2\pi nx)$ as a geometric series, it is one of the ways to study this summation.
So, it can be calculated as

$$\begin{aligned} \sum_{m=0}^N \cos 2\pi mx &= \mathbf{Re}\left(\sum_{m=0}^N e^{i2\pi mx}\right) \\ \sum_{m=1}^N \cos 2\pi mx &= \mathbf{Re}\left(\sum_{m=0}^N e^{i2\pi mx}\right) - 1 \\ &= \mathbf{Re}\left(\frac{e^{i(N+1)2\pi x}-1}{e^{i2\pi x}-1}\right) - 1 \\ &= \mathbf{Re}\left(\frac{e^{i(N+1)x}}{e^{i2\pi x}-1}\right) + \mathbf{Re}\left(\frac{-e^{i2\pi x}}{e^{i2\pi x}-1}\right) \end{aligned} \tag{3.15}$$

Then, for $N\to\infty$, the first term,

$$\begin{aligned} I_1 &= \lim_{N\to\infty}\mathbf{Re}\left(\frac{e^{i(N+1)2\pi x}}{e^{i2\pi x}-1}\right) \\ &= \lim_{N\to\infty}\frac{\sin((N+\frac{1}{2})2\pi x)}{2\sin(\pi x)} \\ &= \frac{1}{2}\delta(x) \qquad \text{for } (x=n) \end{aligned} \tag{3.16}$$

the second integral

$$
\begin{aligned}
I_2 &= \lim_{N\to\infty} \mathbf{Re}\left(\frac{-e^{i2\pi x}}{e^{i2\pi x}-1}\right) \\
&= \lim_{N\to\infty} \mathbf{Re}\left(\frac{e^{i\frac{2\pi x}{2}}}{-2i\sin(\pi x)}\right) \qquad (3.17)\\
&= -\frac{\sin(\pi x)}{2\sin(\pi x)} = -\frac{1}{2} \qquad \sin\pi x \neq 0 \; or \; (x \neq \mathbb{N}) \; \equiv |\cos 2\pi x| < 1
\end{aligned}
$$

We notice that the first integral $I_1 = \frac{1}{2}\delta(x)$ in right-hand-side is defined only for $x = \mathbb{N}$ $(integer)$ otherwise it will be zero. On the other hand, the second integral $I_2 = -\frac{1}{2}$ is defined for $x \neq \mathbb{N}$. So I_1 and I_2 are defined into two different domains, and if we define $x = [x] + \{x\}$ in the interval $[n, n+1)$, where $[x]$ and {x} are the integral and fractional parts of x respectively. Then the series could be expressed as two independent functions,

$$
2\sum_{m=1}^{\infty}\cos(2\pi m x) = \begin{cases} \delta(x), & (x=[x]) \equiv (\cos 2\pi x = 1) \\ \\ -1, & (x=\{x\}) \equiv |\cos 2\pi x| < 1 \end{cases} \qquad (3.18)
$$

So, $\mathbb{Z}$eta function can be re-written as
(3.19)

$$
\zeta(s) = \sum_{n=1}^{\infty}\int_0^{\infty} x^{-s}\,\delta(x-n)\,dx + \int_0^{\infty} x^{-s}\,\left[2\sum_{m=1}^{\infty}\cos(2\pi m x)\right]_{|\cos 2\pi x|<1} dx
$$

For the second integral in the right-hand-side, we have
(3.20)

$$
\begin{aligned}
\int_0^{\infty} x^{-s}\times(-1)\,dx &= \int_0^{\infty} x^{-s}\,\left[2\sum_{m=1}^{\infty}\cos(2\pi m x)\right]_{|\cos 2\pi x|<1} dx \\
&= \int_0^{\infty} x^{-s}\,\underbrace{\left(\sum_{m=1}^{\infty} e^{i2\pi m x} + \sum_{m=1}^{\infty} e^{-i2\pi m x}\right)}_{|e^{i2\pi x}|<1} dx \qquad \text{(by setting } x \to 2\pi x) \\
&= (2\pi)^{s-1}\left[\int_0^{\infty} x^{-s}\,\frac{1}{e^{-ix}-1}\,dx + \int_0^{\infty} x^{-s}\frac{1}{e^{ix}-1}\,dx\right] \qquad |x|<1
\end{aligned}
$$

on setting $y = -ix$ in first integral , and $u = ix$ in the second one, we get
(3.21)

$$
\begin{aligned}
\int_0^{\infty} x^{-s}\times(-1)\,dx &= (2\pi)^{s-1}\left[(-i)^{s-1}\int_0^{-i\infty} y^{-s}\frac{1}{e^{y}-1}\,dy + (i)^{s-1}\int_0^{i\infty} u^{-s}\frac{1}{e^{u}-1}\,du\right] \\
&= (2\pi)^{s-1}\left[(-i)^{s-1}\int_0^{-i\infty} y^{-s}\sum_{n=1}^{\infty} e^{-ny}\,dy + (i)^{s-1}\int_0^{i\infty} u^{-s}\sum_{n=1}^{\infty} e^{-nu}\,du\right] \\
&= (2\pi)^{s-1}\sum_{n=1}^{\infty} n^{s-1}\left[(-i)^{s-1}\int_0^{-i\infty} y^{-s}e^{-y}\,dy + (i)^{s-1}\int_0^{i\infty} u^{-s}e^{-u}\,du\right]
\end{aligned}
$$

The integral in right-hand-side can be solved by selecting an appropriate contour, and it would equal $\Gamma(1-s)$ for $0 < \Re(s) < 1$,[4] then,

(3.22)
$$\int_0^\infty x^{-s} \times (-1)\, dx = (2\pi)^{s-1} \left[(-i)^{s-1} + (i)^{s-1}\right] \Gamma(1-s) \left(\sum_{n=1}^\infty n^{s-1}\right)$$
$$= 2(2\pi)^{s-1} \sin\left(\frac{\pi}{2}s\right)\Gamma(1-s) \left(\sum_{n=1}^\infty n^{s-1}\right) \qquad 0 < \Re(s) < 1$$

This is the Zeta functional equation and this part $2(2\pi)^{s-1} \sin\left(\frac{\pi}{2}s\right)\Gamma(1-s)$ can be expressed as a Mellin transform of $(\cos 2\pi x)$ for $0 < \Re(s) < 1$, and this term can be written as

(3.23)
$$\int_0^\infty x^{-s} \times (-1)\, dx = \left(\sum_{m=1}^\infty m^{s-1}\right) 2 \left(\int_0^\infty x^{-s} \cos(2\pi x)\, dx\right) \qquad 0 < \Re(s) < 1$$

or by setting $x \to mx$, then

$$\int_0^\infty x^{-s} \times (-1)\, dx = 2 \sum_{m=1}^\infty \int_0^\infty x^{-s} \cos(2\pi m x)\, dx \qquad 0 < \Re(s) < 1 \tag{3.24}$$

Then,

$$\boxed{\zeta(s) = \sum_{n=1}^\infty \int_0^\infty x^{-s}\, \delta(x-n)\, dx + 2 \sum_{m=1}^\infty \int_0^\infty x^{-s} \cos(2\pi m x)\, dx} \tag{3.25}$$

Therefore, the condition of $x = \{x\}$ in the integral can be achieved by making the integration first then summation, but if the summation is taken first, then the kernel should be expressed as (-1). It seems that the convergence in the strip region is related to the term-wise function in Zeta. Now, back to the "Fourth test". For simplicity $\zeta(s)$ will be defined by two functions $\zeta(s)_{dis}$ (discontinuous kernel) and $\zeta(s)_{con}$ (continuous kernel) for the kernels $\delta(x-n)$ and $\cos 2\pi m x$ respectively, then

$$\zeta(s) = \zeta(s)_{dis} + \zeta(s)_{con} \tag{3.26}$$

where,

(3.27)
$$\zeta(s)_{dis} = \sum_{n=1}^\infty \int_0^\infty x^{-s} \delta(x-n)\, dx \qquad \text{and} \qquad \zeta(s)_{con} = 2 \sum_{m=1}^\infty \int_0^\infty x^{-s} \cos 2\pi m x\, dx$$

Thus,

(3.28)
$$\left\{2(2\pi)^{-s} \cos\left(\frac{\pi s}{2}\right)\Gamma(s)\right\} \zeta(s) = \left\{2(2\pi)^{s-1} \cos\left(\frac{\pi s}{2}\right)\Gamma(s)\right\} \left(\zeta(s)_{dis} + \zeta(s)_{con}\right)$$

---

[4] $\int_0^{-i\infty} x^{-s} e^{-x}\, dx$ (or) $\int_0^{i\infty} x^{-s} e^{-x}\, dx = \int_0^{\infty} x^{-s} e^{-x}\, dx = \Gamma(1-s) \qquad$ for $\quad 0 < \Re(s) < 1$

Hence for $\zeta(s)_{dis}$, we get
(3.29)

$$
\begin{aligned}
2(2\pi)^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)_{dis} &= 2\left(\int_0^\infty x^{s-1}\ \cos 2\pi x\ dx\right)\left(\sum_{n=1}^\infty \int_0^\infty u^{-s}\ \delta(u-n)\ du\right) \\
&= 2\sum_{n=1}^\infty \int_0^\infty x^{s-1}\ \left(\int\limits_0^\infty \cos(2\pi ux)\delta(u-n)\ dt\right)\ dx \\
&= 2\sum_{n=1}^\infty \int_0^\infty x^{s-1}\ \cos 2\pi nx\ dx \\
&= \zeta(1-s)_{con}
\end{aligned}
$$

and for $\zeta(s)_{con}$, we obtain
(3.30)

$$
\begin{aligned}
2(2\pi)^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)_{con} &= 2\left(\int_0^\infty x^{s-1}\ \cos 2\pi x\ dx\right)\left(2\sum_{m=1}^\infty \int_0^\infty u^{-s}\cos(2\pi mu)\ du\right) \\
&= 2\sum_{m=1}^\infty \int_0^\infty x^{s-1}\left(2\int\limits_0^\infty \cos(2\pi ux)\cos(2\pi mu)\ du\right)\ dx \\
&= 2\sum_{m=1}^\infty \int_0^\infty x^{s-1}\left(\int\limits_0^\infty [\cos 2\pi u(x+m)+\cos 2\pi u(x-m)]\ du\right)\ dx \\
&= 2\sum_{m=1}^\infty \int_0^\infty x^{s-1}\frac{1}{2}\times[\delta(x+m)+\delta(x-m)]\ dx \\
&= \sum_{m=1}^\infty \int\limits_0^\infty x^{s-1}\ \delta(x-m)\ dx \\
&= \zeta(1-s)_{dis}
\end{aligned}
$$

hence
(3.31)

$$
\begin{aligned}
2(2\pi)^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s) &= 2\sum_{n=1}^\infty \int_0^\infty x^{s-1}\cos(2\pi nx)\ dx + \sum_{m=1}^\infty \int\limits_0^\infty x^{s-1}\delta(x-m)\ dx \\
&= \zeta(1-s)_{con}+\zeta(1-s)_{dis} \\
&= \zeta(1-s)
\end{aligned}
$$

That is the expected result.
The Zeta function in this form will be named *Zeta-cosine form*, and their integrals the *Zeta integral-pair* or the *Zeta function-pair*.

Remark 3.1.
First: What is the difference between this form of $\zeta(s)$ and the origin one in 2.17? In fact, if we try to solve the integral of 2.17, in the complex domain it will be solved for the condition $|\cos(2\pi x)|<1$ and exclude all poles which exist for all the values of $x=n$.

Whereas in this formula, the poles are taken into account, and this is expressed by the delta function at $x = n$, that is the first integration in the right-hand-side.
Remark 3.2.
Second: Both integrals represent $\zeta(s)$ independently. So, the poles carry the same information as the other part of the function. In other words, the integral of $\zeta(s)$ can be solved for the values $|\cos(2\pi x)| = 1$ which represent the poles, and express it, as same as, the solution of the integral for the $|\cos(2\pi x)| < 1$.
Remark 3.3.
Third: The important remark here is that, there is a relation between the *Zeta function-pair* where, each of them is the *Fourier transform* of the other.

3.5. **The $\mathbb{Z}$eta Self-Operator.**

$\mathbb{Z}$eta self-operator is defined as the operator which converts $\zeta(s)$ to its conjugate $\zeta(1-s)$ and vise versa, and *"self"* because it is part from the $\mathbb{Z}$eta functional equation [5]. It will be denoted by $\mathcal{S}$, it takes two shapes, either $\mathcal{S}_s$ or $\mathcal{S}_{1-s}$. Their equations are

$$\begin{aligned} \mathcal{S}_s &= 2(2\pi)^{-s}\cos\left(\frac{\pi}{2}s\right)\Gamma(s) \qquad \text{or} \\ \mathcal{S}_{1-s} &= 2(2\pi)^{s-1}\sin\left(\frac{\pi}{2}s\right)\Gamma(1-s) \end{aligned} \tag{3.32}$$

They take the integral forms

$$\mathcal{S}_s = 2\int_0^\infty x^{s-1}\ \cos(2\pi x)\ dx \quad \text{(or)} \quad \mathcal{S}_{1-s} = 2\int_0^\infty x^{-s}\ \cos(2\pi x)\ dx \tag{3.33}$$

The important relations;

$$\zeta(1-s) = \mathcal{S}_s\zeta(s) \tag{3.34}$$

And for $\zeta(s) = \zeta(s)_{dis} + \zeta(s)_{con}$ in 3.25, we have

$$\mathcal{S}_s\zeta(s)_{dis} = \zeta(1-s)_{con} \quad \text{and} \quad \mathcal{S}_s\zeta(s)_{con} = \zeta(1-s)_{dis} \tag{3.35}$$

3.5.1. *The self-operator as a Unitary operator.*

The self-operator in general has a relation

$$\mathcal{S}_s\ \mathcal{S}_{1-s} = 1 \tag{3.36}$$

And for $s = \frac{1}{2} + it$ it will be *a unitary operator*, where;

$$\left(2\int_0^\infty x^{-\frac{1}{2}+it}\ \cos(2\pi x)\ dx\right)^* = 2\int_0^\infty x^{-\frac{1}{2}-it}\ \cos(2\pi x)\ dx \tag{3.37}$$

And

$$\left(2\int_0^\infty x^{-\frac{1}{2}+it}\ \cos(2\pi x)\ dx\right)^{-1} = 2\int_0^\infty x^{-\frac{1}{2}-it}\ \cos(2\pi x)\ dx \tag{3.38}$$

Hence

$$(\mathcal{S}_{\frac{1}{2}+it})^* = (\mathcal{S}_{\frac{1}{2}+it})^{-1} \tag{3.39}$$

---

[5] It can be also named the convert operator because it converts $\zeta(s)_{dis}$ to $\zeta(s)_{con}$ and vise versa,

In the case of unitary we will denote it by $\mathcal{S}$ symbol without subscribe.

4. Zeta function in the strip region

In this section we will start by proving that Zeta function in strip region is only defined for $\Re(s) = 1/2$ for two functions: first for the *Zeta-cosine* function ,and second in general for Zeta with conjugating function $f(x)$.
Then, going more deeply in Zeta function to explore its properties. Finally deducing a new formula *Zeta-sine* form which is used to prove the Riemann Hypothesis.

Theorem 4.1. *For $s \in \mathbb{C}$, we have*

(4.1)
$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s}\, \delta(x-n)\, dx + 2 \sum_{m=1}^{\infty} \int_0^{\infty} x^{-s} \cos(2\pi m x)\, dx \qquad 0 < \Re(s) < 1$$

is only defined for $\Re(s) = \frac{1}{2}$

Proof. As a result of 3.31 we get

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s}\, \delta(x-n)\, dx + 2 \sum_{m=1}^{\infty} \int_0^{\infty} x^{-s} \cos(2\pi m x)\, dx$$

(4.2) and

$$\zeta(1-s) = 2 \sum_{n=1}^{\infty} \int_0^{\infty} x^{s-1} \cos(2\pi n x)\, dx + \sum_{m=1}^{\infty} \int_0^{\infty} x^{s-1}\, \delta(x-m)\, dx$$

So, n and m are equivalent and therefore there is a term-wise equality between both functions of Zeta. Then we can write

$$\sum_{r=1}^{\infty} : \quad \int_0^{\infty} x^{-s}\, \delta(x-r)\, dx = 2 \int_0^{\infty} x^{-s} \cos(2\pi r x)\, dx \tag{4.3}$$

for $r = 1$ we get

$$\begin{aligned} \int_0^{\infty} x^{-s}\, \delta(x-1)\, dx &= 2 \int_0^{\infty} x^{-s} \cos(2\pi x)\, dx \\ 1^{-s} &= 2 \int_0^{\infty} x^{-s} \cos(2\pi x)\, dx \end{aligned} \tag{4.4}$$

The Left-hand-side will be equal 1 for any value of s. Whereas the right-hand-side is solved as $2(2\pi)^{-s} \cos(\frac{\pi s}{2})\Gamma(s)$ and this term is equal 1 only for $\Re(s) = 1/2$ □

Theorem 4.2. *For $s \in \mathbb{C}$,*

(4.5)
$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s}\, \delta(x-n)\, dx + 2 \sum_{m=1}^{\infty} \int_0^{\infty} x^{-s} \cos(2\pi m x)\, dx \qquad 0 < \Re(s) < 1$$

The combination

$$\left[\int_0^{\infty} x^{s-1} f(x)\, dx\right] \zeta(s) = \mathbb{F}(s)\zeta(s) \tag{4.6}$$

is only defined for $\Re(s) = \frac{1}{2}$ with condition $f(0) = \mathcal{F}(0)$. where $\mathcal{F}(x)$ and $\mathbb{F}(s)$ are the Fourier and the Mellin transforms respectively.

Proof. The approach here is to study the effect of the operators $\mathcal{S}_s\mathcal{S}_{1-s}$ on the combination $\mathbb{F}(s)\zeta(s)$, that will be the key of the proof.
By conjugating $\zeta(s)$ with $f(x)$ function in the form $\mathbb{F}(s)$, we get
(4.7)

$$\begin{aligned}
&\Big(\int_0^\infty x^{s-1} f(x)\, dx\Big)\zeta(s)\\
&\quad= \left(\int_0^\infty x^{s-1} f(x)\, dx\right)\left(\sum_{n=1}^\infty \int_0^\infty t^{-s}\, \delta(t-n)\, dt + 2\sum_{m=1}^\infty \int_0^\infty t^{-s} \cos(2\pi m t)\right)\, dt\\
&\quad= \int_0^\infty x^{s-1} \sum_{n=1}^\infty \int_0^\infty f(xt)\, \delta(t-n)\, dt\, dx + \int_0^\infty x^{s-1}\, 2\sum_{m=1}^\infty \int_0^\infty f(xt)\cos(2\pi m t)\, dt\, dx\\
&\quad= \int_0^\infty x^{s-1} \left[\sum_{n=1}^\infty f(nx)\right]\, dx + \int_0^\infty x^{s-1}\, \frac{1}{x}\left[\sum_{m=1}^\infty \mathcal{F}(\frac{m}{x})\right]\, dx
\end{aligned}$$

Now, we are going to study the effect of the term $(\mathcal{S}_s\mathcal{S}_{1-s}) = 1$ (the double self-operators, it will be considered as the identity operator) on $\mathbb{F}(s)\zeta(s)$ and proves that

$$(\mathcal{S}_s\mathcal{S}_{1-s})\mathbb{F}(s)\zeta(s) = \int_0^\infty x^{-s}\, \frac{1}{x}\left[\sum_{n=1}^\infty f(\frac{n}{x})\right]\, dx + \int_0^\infty x^{-s}\, \left[\sum_{m=1}^\infty \mathcal{F}(mx)\right]\, dx$$

We conclude that the identity operation makes a reciprocal process ($x \to 1/x$) for the integrands of the function $\mathbb{F}(s)\zeta(s)$. To prove that, we have

$$(\mathcal{S}_s\mathcal{S}_{1-s})\mathbb{F}(s)\zeta(s) = [\mathcal{S}_{1-s}\mathbb{F}(s)]\, [\mathcal{S}_s\, \zeta(s)]$$

$$\begin{aligned}
\mathcal{S}_s\, \zeta(s) &= \zeta(1-s)\\
&= 2\sum_{n=1}^\infty \int_0^\infty x^{s-1}\, \cos(2\pi n x)\, dx + \sum_{m=1}^\infty \int_0^\infty x^{s-1}\, \delta(x-m)\, dx
\end{aligned} \tag{4.8}$$

and

$$\begin{aligned}
\mathcal{S}_{1-s}\, \mathbb{F}(s) &= \left(2\int_0^\infty x^{-s}\, \cos(2\pi x)\, dx\right)\left(\int_0^\infty u^{s-1} f(u)\, du\right)\\
&= 2\int_0^\infty x^{-s} \int_0^\infty f(u)\cos 2\pi u x\, du\, dx\\
&= \int_0^\infty x^{-s}\mathcal{F}(x)\, dx
\end{aligned} \tag{4.9}$$

Then
(4.10)

$$\begin{aligned}
&[\mathcal{S}_{1-s}\mathbb{F}(s)]\, [\mathcal{S}_s\, \zeta(s)]\\
&\quad= \left(\int_0^\infty x^{-s}\mathcal{F}(x)\, dx\right)\left(2\sum_{n=1}^\infty \int_0^\infty t^{s-1}\, \cos(2\pi n t)\, dt + \sum_{m=1}^\infty \int_0^\infty t^{s-1}\, \delta(t-m)\, dt\right)\\
&\quad= \int_0^\infty x^{-s}\, \frac{1}{x}\left[\sum_{n=1}^\infty f(\frac{n}{x})\right]\, dx + \int_0^\infty x^{-s}\, \left[\sum_{m=1}^\infty \mathcal{F}(mx)\right]\, dx
\end{aligned}$$

Therefore x (and/or) $1/x$ are the same and exactly equivalent in the integral form of $\mathbb{F}(s)\, \zeta(s)$

Now, we are going proof the theory for $f(x) = e^{-\pi x^2}$, so we have

$$\begin{aligned} \mathbb{F}(s) &= \int_0^\infty x^{s-1} e^{-\pi x^2} = \frac{1}{2}\pi^{\frac{-s}{2}}\Gamma(\frac{s}{2}) \\ \mathcal{F}(e^{-\pi u^2}) &= 2\int_0^\infty e^{-\pi u^2} \cos(2\pi u x) du = e^{-\pi x^2} = f(x) \end{aligned} \tag{4.11}$$

Hence

$$\frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \int_0^\infty x^{s-1}\left(\sum_{n=1}^{\infty} e^{-\pi n^2 x^2}\right) dx + \int_0^\infty x^{s-1}\left(\sum_{m=1}^{\infty}\frac{1}{|x|} e^{-\frac{\pi m^2}{x^2}}\right) dx \tag{4.12}$$

By applying the identity operator, we get

$$\begin{aligned} (\mathcal{S}_s\mathcal{S}_{1-s})\left[\frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)\right] &= \int_0^\infty x^{-s}\left(\sum_{n=1}^{\infty}\frac{1}{|x|} e^{-\frac{\pi n^2}{x^2}}\right) dx + \int_0^\infty x^{-s}\left(\sum_{m=1}^{\infty} e^{-\pi m^2 x^2}\right) dx \\ &= \left[\frac{1}{2}\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)\right] \end{aligned} \tag{4.13}$$

This is maybe confused by the symmetric relation.

$$\left[\frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)\right] = \left[\frac{1}{2}\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)\right] \tag{4.14}$$

But,

$$(\mathcal{S}_s\mathcal{S}_{1-s})\left[\frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)\right] = \left[\frac{1}{2}\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)\right] \tag{4.15}$$

is an identity relation and in this case the variable s must be identically equal $1-s$, and that leads to $\Re(s) = 1/2$.

Another way:

The Poisson summation rule is

$$\sum_{n=-\infty}^{\infty} e^{-\pi n^2 x^2} = \frac{1}{x}\sum_{m=-\infty}^{\infty} e^{-\frac{\pi m^2}{x^2}} \tag{4.16}$$

Then,

$$1 + 2\sum_{n=1}^{\infty} e^{-\pi n^2 x^2} = \frac{1}{x}\, 2\sum_{m=1}^{\infty} e^{-\frac{\pi m^2}{x^2}} + \frac{1}{x} \tag{4.17}$$

Substituting in 4.12, then

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \int_0^\infty x^{s-1}\left(2\sum_{n=-\infty}^{\infty} e^{-\pi n^2 x^2} - 1\right) dx + \int_0^\infty x^{s-1}\left(\frac{2}{x}\sum_{m=-\infty}^{\infty} e^{-\frac{\pi m^2}{x^2}} - \frac{1}{x}\right) dx \tag{4.18}$$

Since, both integrals in the right-hand-side are equal, then we must have

$$\int_0^\infty x^{s-1}\, dx = \int_0^\infty x^{s-1}\,\frac{1}{x}\, dx \tag{4.19}$$

By setting $y \to 1/x$ in the right-hand-side integral, we get

$$\int_0^\infty x^{s-1}\, dx = \int_0^\infty y^{-s}\, dy \tag{4.20}$$

therefore the only solution is $\Re(s) = 1/2$.
Or by calculating the integrals, we have

$$\int_0^\infty x^{s-1}\, dx = \int_0^\infty x^{s-1}\, \frac{1}{x}\, dx \tag{4.21}$$

or

$$\begin{aligned} \lim_{a\to\infty} \int_{\frac{1}{a}}^{a} x^{s-1}\, dx &= \int_{\frac{1}{a}}^{a} x^{s-2}\, dx \\ \lim_{a\to\infty} \frac{x^s}{s}\Big|_{\frac{1}{a}}^{a} &= \frac{x^{s-1}}{s-1}\Big|_{\frac{1}{a}}^{a} \\ \lim_{a\to\infty} \frac{a^s - a^{-s}}{s} &= \frac{a^{s-1} - a^{-(s-1)}}{s-1} \\ \frac{s-1}{s} &= \lim_{a\to\infty} \left| \frac{a^{s-1} - a^{-(s-1)}}{a^s - a^{-s}} \right| \end{aligned} \tag{4.22}$$

By taking the largest power for both numerator and denominator in the region $0 < \Re(s) < 1$, then

$$\frac{s-1}{s} = \lim_{a\to\infty} \left| \frac{a^{s-1} - a^{-(s-1)}}{a^s - a^{-s}} \right| \rightsquigarrow \lim_{a\to\infty} \left| \frac{-a^{-(s-1)}}{a^s} \right| = -\lim_{a\to\infty} \left| a^{-2s+1} \right| \tag{4.23}$$

The value of the limit can't be ∞ or 0 because $s \neq 0$ *or* 1, therefore, there is only one solution which is $-2s + 1 = 0$, or $\Re(s) = \frac{1}{2}$.

Another approach

For the general function we have the Poisson general summation formula [4]

$$\frac{1}{2} f(0) + \sum_{n=1}^{\infty} f(nx) = \frac{1}{2}\frac{\mathcal{F}(0)}{x} + \sum_{m=1}^{\infty} \frac{1}{x}\mathcal{F}(\frac{m}{x}) \tag{4.24}$$

where

$$\begin{aligned} \mathcal{F}(x) &= 2\int_0^\infty \cos 2\pi x t\; f(t)\; dt \\ f(x) &= 2\int_0^\infty \cos 2\pi x t\; \mathcal{F}(t)\; dt \end{aligned} \tag{4.25}$$

By applying the formula for the kernel functions in 4.7, we get

$$\begin{aligned} \mathbb{F}(s)\zeta(s) = \int_0^\infty x^{s-1} \left[\sum_{n=1}^{\infty} f(nx) \; + \frac{1}{2}\int_0^\infty x^{s-1} f(0) \; - \frac{1}{2}\int_0^\infty x^{s-1} f(0) \right] dx + \\ \int_0^\infty x^{s-1} \left[\frac{1}{x}\sum_{m=1}^{\infty} \mathcal{F}(\frac{m}{x}) + \frac{1}{2}\int_0^\infty x^{-s}\frac{\mathcal{F}(0)}{x} \; - \frac{1}{2}\int_0^\infty x^{-s}\frac{\mathcal{F}(0)}{x} \right] dx \end{aligned} \tag{4.26}$$

Then, we get the condition

$$\frac{1}{2}\int_0^\infty x^{s-1}\; f(0)\; dx \;\; = \frac{1}{2}\int_0^\infty x^{s-1}\; \frac{\mathcal{F}(0)}{x}\; dx \tag{4.27}$$

We get the same relation 4.21, as well as, another condition $f(0) = \mathcal{F}(0)$ [6] . In this approach we will use this equality without solving it. According to the identity operation for $\mathbb{F}(s)\zeta(s)$ we have

(4.29)

$$
\begin{aligned}
(\mathcal{S}_s\mathcal{S}_{1-s}) \int_0^\infty x^{-s} \left[\sum_{n=1}^\infty f(nx)\right] dx &= \int_0^\infty x^{s-1} \left[\frac{1}{x}\sum_{n=1}^\infty f(\frac{n}{x})\right] dx \\
&= \int_0^\infty x^{s-1} \left[\sum_{m=1}^\infty \mathcal{F}(mx)\right] dx + \frac{1}{2}\int_0^\infty x^{s-1}\left(f(0) - \frac{\mathcal{F}(0)}{x}\right) dx \\
&= \int_0^\infty x^{s-1} \left[\sum_{m=1}^\infty \mathcal{F}(mx)\right] dx \\
&= \int_0^\infty x^{-s} \left[\sum_{m=1}^\infty \frac{1}{x}\,\mathcal{F}(\frac{m}{x})\right] dx
\end{aligned}
$$

Where, the integral $\int_0^\infty x^{s-1}\left(f(0) - \frac{\mathcal{F}(0)}{x}\right) dx = 0$ since 4.27.

By conjugate $\mathbb{F}(s)\zeta(s)$ by only one $\mathcal{S}_{1-s}$ operator, then

(4.30)

$$
\begin{aligned}
&\left(2\int_0^\infty x^{-s}\cos 2\pi x\, dx\right)\left(\int_0^\infty u^{s-1}\sum_{n=1}^\infty f(nu)\, du\right) \\
&= \sum_{n=1}^\infty 2\int_0^\infty x^{-s} \int_0^\infty \cos(2\pi ux) f(nu)\, du\, dx \quad \text{(by substitution } y = \frac{x}{n^2}\text{, then)} \\
&= \sum_{n=1}^\infty 2\int_0^\infty y^{-s}\, n^{(-2s+2)} \int_0^\infty \cos(2\pi n^2 uy) f(nu)\, du\, dy \quad \text{(by substitution } nu = v\text{)} \\
&= \sum_{n=1}^\infty 2\int_0^\infty y^{-s}\, n^{(-2s+1)} \int_0^\infty \cos(2\pi nyv)\, f(v)\, dv\, dy \quad \text{(Fourier for Gaussian see 4.11)} \\
&= \int_0^\infty y^{-s}\sum_{n=1}^\infty n^{(-2s+1)}\mathcal{F}(ny)\, dy \\
&= \int_0^\infty w^{s-1}\sum_{n=1}^\infty n^{(-2s+1)}\,\frac{1}{w}\mathcal{F}(\frac{n}{w})\, dw
\end{aligned}
$$

Since,

$$\int_0^\infty u^{s-1}\sum_{n=1}^\infty f(nu)\, du = \int_0^\infty u^{s-1}\,\frac{1}{u}\sum_{n=1}^\infty \mathcal{F}(\frac{n}{u})\, du$$

Hence, to achieve this equality, the only solution is $\Re(s) = 1/2$, where $n^{(-2s+1)} = 1$, and $2\int_0^\infty x^{-s}\cos 2\pi x\, dx = 1$.

---

[6]this condition is equivalent to

$$\int_0^\infty \mathcal{F}(x)\, dx = \int_0^\infty f(x)\, dx \tag{4.28}$$

Remark 4.3.
The important note here is that, this proof depends on both functions for Zeta. It is the tied relation between them, this connection enforces both integrals to be defined only in $\Re(s) = 1/2$. So we get this value because both side should be equal each other, not because of solving the equation or taking into account the conditions of convergence. So the source of the hardness of this problem was the trying to solve it by one leg. Which means by one integral, and that is the difference between Zeta function in this new form and all other forms.

□

Corollary 2. *For $s = 1/2 \pm t$ in the $\zeta(s)$ the self-operator $\mathcal{S}_s$ is always equal* 1 *regardless the t value.*

Proof. Since Zeta function is term-wise equality, then for the first term $(n = m = 1)$ we have

$$\int_0^\infty x^{-1/2+it}\,\delta(x-1)\,dx = 2\int_0^\infty x^{-1/2+it}\cos(2\pi x)\,dx$$

Since the right-hand-side is equal to $\mathcal{S}_{1/2+it}$ and the left-hand-side will be solved as $(1)^{-1/2}\,(1)^{+it} = 1$ regardless the t value, this result should be the same for the right-hand-side. □

4.1. **The relation between Zeta function-pair domains.**

In 3.25, we say that the x variable in the kernel functions is represented by two ways $x = [x]$ and $x = \{x\}$ for the first and second integrals respectively, so these variables represent the domains of Zeta in its integrals. We will prove that these domains span two-dimensions space equivalent to a complex plane. In other words, if $[x] \equiv x$, then $\{x\} \equiv ix$ and vise versa, but this shape of equivalence is right only for $\Re(s) = 1/2$.
To proof that,
We started the proof in theorem 2.1 by the equation

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty x^{s-1}\sum_{n=1}^{\infty} e^{-nx}\,dx \tag{4.31}$$

At the end we get the function

$$\zeta(s) = \frac{2^{s-1}}{s-1} + \int_{\frac{1}{2}}^\infty x^{-s}\,2\sum_{m=1}^{\infty}\cos 2\pi m x\,dx \tag{4.32}$$

As mentioned before, this function can be re-written by two different ways,
First:

$$\begin{aligned}\zeta(s) &= \int_{\frac{1}{2}}^\infty x^{-s}\times(1)\,dx + \sum_{n=1}^{\infty}\int_{\frac{1}{2}}^\infty x^{-s}\,\delta(x-n)\,dx - \int_{\frac{1}{2}}^\infty x^{-s}\times(1)\,dx\\ &= \sum_{n=1}^{\infty}\int_{\frac{1}{2}}^\infty x^{-s}\,\delta(x-n)\,dx \quad \text{or}\\ &= \sum_{n=1}^{\infty}\int_0^\infty x^{-s}\,\delta(x-n)\,dx\end{aligned} \tag{4.33}$$

Therefore, for $\Re(s) = 1/2$, we get

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-1/2-it}\, \delta(x-n)\, dx \quad = \frac{1}{\sqrt{\pi}} \int_0^{\infty} x^{-1/2+it} \sum_{n=1}^{\infty} e^{-nx}\, dx \qquad (x = [x]) \tag{4.34}$$

Second, as in the corollary 1, where

$$\zeta(s) = \int_0^{\infty} x^{-s}\, 2 \sum_{m=1}^{\infty} \cos(2\pi m x)\, dx \qquad (x = \{x\})$$

by substituting $x = (2\pi)(iy)$, then

$$\begin{aligned}
\zeta(s) &= 2\mathbf{Re} \int_0^{\infty} x^{-s} \sum_{m=1}^{\infty} e^{i2\pi m x}\, dx \qquad (|x| < 1)\\
&= 2^s\, \pi^{s-1} \sin(\frac{\pi s}{2}) \int_0^{i\infty} y^{-s} \sum_{m=1}^{\infty} e^{-my}\, dy\\
&= 2^s\, \pi^{s-1} \sin(\frac{\pi s}{2}) \int_0^{\infty} y^{-s} \sum_{m=1}^{\infty} e^{-my}\, dy \qquad (\text{and for } \Re(s) = 1/2)\\
&= \frac{1}{\sqrt{\pi}} \int_0^{\infty} y^{-\frac{1}{2}-it} \sum_{m=1}^{\infty} e^{-my}\, dy \qquad (x = \{x\} = 2\pi i y)
\end{aligned}$$

So we get the same form in 4.34.
Since *Zeta-cosine* consists of the two functions, therefor it is defined into their dimensions which are similar to the complex plane, so they are orthonormal.

4.2. **The reciprocal x in the Zeta function.**

It is obvious from the previous theory [4.2] that there is an equivalence between the operation of the double self-operators and the reciprocal x in the 4.15 where [7]

$$(\mathcal{S}_s \mathcal{S}_{1-s})\xi(s) = \xi(1-s) \tag{4.35}$$

In Zeta function we only have the relation with one self-operator $\mathcal{S}_s\zeta(s) = \zeta(1-s)$. In this section, we will try to simulate the operator process by reciprocal x in $\zeta(s)$ to get $\zeta(1-s)$. We will study $\zeta(s)$ for $s \in \Re$, and setting $x \to 1/x$ in $\zeta(s)$, and noticing that $\zeta(k) \to \zeta(1-k)$, $k = 0, 1, 2, \ldots$. In addition this will give us a clue a bout the meaning of the $\zeta(-k)$ values.
One of an interesting paper that gives an idea about the values of $\zeta(-k)$ is the Minac paper [see [9]]. He devised a new method to calculate $\zeta(-k)$ using a trick. The success of this method is one of the $\mathbb{Z}$eta function puzzles, no explanation has been provided yet for this method. This method shads some light on the meaning of these values. It can be summarized as follows;
The Bernoulli's polynomials is defined as [13]

$$\begin{aligned}
S_m(n) = 0^m + 1^m + \ldots(n-1)^m &= \sum_{k=0}^{n-1} k^m\\
&= \frac{B_{m+1}(n) - B_{m+1}}{m+1}
\end{aligned} \tag{4.36}$$

[7] $\frac{\xi(s)}{s(s-1)} = \frac{1}{2}\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)$.

Now, if we replace the argument n in $S_m(n)$ by a variable x and then the corresponding function is integrated over the interval $[0,1]$, we get
(4.37)

$$1 + ... + 1 = n - 1 \rightsquigarrow x - 1 \rightsquigarrow \int_0^1 (x-1)\, dx = -\frac{1}{2} = \zeta(0)$$

$$1 + ... + (n-1) = \frac{n(n-1)}{2} \rightsquigarrow \frac{x(x-1)}{2} \rightsquigarrow \int_0^1 \frac{x(x-1)}{2}\, dx = -\frac{1}{12} = \zeta(-1)$$

....

$$S_m(n) = 0^m + 1^m + ...(n-1)^m \rightsquigarrow S_m(x) \rightsquigarrow \int_0^1 S_m(x)\, dx = -\frac{B_{m+1}}{m+1} = \zeta(-m)$$

The puzzle here is how to explain this converting of $n \to x$ then taking the integral from $0 \to 1$ to get $\zeta(-k)$ values ?

For this study, we will take $\zeta(s)_{dis}$ form, the limits of the integral will be from $1 \to \infty$, because delta function originally starting from $n = 1$, so we have

$$\zeta(s) = \sum_{n=1}^{\infty} \int_1^{\infty} x^{-s}\, \delta(x-n)\, dx \qquad n \neq 0 \tag{4.38}$$

By solving this integral for $(s = k), k = 0, 1, 2...$ in direct way, we get $\sum\limits_{n=1}^{\infty} n^{-k}$, this is the Zeta regular series which gives its regular values.
Now, by setting $x \to \frac{1}{x}$, then

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^1 x^{s-2} \delta(\frac{1}{x} - n)\, dx$$

To calculate $\delta(\frac{1}{x} - n)$ we will use the rule of The Heaviside step function $H(x)$.

$$H(g(x)) = \begin{cases} H(x-x_1) - H(x-x_2) + ... - (-)^n H(x-x_n), & g'(x_1) > 0 \\ 1 - \{H(x-x_1) - H(x-x_2) + ... - (-)^n H(x-x_n)\}, & g'(x_1) < 0 \end{cases}$$

where

$$g(x) = g_0(x-x_1)(x-x_2)...(x-x_n)$$

with $x_1 < x_2 < ... < x_n$. Starting with Heaviside step function $H(\frac{1}{x} - n)$ for $g(x) = \frac{1}{x}$

$$H(\frac{1}{x} - n) = 1 - H(x - \frac{1}{n})$$

by differentiation entails

$$\frac{-1}{x^2}\delta(\frac{1}{x} - n) = -\delta(x - \frac{1}{n})$$
$$\delta(\frac{1}{x} - n) = x^2 \delta(x - \frac{1}{n})$$

hence

$$\begin{aligned} \zeta(s) &= \sum_{n=1}^{\infty} \int_0^1 x^{s-2}\, x^2 \delta(x - \frac{1}{n})\, dx \\ &= \sum_{n=1}^{\infty} \int_0^1 x^s \delta(x - \frac{1}{n})\, dx \end{aligned} \tag{4.39}$$

We need a trick here, we will consider $\delta(x-\frac{1}{n})$ as a function $f(x,n)$, as in general case, so we have two operations: summation and integration, and in this case $\delta(x-\frac{1}{n})$ function will be as a switch between x and $\frac{1}{n}$ variable. We will take the summation first, then

$$\begin{aligned}\zeta(k) &= \int_0^1 \sum_{n=1}^{\infty} x^k \delta(x-\frac{1}{n})\,dx \\ &= \int_0^1 \sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^k \delta(x-\frac{1}{n})\,dx \\ &= \lim_{N\to\infty} \int_0^1 \sum_{n=1}^{N} \left(\frac{1}{n}\right)^k \delta(x-\frac{1}{n})\,dx\end{aligned} \tag{4.40}$$

Here is the idea, we will consider $\left(\frac{1}{n}\right)$ as a fraction element $0 \le \{u_n\} < 1$, so the fraction starts from 0 to ∞, hence the counter n will change the staring count to be from $0 \to N-1$ ($N \to \infty$). $\zeta(k)$ in the left-hand-side is unknown because we don't know which value will be expressed, so we will write it as $\zeta(?)$, therefore

$$\begin{aligned}\zeta(?) &= \lim_{N\to\infty} \int_0^1 \sum_{u_n=0}^{N-1} (u_n)^k \delta(x-u_n)\,dx \\ &= \lim_{N\to\infty} \int_0^1 S_k(N)\,\delta(x-N)\,dx \\ &= \int_0^1 S_k(x)\,dx \\ &= -\frac{B_{k+1}}{k+1} \\ &= \zeta(?)\end{aligned} \quad , \tag{4.41}$$

There is an important remark. In the last step, if $k=2$ then, we have $\zeta(2)$, and its corresponding value will be $\zeta(-1)$, but the last equation gives us $-\frac{B_{2+1}}{2+1}$ which is the value of $\zeta(-2)$, and that means k in the last equation should be shifted by 1 to be $k-1$, in other words, we have to do this

$$\begin{aligned}\zeta(?) &= \lim_{N\to\infty} \int_0^1 \sum_{u_n=0}^{N-1} (u_n)^{k-1} \delta(x-u_n)\,dx \\ &= \lim_{N\to\infty} \int_0^1 S_{k-1}(N)\,\delta(x-N)\,dx \\ &= \int_0^1 S_{k-1}(x)\,dx \\ &= -\frac{B_k}{k} \\ &= \zeta(1-k)\end{aligned} \quad , \tag{4.42}$$

Therefore, two things should be modified when $\left(\frac{1}{n}\right)$ is considered as a fraction: First, shifting the counter to start from 0 instead 1 and secondly, $k \to k-1$.
So, we conclude that $\mathbb{Z}$eta function reveals another type of a conjugate relation similar to the conjugation of the complex number, which means for σ there is

$\bar{\sigma} = 1 - \sigma$, or for $s = \sigma + it$ there is $\bar{s} = \bar{\sigma} - it$, and the equivalence $s \equiv \bar{s}$ is achieved in Zeta function form

$$\mathbb{F}(s)\zeta(s) = \int_0^\infty x^{-s} \sum_{n=1}^{\infty} f(nx)\ dx + \int_0^\infty x^{-\bar{s}} \sum_{m=1}^{\infty} \mathcal{F}(mu)\, du \tag{4.43}$$

But this equivalent is only in the strip region $0 < \Re(s) < 1$.
Remark 4.4.
Here, we notice that $(1-\sigma)$ is taken as one variable, which leads to the conclusion that the point $(\sigma = 1/2)$ is mostly a virtual point.

4.3. **The flipping and the loop in the Zeta function.**
We can summarize this part by the two equations
(4.44)

$$\mathbb{F}(s)\zeta(s) = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{+it} \left[\sum_{r=1}^{\infty} f(rx)\right] dx + \int_0^\infty \frac{1}{\sqrt{x}}\, x^{+it} \left[\sum_{r=1}^{\infty} \mathcal{F}\left(\frac{r}{x}\right)\right] dx$$
$$\mathcal{S}\mathbb{F}(s)\zeta(s) = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it} \left[\sum_{r=1}^{\infty} \mathcal{F}(rx)\right] dx + \int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it} \left[\sum_{r=1}^{\infty} f\left(\frac{r}{x}\right)\right] dx$$

The $\mathcal{S}$ operator makes two things, flipping the sign of the imaginary variable t and transform each function to its Fourier form. This transform makes an exchange among their variables.

Since, they are defined into two perpendicular domains similar to the complex plane, then the effect of the operator on the Zeta function-pair will make them as a moving in a closed path. This idea is illustrated in the Figure 1.
By applying the operator again we get the first form. So we get a complete loop by $\mathcal{S}^2$ processes.
Maybe the proving of the previous theory is enough to prove the Riemann hypothesis, since $\mathbb{Z}$eta is defined only for $\Re(s) = 1/2$, then all zeros will be there. But in the next section we will prove that there is a big zero for $\mathbb{Z}$eta in $\Re(s) = 1/2$, that requires a new form for $\mathbb{Z}$eta.

4.4. **Zeta-sine form.**
In this part we're going to study another form for $\zeta(s)$. It will be named the *Zeta-sine* form which is opposite to *Zeta-cosine* form in 3.25. This form has an advantage than the *Zeta-cosine* because it convergences clearly in the strip region.

Lemma 4.5. *For $s \in \mathbb{C}$ we have*

$$\zeta(s) = \frac{2^{s-1}}{s-1} + \int_{\frac{1}{2}}^\infty x^{-s}\, 2 \sum_{m=1}^{\infty} \cos 2\pi m x\, dx$$

Proof that

$$\zeta(s) = -s \int_0^\infty x^{-s-1} \left(\frac{1}{2} - 2\sum_{m=1}^{\infty} \frac{\sin 2\pi m x}{2\pi m}\right) dx \qquad 0 < \Re(s) < 1$$

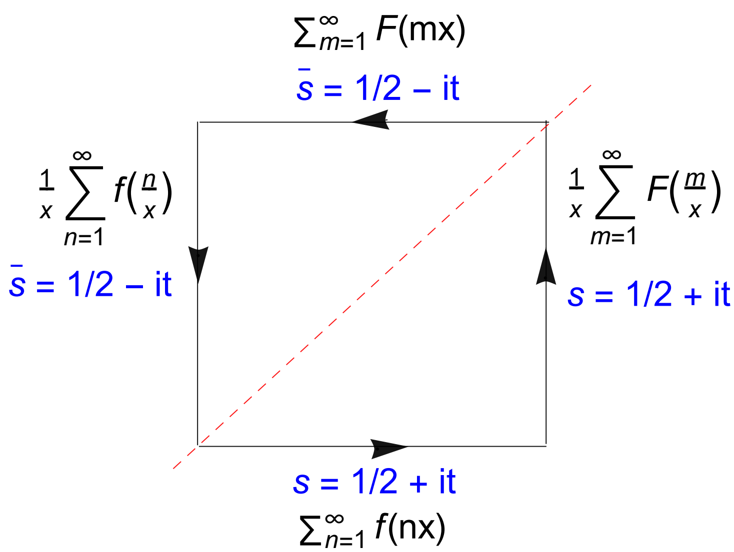

FIGURE 1. The closed loop in Zeta function

Proof.
Since

$$\zeta(s) = \frac{2^{s-1}}{s-1} + \int_{\frac{1}{2}}^{\infty} x^{-s}\, 2\sum_{m=1}^{\infty} \cos 2\pi m x \, dx$$

$$\zeta(s) = \frac{2^{s-1}}{s-1} + \left(x^{-s}\, 2\sum_{m=1}^{\infty} \frac{\sin 2\pi m x}{2\pi m}\Big|_{\frac{1}{2}}^{\infty} + s\int_{\frac{1}{2}}^{\infty} x^{-s-1}\, 2\sum_{m=1}^{\infty} \frac{\sin 2\pi m x}{2\pi m}\, dx \right)$$

we have (see [1])

$$2\sum_{m=1}^{\infty} \frac{\sin 2\pi m x}{2\pi m} = \frac{1}{2} - x \qquad 0 < x < 1$$

The series is bounded, and equal to zero at $x = \frac{1}{2}$, hence

$$\begin{aligned}\zeta(s) &= \frac{2^{s-1}}{s-1} + s\int_{\frac{1}{2}}^{\infty} x^{-s-1}\, 2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}\,dx \qquad \Re(s) > 0\\ &= \frac{2^{s-1}}{s-1} + \left(\frac{1}{2}\right)^{-s+1} - \left(\frac{1}{2}\right)^{-s+1} + s\int_{\frac{1}{2}}^{\infty} x^{-s-1}\, 2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}\,dx\\ &= \left(\frac{2^{s-1}}{s-1} + 2^{s-1}\right) - s\int_{\frac{1}{2}}^{\infty} x^{-s-1}\left(\frac{1}{2} - 2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}\right)dx\\ &= \left(\frac{s}{s-1}2^{s-1}\right) - s\int_{\frac{1}{2}}^{\infty} x^{-s-1}\left(\frac{1}{2} - 2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}\right)dx\end{aligned}$$

Since,

(4.45)

$$\begin{aligned}-s\int_{0}^{\frac{1}{2}} x^{-s-1}\left(\frac{1}{2} - 2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}\right)dx &= -s\int_{0}^{\frac{1}{2}} x^{-s-1}\left(\frac{1}{2} - \left[\frac{1}{2} - x\right]\right)dx\\ &= \left(\frac{s}{s-1}2^{s-1}\right) \qquad \Re(s) < 1\end{aligned}$$

Then the equation follow.

□

The kernel function is expressed as

$$2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m} = \frac{1}{2} - x \qquad 0 < x < 1$$

The compact form of this summation, for a long x, has a periodic property (Figure 2), and the function in this case can be represented by two ways. First

$$2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m} = \frac{1}{2} - (x - n) \qquad n < x < n+1 \tag{4.46}$$

If we solve the integration for this form, we get

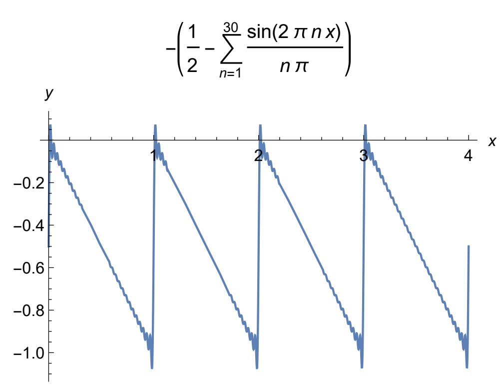

FIGURE 2. The real representation of the *Zeta-sine* kernel

$$\begin{aligned}\zeta(s) &= -s\int_0^\infty x^{-s-1}\ x\ dx + s\sum_{n=1}^\infty n\int_n^{n+1} x^{-s-1}\ \ dx \\ &= -s\int_0^\infty x^{-s-1}\ x\ dx + \sum_{n=1}^\infty n\left(\frac{1}{n^s} - \frac{1}{(n+1)^s}\right)\end{aligned} \tag{4.47}$$

The second integral represents $\zeta(s)$ in its series form and converge, as usual, for $\Re(s) > 1$. These terms in this representation are encoded by the periodic property of the ramp function x. Whereas, the first integral “officially“ equal zero [10.5 Edwards [3]]. But in fact, this integral is the same as the integral corresponding to the integral $\int_0^\infty x^{-s}\,(-1)\,dx$ in 3.1, it is the responsible term for conversion $\zeta(s)$ in the strip region $0 < \Re(s) < 1$.
Second: The other approach will encode this periodicity by using the unit step function as an auxiliary function (figure 3) , taking the form

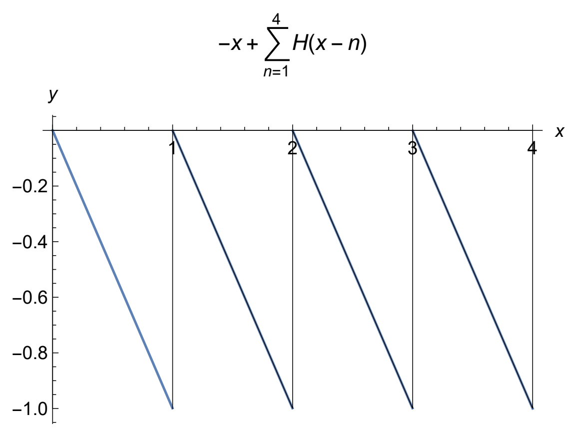

Figure 3. The approximate shape, here the periodicity is represented by using the unit step function.

(4.48)

$$\begin{aligned}\zeta(s) &= -s\int_0^\infty x^{-s-1}\ \left(\frac{1}{2} - 2\sum_{m=1}^\infty \frac{\sin 2\pi m x}{2\pi m}\right) dx \qquad 0 < \Re(s) < 1 \\ &= -s\sum_{n=1}^\infty\int_0^\infty x^{-s-1}\ \left(\frac{1}{2} - \left[(\frac{1}{2} - x) + H(x-n))\right]\right) dx \qquad n \le x \le (n+1) \\ &= -s\int_0^\infty x^{-s}\ dx + s\ \sum_{n=1}^\infty\int_n^{n+1} x^{-s-1} H(x-n)\ dx \\ &= -s\int_0^\infty x^{-s}\ dx + s\ \sum_{n=1}^\infty\int_n^\infty x^{-s-1}\ dx \\ &= -s\int_0^\infty x^{-s}\ dx + \sum_{n=1}^\infty n^{-s}\end{aligned}$$

Again, the first integral “officially” equal zero, and the next integral is the $\zeta(s)$ series.

We notice that, *Zeta-sine* form, in either way, gives one series representation for $\zeta(s)$ whereas, in its corresponding *Zeta-cosine*, both integrals express this series. So, we claim that the approximation form of *Zeta-sine* is not complete or there is a missing part ,and that what we are going to reveal.

4.4.1. *A new representation for Zeta sine form.*
The series $\sum_{m=1}^{\infty}\frac{\sin mx}{m}$ has been studied many times under the name of the **Gibbs phenomenon** [5], [2], [1]. All these studies defined this series as follows,
For the interval $(0,2\pi)$, we have

$$f(x)=\sum_{m=1}^{\infty}\frac{\sin mx}{m}=\begin{cases} f(0)=f(2\pi)=0, & x=0,2\pi \\ f(x)=\frac{1}{2}(\pi-x) & 0<x<2\pi \end{cases}$$

Or

$$2\sum_{m=1}^{\infty}\frac{\sin 2\pi mx}{2\pi m}=\begin{cases} 0, & x=0,1 \\ \\ \frac{1}{2}-x & 0<x<1 \end{cases} \tag{4.49}$$

By comparing the *Zeta-sine* function with *Zeta-cosine*, there is a missing part which is corresponding to $\delta(x-n)$ function in $2\sum_{m=1}^{\infty}\cos 2\pi mx$. To illustrate that, first: We will derive $\sum_{m=1}^{\infty}\frac{\sin mx}{m}$ as a power series under the condition $|e^{ix}|<1$, hence

$$\begin{aligned}\sum_{m=1}^{\infty}\frac{\sin mx}{m} &= \mathbf{Im}\left(\sum_{m=1}^{\infty}\frac{e^{imx}}{m}\right)\\ &= \mathbf{Im}\left(-\ln(1-e^{ix})\right)\\ &= -\mathbf{Im}\left(\ln(-e^{i\frac{x}{2}})(e^{i\frac{x}{2}}-e^{-i\frac{x}{2}})\right)\\ &= -\mathbf{Im}\left(\ln(-2i\sin(\frac{x}{2}))(e^{i\frac{x}{2}})\right)\\ &= -\mathbf{Im}\left(\ln(2\sin(\frac{x}{2}))+\ln(e^{-\frac{\pi}{2}}e^{i\frac{x}{2}})\right)\\ &= -\mathbf{Im}\left(\ln(2\sin(\frac{x}{2}))+\ln(e^{-\frac{1}{2}(\pi-x)})\right)\\ &= \frac{1}{2}(\pi-x) \qquad\qquad |e^{ix}|<1\end{aligned} \tag{4.50}$$

now, if we want to derive $\sum_{m=1}^{\infty}\cos mx$ from $\sum_{m=1}^{\infty}\frac{\sin mx}{m}$, since

$$\begin{aligned}\sum_{m=1}^{\infty}\cos mx &= \frac{d}{dx}\sum_{m=1}^{\infty}\frac{\sin mx}{m}\\ &= \frac{d}{dx}\mathbf{Im}\left(\sum_{m=1}^{\infty}\frac{e^{imx}}{m}\right)\\ &= \frac{d}{dx}\mathbf{Im}\left(-\ln(1-e^{ix})\right)\\ &= \frac{d}{dx}\left(\frac{1}{2}(\pi-x)\right)\\ &= -\frac{1}{2}\end{aligned} \tag{4.51}$$

under the same condition $|e^{ix}| < 1$, we want to estimate $\sum_{m=1}^{\infty} \cos mx$, hence

$$\sum_{m=1}^{\infty} \cos mx = \mathbf{Re}\left(\sum_{m=1}^{\infty} e^{imx}\right) = \mathbf{Re}\left(\frac{e^{ix}}{1-e^{ix}}\right) = -\frac{1}{2} \qquad |e^{ix}| < 1 \tag{4.52}$$

That means the formula of $\sum_{m=1}^{\infty} \frac{\sin mx}{m} = \frac{1}{2}(\pi - x)$ corresponds only to the term $(-\frac{1}{2})$ in the $\sum_{m=1}^{\infty} \cos mx$.
Second: If we estimate $\sum_{m=1}^{\infty} \frac{\sin mx}{m}$ from $\sum_{m=1}^{\infty} \cos mx$ but for all x values, as in 3.15, 3.16 and 3.17, we get

$$\begin{aligned}\sum_{m=1}^{N} \frac{\sin mx}{m} &= \int \sum_{m=1}^{N} \cos mx \, dx \\ &= \int \mathbf{Re}\left(\frac{e^{i(N+1)x}}{e^{ix}-1}\right) dx + \int \mathbf{Re}\left(\frac{e^{ix}}{e^{ix}-1}\right) dx\end{aligned}$$

for $(N \to \infty)$

$$\lim_{N\to\infty} \sum_{m=1}^{N} \frac{\sin mx}{m} = \lim_{N\to\infty} \int \frac{\sin\left((N+\frac{1}{2})x\right)}{2\sin\frac{1}{2}x} dx + \frac{1}{2}(\pi - x) \tag{4.53}$$

The integral in the right-hand-side represents $\int \delta(x)$ and gives a unit step function $H(x)$, but precisely, it is not jumping between two points with nothing between them, but it is a continuous function that is defined for a very narrow interval because as a result of "Riemann-Lebesgue lemma"[8] the integral in 4.53 in right-hand-side is vanished except at the points that the function $\dfrac{1}{2\sin\left(\frac{1}{2}x\right)}$ is not continuous.
In other words, if we calculate the integral for $-\pi < x < \pi$, we get

$$\int_{-\pi}^{\pi} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx = \int_{-\pi}^{-\epsilon} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx + \int_{-\epsilon}^{+\epsilon} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx + \int_{+\epsilon}^{\pi} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx \tag{4.54}$$

Since the $\frac{1}{\sin\left(\frac{1}{2}x\right)}$ is continuous for all interval $(-\pi, \pi)$ except at $x = 0$, hence, the first and the third terms in the right-hand-side equal zero, then

$$\int_{-\pi}^{\pi} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx = \int_{-\epsilon}^{+\epsilon} \frac{\sin\left((N+\frac{1}{2})x\right)}{\sin\left(\frac{1}{2}x\right)} dx \tag{4.55}$$

It can be described as a function

$$g(x) = \frac{1}{2\epsilon}x \qquad -\epsilon \le x \le +\epsilon \quad (\epsilon << 1) \tag{4.56}$$

And for the function $\left(\frac{1}{2} - 2\sum_{m=1}^{\infty} \frac{\sin 2\pi mx}{2\pi m}\right)$ the $g(x)$ function will be as a vertical line $\mathbf{1}_n(x)$, and defined only for $x = n$ as

$$\mathbf{1}_n(x-n) \begin{cases} 1, & x = n \quad \text{(n=1,2...)}, \\ 0, & x \neq n \end{cases} \tag{4.57}$$

[8]For $f(x)$ is a Riemann integrable on $[a, b]$, then

$$\lim_{N\to\infty} \int_a^b f(x)\cos(Nx)\,dx = 0 \,, \quad \lim_{N\to\infty} \int_a^b f(x)\sin(Nx)\,dx = 0 \,, \quad \text{and} \lim_{N\to\infty} \int_a^b f(x)e^{iNx}\,dx = 0$$

This result could be concluded by another way, by making integral by parts for 3.25 then we get,

(4.58)

$$\begin{aligned}\zeta(s) &= \sum_{n=1}^{\infty}\int_0^{\infty} x^{-s}\,\delta(x-n)\,dx + 2\sum_{m=1}^{\infty}\int_0^{\infty} x^{-s}\cos(2\pi m x)\,dx\\ &= \sum_{n=1}^{\infty} x^{-s}H(x-n)\Big|_n^{n+1} - s\int_0^{\infty} x^{-s-1}\,H(x-n)\,dx + 2\sum_{m=1}^{\infty} x^{-s}\frac{\sin 2\pi m x}{2\pi m}\Big|_0^{\infty} - s\int_0^{\infty} x^{-s-1}\frac{\sin 2\pi m x}{2\pi m}\,dx\\ &= -s\left[\sum_{n=1}^{\infty}\int_0^{\infty} x^{-s-1}\,H(x-n)\,dx + \sum_{m=1}^{\infty}\int_0^{\infty} x^{-s-1}\frac{\sin 2\pi m x}{\pi m}\,dx\right]\end{aligned}$$

Where, $x^{-s}\frac{\sin 2\pi m x}{2\pi m}\Big|_0^{\infty} = 0$ for $0 < \Re(s) < 1$, and $H(x-n)$ is the step function defined as

$$H(x-n) = \begin{cases} 1, & x \leq n\\ 0, & x > n\end{cases} \tag{4.59}$$

By re-writing the function as

$$\zeta(s) = -s\left[\sum_{n=1}^{\infty}\int_0^{\infty} x^{-s}\,\frac{H(x-n)}{x}\,dx + \sum_{m=1}^{\infty}\int_0^{\infty} x^{-s}\frac{\sin 2\pi m x}{\pi m x}\,dx\right] \tag{4.60}$$

Then applying the self-operator $\mathcal{S}_s$ we get

$$\begin{aligned}\mathcal{S}_s\frac{\zeta(s)}{-s} = &\sum_{n=1}^{\infty}\int_0^{\infty} x^{s-1}\left(2\int_0^{\infty}\cos(2\pi u x)\frac{H(u-n)}{u}\,du\right)\,dx +\\ &\sum_{m=1}^{\infty}\int_0^{\infty} x^{s-1}\left(2\int_0^{\infty}\cos(2\pi u x)\,\frac{\sin 2\pi m u}{\pi m u}\,du\right)\,dx\end{aligned} \tag{4.61}$$

The Fourier transform in the second integral

$$2\int_0^{\infty}\cos(2\pi u x)\,\frac{\sin 2\pi m u}{\pi m u}\,du = \frac{H(x-m)}{m}$$

According to the first integral, this result should equal $\frac{H(x-m)}{x}$ which means, this step function is only defined for $x = m$, and therefore it takes the same representation of 4.57.

This function is periodic as same as the ramp function x, but does not belong to its domain. We will describe these domains as $x-y$ plan and define Zeta as follows;

It is a periodic function, each period can be described as a cell, one cell for a period, each cell consists of two functions in two different domains, from left-to-right, in $x-y$ plan , the ramp function x moves from $n \to n+1$ by a -ve slope, it goes down gradually then goes up vertically on the $\mathbf{1}_n$ function at $n+1$ in the +ve y-axis. We will consider that the y-axis is divided into periods as the same as x-axis which is $[n, n+1]$, and the function $\mathbf{1}_n$ is varying on the +ve y-axis from n to $n+1$. So the complete period in each cell will be achieved in both x and y domains. Now, because of these two functions are defined into two different domains, then each one of them should independently represents $\zeta(s)$ separately in its own domain. The function will take the form

$$\zeta(s) = -s \int_0^\infty x^{-s-1} \left(\frac{1}{2} - 2 \sum_{m=1}^\infty \frac{\sin 2\pi m x}{2\pi m} \right) dx \qquad 0 < \Re(s) < 1$$
$$= -s \sum_{n=1}^\infty \int_0^\infty x^{-s-1} [\frac{1}{2} - (\frac{1}{2} - (x-n))\, dx + s \sum_{n=1}^\infty \int_0^\infty y^{-s-1} (\mathbf{1}_n - n)\, dy$$

Then
(4.62)

$$\zeta(s) = \left[s \sum_{n=1}^\infty n \int_n^{n+1} x^{-s-1}\, dx - s \int_0^\infty x^{-s-1}\, x\, dx \right] + \left[s \int_0^\infty y^{-s-1}\, \mathbf{1}_n\, dy - s \sum_{n=1}^\infty n \int_n^{n+1} y^{-s-1}\, dy \right]$$

In order to make a right comparison between the two functions, they should be defined in the same domain, this can be achieved by make the substitution $y = 1/x$ in the next integral in the right-hand-side then
(4.63)

$$\zeta(s) = \left[\sum_{n=1}^\infty n \left(\frac{1}{n^s} - \frac{1}{(n+1)^s} \right) - s \int_0^\infty x^{-s-1}\, x\, dx \right] + \left[s \int_0^\infty x^{s-1}\, dx - s \sum_{n=1}^\infty n \int_{\frac{1}{n+1}}^{\frac{1}{n}} x^{s-1}\, dx \right]$$
$$= \left[\sum_{n=1}^\infty n \left(\frac{1}{n^s} - \frac{1}{(n+1)^s} \right) \right] + \left[-s \int_0^\infty x^{-s}\, dx + s \int_0^\infty x^{s-1}\, dx \right] - \left[\sum_{n=1}^\infty n \left(\frac{1}{n^s} - \frac{1}{(n+1)^s} \right) \right]$$

So $\zeta(s)$ equals zero for $\Re(s) = 1/2$, and this is the big zero for Riemann hypothesis, that is the complete proof for the Riemann hypothesis.

Remark 4.6.
As we notice, the terms of the series cancel each other out in both functions regardless the s value, these series represent the idea of the periodicity for the kernel function. So we concluded that all the properties of Zeta function actually achieved **locally** per period, it appears that this part directly relates to its definition in the strip region $0 < \Re(s) < 1$, considering that it can be extended **globally** to all s-domain by the periodic property of the kernel.

Remark 4.7.
This equation actually says that there are **no real zeros** because the Zeta function-pair does not share its definition domains, even if we have an equivalent relation to compute Zeta.

5. Zeta function zeros

For the t values that make Zeta-series equal zero, they give another meaning in the Zeta (as a function) because these values affect on Zeta in different way. As an instance, the summation operation for $\sum \delta(x-n)$ gets again $\sum \delta(x-n)$ as it is in a separated form.

The interesting things is that the representation part of the periodic property in the kernel is extracted in a form that takes exactly the Zeta-series, which is $\sum_{n=1}^{\infty} n\left(\frac{1}{n^s} - \frac{1}{(n+1)^s}\right)$ and that will be equal zero for these t values. The remaining part of Zeta function, in this case, will be only in the variable x from $0 \to \infty$ as smooth continuous function.

6. The Conclustion

Zeta function in the strip region is a formula that describes a symmetric relation between two sub-functions, this relation has two equivalent forms, and together these forms describe a closed loop.

Appendix

6.1. Derivative $\zeta(2k)$ from $\zeta(s)_{con}$.

As we calculate $\zeta(s)$ for positive and negative s from $\zeta(s)_{dis}$, we can do the same thing for the $\zeta(s)_{con}$, here we will deduce the $\zeta(2k)$ where,

(6.1)

$$\begin{aligned}
\zeta(s)_{con} &= 2\sum_{m=1}^{\infty}\int_0^{\infty} x^{-s}\cos(2\pi m x)\,dx \\
&= 2(2\pi)^{s-1}\int_0^{\infty} x^{-s}\sum_{m=1}^{\infty}\cos(2\pi m x)\,dx \\
&= 2(2\pi)^{s-1}\int_0^{\infty} x^{-s}\,\frac{1}{2}\left(\frac{1}{e^{-ix}-1}+\frac{1}{e^{ix}-1}\right)\,dx \qquad \text{where } |x|<1 \\
&= (2\pi)^{s-1}\left[(-1)^{s+1}\int_0^{-\infty}\frac{x^{-s}}{e^{ix}-1}\,dx + \int_0^{\infty}\frac{x^{-s}}{e^{ix}-1}\,dx\right] \\
&= (2\pi)^{s-1}\left[(-1)^{s+1}\int_0^{-\infty}\frac{(i)^{s-1}(ix)^{-s}}{e^{ix}-1}\,d(ix) + \int_0^{\infty}\frac{(i)^{s-1}(ix)^{-s}}{e^{ix}-1}\,d(ix)\right] \\
&= (2\pi)^{s-1}(i)^{s-1}\left[(-1)^{s+1}\int_0^{-\infty}\frac{(ix)^{-s}}{e^{ix}-1}\,d(ix) + \int_0^{\infty}\frac{(ix)^{-s}}{e^{ix}-1}\,d(ix)\right]
\end{aligned}$$

for $s = 2k$, $(k = 1, 2, 3, ...)$, then

(6.2)

$$\begin{aligned}
\zeta(2k)_{con} &= (2\pi)^{2k-1}(-1)^k(-i)\left(-\int_0^{-\infty}\frac{(ix)^{-2k}}{e^{ix}-1}\,d(ix) + \int_0^{\infty}\frac{(ix)^{-2k}}{e^{ix}-1}\,d(ix)\right) \\
&= (2\pi)^{2k-1}(-1)^k(-i)\left(\int_{-\infty}^{0}\frac{(ix)^{-2k}}{e^{ix}-1}\,d(ix) + \int_0^{\infty}\frac{(ix)^{-2k}}{e^{ix}-1}\,d(ix)\right)
\end{aligned}$$

Since,

(6.3)

$$\frac{z}{e^z-1} = \sum_{m=0}^{\infty}\frac{B_m z^m}{m!} \qquad |z|<1 \quad \text{(this condition is achieved for } (ix) \text{ in the right-hand-side)}$$

hence,

(6.4)

$$\zeta(2k)_{con} = (2\pi)^{2k-1}(-1)^k(-i)\sum_{m=0}^{\infty}\frac{B_m}{m!}\left(\int_{-\infty}^{0}\frac{(ix)^{m-2k}}{(ix)}\,d(ix) + \int_0^{\infty}\frac{(ix)^{m-2k}}{(ix)}\,d(ix)\right)$$

and for $m = 2k$, then

$$\zeta(2k)_{con} = (2\pi)^{2k-1}(-1)^k(-i)\frac{B_{2k}}{2k!}\left(\int_{-\infty}^{0}\frac{1}{(ix)}\,d(ix) + \int_{0}^{\infty}\frac{1}{(ix)}\,d(ix)\right) \tag{6.5}$$

By computing the complex integral

$$\oint\frac{dz}{z} \tag{6.6}$$

For the path in (Figure 4), then we get

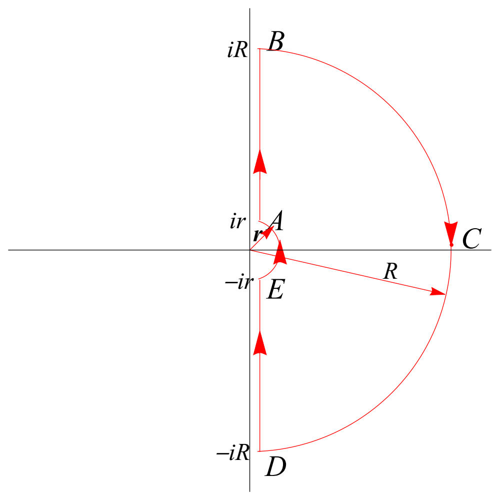

FIGURE 4. Path integral for case of (k is an odd number)

$$\oint\frac{dz}{z} = \int_{r}^{R}\frac{d(iy)}{iy} + \int_{BCD}\frac{d(Re^{i\theta})}{(Re^{i\theta})} + \int_{-R}^{-r}\frac{d(iy)}{iy} + \int_{EA}\frac{d(re^{i\theta})}{(re^{i\theta})} = 0 \tag{6.7}$$

hence for $(R \to \infty)$ and $(r \to 0)$, we obtain

$$\int_{BCD}\frac{d(Re^{i\theta})}{(Re^{i\theta})} \to 0 \qquad \text{and} \qquad \int_{EA}\frac{d(re^{i\theta})}{(re^{i\theta})} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i d\theta = i\pi \tag{6.8}$$

then

$$\int_{0}^{\infty}\frac{diy}{iy} + \int_{-\infty}^{0}\frac{diy}{iy} = -i\pi \tag{6.9}$$

therefore

$$\begin{aligned}\zeta(2k)_{con} &= (2\pi)^{s-1}(-1)^k(-i)\frac{B_{2k}}{2k!}\,(-i\pi)\\ &= (-1)^{k+1}\frac{(2\pi)^{2k}}{2(2k)!}B_{2k}\end{aligned} \tag{6.10}$$

PART2: PHYSICS PART

6.2. **Introduction.**

6.2.1. *Between the Quantum and the Classical physics.*
From our point of view, we see that the discontinuity and the continuity nature of the energy is the core of the problem between the "Quantum" and the "Classical" physics respectively.

Mathematically, Differential equations is the main player in the study of everything continuous and smooth, that is perfectly compatible with classical physics. Whereas, the Quantum mechanics lives more in the world of the Linear algebra and Matrices. We can do the calculations, but we can not understand the whole picture. In the end, these two worlds remain mathematically separated.

6.2.2. *What is new in the Zeta function?*
The Zeta function reveals the hidden relationship between the continuity and discontinuity in mathematics in an abstract way, explaining how can a function hold these properties together to be continuous and discontinuous at the same time.
In fact, we dominate it to be a model can unify all the properties of matter in both Classical and Quantum physics. As an instance, the Zeta function on one hand can describe the properties of the space-time curvature and on the other hand, it can describe the main properties of a particle in Quantum mechanics (QM), so it gives a model for the Graviton particle.

We will start by introducing a new theory, and describe a new perspective for the physics quantities, like the matter, the energy, the space and the time by defining the concepts of these quantities and try to express it in the model. This theory is based on the Zeta function equation, and by this theory, we will understand the behavior of the matter and energy, and their mechanism of the motion and translation. According to this description, we will try to answer some unsolved questions in physics which are still open like the entangled phenomenon and how the connection happens instantaneously between their objects. then we will suggest an explanation to the all outcomes of the double slits experiment and The Stern-Gerlach experiment.
After that, we will start to discuss the idea of the time and the space in the new model, and according to this we will proof that the $\mathbb{Z}$eta function achieves the condition of the Lorentz invariant, as well as, it gives an answer to the reason of the negative sign of the time t in the relation $(x^2 - t^2)$. After that, explain how is Zeta function can achieve the main conditions of the General Relativity theory. At the end we will answer the question about a new perspective for the Gravity force and its source.

7. A unification model for the Classical and the Quantum physics

The starting point will define matter and space-time concepts in the Zeta model equation.

7.1. **The matter-space and the space-time concepts in the Zeta model.**
This model provides a picture for a *dynamic matter-space-time.*

7.1.1. *The idea of matter-space:*
In the $\mathbb{Z}$*eta-cosine* model. [9] we have

$$\sum_{n=1}^{\infty} n^{-s} = \sum_{n=1}^{\infty} \int_0^{\infty} \frac{1}{\sqrt{x}}\, x^{-it}\, \delta(x-n)\, dx - \int_0^{\infty} \frac{1}{\sqrt{x}}\, x^{-it} \times (\mathbf{1})\, dx$$

or

$$\sum_{n=1}^{\infty} n^{-s} = \sum_{n=1}^{\infty} \int_0^{\infty} \frac{1}{\sqrt{x}}\, x^{-it}\, \delta(x-n)\, dx + \quad 2 \sum_{n=1}^{\infty} \int_0^{\infty} \frac{1}{\sqrt{x}}\, x^{-it}\, \cos(2\pi n x)\, dx$$

The left-hand-side represents the pure matter [10] which is split into *space* and *matter* or space-matter structure in the right-hand-side [11]. They are represented by the variables n and x respectively.
They are always associate together, mathematically this idea is represented by the term (nx) in the kernel functions, and that provides the idea of the totally merge between them or the space-matter concept.
The idea of *matter-space-time*: The matter or the energy takes two shapes locality (discontinuity) and non-locality (continuity) forms which are represented in the right-hand-side by the kernel functions $\delta(x-n)$ and $\mathbf{1}$ function respectively. They are a result for combining of an infinite number of waves $\sum \cos(2\pi nx)$, we will consider here the energy is represented by every single one of them,
this gathering happens in two ways; **First:** increases (rises) vertically at one point (synchronized in phase) and that produces the $\delta(x-n)$ function, it will be a model for a point of mass, **The concept of *time*** can be realized by the direction of the increasing rate of this energy (not transmitted) which is the vertical direction, so we will assume that the time axis is the vertical axis.
Second: Assembling waves but not in-phase, that is performed in another domain, we will assume that domain defines the *Space*. In this space the matter takes another equivalent shape. It is spreading and it takes a direction perpendicular to the first one, and that achieves the idea of the orthogonality between the pure space and the pure time, and their connection gives the idea of the *spacetime* as one word[12].
The *anti-matter* idea: The spreading matter in the space takes an opposite sign relative to its equivalent in time direction, that may gives the meaning of the "anti" or anti-matter. The coupling function $f(x)$ with Zeta in this model is read in the left-hand-side as a changing in space, whereas in the right-hand-side it takes its massive form or as a particle that has an effective mass, $\sum f(nx)$.

The *dynamics* in the model:
This model also provides a self-dynamic picture, this is achieved by the self-operator

[9] All the equations are defined in the strip region, and $s = 1/2 \pm it$ even if we did not mention that

[10] we mean by "pure" the rest mass

[11] space: is the boundless three-dimensional extent in which objects and events have relative position and direction (https://en.wikipeida.org/wiki/Space), the source of that definition is : *physics and metaphysics theories of space and time- Jennifer trusted*).

[12] So, the space in this context maybe defined as: *Space: a place which the matter can exist and transmit.*,

which is coupling with $\mathbb{Z}$eta as an identity operator, and for $\zeta(s)$ we always had $\zeta(1-s)$ this conjugation is generated by the operation

$$\mathcal{S}_s\zeta(s) = \zeta(1-s) \tag{7.1}$$

so we always have this process, and in this process the matter in the delta function is converted to the space and the matter in the space converted to a new delta function.

It looks like a heartbeat. That gives also the concept of velocity (v) which represents the velocity change between the both forms of matter or the velocity of the matter translation.

Noting that, the functions did not convert to the same pair between each other but to a new similar pair, that is because these actions are doing simultaneously for both functions, so it is not possible to make two different actions instantaneous for the same function. that means this process produces a new couple of functions and that gives the concept of the dynamic.

Therefore, the Zeta equation can provide a dynamic model for the matter, space and time or *the matter-space-time dynamic* equation. So this is the whole picture of that model for the new perspective.

7.2. **The model perspective relative to the Quantum mechanics.**

In this section we will review the most important principles of Quantum Mechanics and know how the new model explains them.

7.2.1. *The proof of Plank's law and the concept of wave-particle duality.*

Wave-particle duality *is the concept in quantum mechanics that every particle or quantum entity may be described as either a particle or a wave.* [13]

This concept is achieved clearly by the kernel functions in the model, where a particle (as delta function) associates by a wave, and we can write

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it}\, \delta(x-r) \;= \int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it}\, 2\cos\left(2\pi r x\right) dx \quad (r\in\mathbb{N}) \tag{7.2}$$

Now, if the Delta function is explained as a localized energy shape E, this energy is quantized and transmitted by quantum number r by the unitary operator (self-operator) action. So we have $E = hr$, (h here will represent the minimum amount of energy which is confined by delta function) assuming ($h = 1$). And this energy is transmitted as two opposite direction plane waves $2\cos 2\pi rx = (e^{2i\pi rx} + e^{-i2\pi rx})$, that proofs the Plank's law.

Additionally, by the paradigm which defines a mass m or a particle as a localized energy shape, that also achieves the relation $E = mc^2$, (c the speed of light) or $E = m$ for ($c = 1$).

Remark 7.1.

1- For the general function $f(x)$ we get the equivalence

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it}\, f(rx) \;= \int_0^\infty \frac{1}{\sqrt{x}}\, x^{-it}\, \frac{1}{x}\mathcal{F}\left(\frac{r}{x}\right)\, dx \quad (r\in\mathbb{N}) \tag{7.3}$$

[13]https://en.wikipedia.org/wiki/Wave-particle_duality

so the rule of "every particle has an associated wave that is not general". It maybe takes another shape, we think the general perspective here, regardless the shape of matter there are **localized and non-localized (in general)** forms represent it. They continually exchange their domains and translated in the same time. Their shapes and the transmission rules obey the Fourier transform rules.
2- For the *Zeta-cosine* model the particle translates as a wave and the wave translates as a particle. This is different in QM, there is no concept about the wave translation as a particle

7.2.2. *The Heisenbergs uncertainty principle.*
Heisenbergs uncertainty principle states that: *If the x-component of the momentum of a particle is measured with an uncertainty Δx , then its x-position cannot, at the same time, be measured more accurately than $\Delta x = h/(2\Delta p_x)$.*
In the model and in 7.2, if a localization energy in delta function represents a particle, so the position of this particle will be determined accurately by the function in the left-hand-side, whereas its momentum $p = \frac{h}{\lambda} = hk$, k is the wave number and $k = r$, so the momentum can be determined accurately by the wave form in the right-hand-side, but the measurement process will select only one of them, in other words, if we are going to determine the position by the function in the left-hand-side, the function in the right-hand-side will collapse in the same moment, and vise versa, if the measuring probe caught the wave (the right-hand-side) and determined its momentum, then the wave will collapse and in the same moment the function in the left-hand-side will be collapsed as well, which means it will not be located.
In mathematics point-of-view, The *uncertainty principle* is defined in the wikipedia as follows; *Mathematically ,in wave mechanics, the uncertainty relation between position and momentum arises because the expressions of the wave-function in the two corresponding orthonormal bases in Hilbert space are Fourier transform of on another (i.e, position and momentum are conjugate variables)*[14]
That is exactly what the model expresses mathematically.
Remark 7.2.
It seems that the uncertainty principle describing the idea of the locality and non-locality states of the matter, and the conjugating quantities can be generalized in that form.

7.2.3. *The symmetric translating conditions between the model and QM:.*
In the model, the s-domain is the domain that keeps the conditions of the symmetries in the translated process, where the energy takes two shapes, mathematically represented as a function and its Fourier transform, the more localized function, the more wideness is spreading. the particle (much more localized as a delta function) translated as a wave, likewise the wave translated as a particle, the movement process takes a quantization shape not infinitesimal one as in QM. It seems like the annihilation and creation process which described by Dirac. It is carried out by a unitary operator that performs the transformation process. The equation can

[14] https://en.wikipedia.org/wiki/Uncertainty_principle

express one particle, in general it takes the shape of

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{it}\, f(rx)\;\, dx = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it}\, \frac{1}{x}\mathcal{F}(\frac{r}{x})\, dx \quad \text{(one particle)}$$

or for many particles (field)

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{it} \sum_{n=1}^{N} f(nx)\;\, dx = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it} \sum_{m=1}^{N} \frac{1}{x}\mathcal{F}(\frac{m}{x})\, dx \qquad \text{(many particles as a filed)}$$

For Classical point of view (macroscopic) we have $N \to \infty$ and both summations in the equation became integrals, then

(7.4)

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{it} \left(\int_0^\infty f(ux)\, du\right)\;\, dx = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it} \left(\int_0^\infty \frac{1}{x}\mathcal{F}(\frac{v}{x})\, dv\right)\; dx$$

$$\int_0^\infty \frac{1}{\sqrt{x}}\, x^{it}\, \frac{1}{x}\left(\int_0^\infty f(w)\, dw\right)\;\, dx = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it} \left(\int_0^\infty \mathcal{F}(y)\, dy\right)\; dx \qquad (w = ux)\ , (y = \frac{v}{x})$$

$$\text{then} \quad \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it}\, \frac{1}{x}\mathcal{F}(0)\;\, dx = \int_0^\infty \frac{1}{\sqrt{x}}\, x^{it}\, f(0)\, dx \qquad (f(0) = \mathcal{F}(0))$$

The total amount of the transmitted energy is represented by the values of $f(0), \mathcal{F}(0)$. This equation represents the initial value problem where the functions independent on its path and that gives the Classical point of view. For this case the kernels of the equation becomes like a particle that moves on a smooth path, and that represents **the spacetime curvature idea**, this will be discussed again in details later.

7.2.4. *superposition and states.*

One of the most strangest properties in Quantum mechanics is the superposition and states idea. Because here we cross the line between two schools, the classical deterministic school and probability school. In fact, the model can link both schools together, that's what would be explained in this section.

***The probability idea*:**

To illustrate the probability idea we need to go to the Zeta picture 3.1 and write it as

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^\infty x^{-s}\, \delta(x-n)\, dx - \sum_{n=1}^{\infty} \int_0^\infty x^{-s} \times \mathbf{1}_n\, dx \tag{7.5}$$

The function $\mathbf{1}_n$ in the right-hand-side, is periodic for (n) and it is constant for all periods so we don't need the summation in this term, but if we notice, we made a probability adding to this term comparable to an algebraic adding to the $\delta(x-n)$ terms in the corresponding integral , in other words, if we adding $\mathbf{1}_n$ as an algebraic which means $1+1+1+... \to \infty$ but what we did ? is $1+1+1+... = 1$ it is a probability adding. so the probability in the equation is related to the periodicity of the kernel function. this periodicity in the other integrals converted to real terms representing states for each period, where (n) in $\delta(x-n)$ represents the energy level (so the energy state) for the particle, so if we have, for instance, $\delta(x-1)+\delta(x-2)+\delta(x-3)$ this system will be described either as 3 particles have different energies or one particle that has three states of energy and we will read it as $\delta(x-1)$ or $\delta(x-2)$ or $\delta(x-3)$, and when we determine (measure) we will get one of them and the system collapses because they are one function. this is the

quantum mechanics point of view, for the classical point of view, these values of energy are adding algebraically to give one particle with 6 units of energy $(1+2+3)$ and for huge numbers of particles $N \to \infty$ we converted it to 7.4, so in summary, in quantum mechanics the $(+)$ means the probability (or), in Classic mechanics, the $(+)$ means an algebraic sign.
Also we can read the function by another way: as $\delta(x-n)$ represents a state n also $\mathbf{1}_n$ represents that state, and the previous equation can be written as

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s}\, \delta(x-n)\, dx + \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s} \left(\sum_{m=1}^{\infty} \cos 2\pi m \right)_n \, dx \tag{7.6}$$

In this form, the meaning for this form, for any state n the particle can take any wave with frequency m by the probability $\cos(2\pi m)$, and the function can be written as

$$\zeta(s) = \int_0^{\infty} x^{-s}\, \delta(x)\, dx + \int_0^{\infty} x^{-s} \left(\sum_{m=1}^{\infty} \cos 2\pi m \right) \, dx \tag{7.7}$$

$\delta(x)$ here is not denoted to $n = 0$, but it denoted to any value of x, and we know $x \in \mathbb{N}$, the kernel function in the second integral in this case will be considered as the probability distribution function (PDF) for the particle, and in case of translation we get $\delta(x-n) \to \cos(2\pi m x)$ so the particle will transmit only by the wave frequency that has the same value of its energy state, $(n = m)$.
So, the big picture is, for every particle we have a corresponding field 7.6 (Dirac Quantum Filed Theory), this particle when it translates, it will take one wave form (wave-particle duality).

7.3. **The Entanglement phenomenon.**

We refer this phenomenon to a new term that will be named *the connection term*, it describes the connection between the shapes of the matter, means its localized and non-localized forms. This term can be extracted from the origin form of the Zeta function. For *Zeta-cosine* we say

$$\begin{aligned} 2\sum_{m=1}^{N} \cos(2\pi m x) &= \lim_{N\to\infty} \frac{\sin\left((N+\frac{1}{2})2\pi x\right)}{\sin(\pi x)} \; - \; 1 \\ &= \lim_{N\to\infty} \frac{\sin(2\pi N x)\cos(\pi x) + \cos(2\pi N x)\sin(\pi x)}{\sin(\pi x)} \; - \; 1 \\ &= \lim_{N\to\infty} \frac{\sin(2\pi N x)}{\tan(\pi x)} + \cos(2\pi N x) \; - \; 1 \end{aligned} \tag{7.8}$$

so we can right Zeta equation as

$$\zeta(s) = \sum_{n=1}^{\infty} \int_0^{\infty} x^{-s}\, \delta(x-n)\, dx + \left[\lim_{N\to\infty} \int_0^{\infty} x^{-s} \cos 2\pi N x \, dx \right] + \sum_{m=1}^{\infty} \int_0^{\infty} x^{-s} \cos 2\pi m \, dx \tag{7.9}$$

This term makes the internal connection between the states inside the function, we define these states as entangled states. In addition, we think that the system of the particles that influenced by each other are entangled and, mathematically, they are belonged to the same s-domain. This term in case of measurement (the wave collapse) [15] it will vanish.

(7.10)
$$\int_0^\infty x^{-s}\cos(2\pi N x)\,dx = N^{s-1}\int_0^\infty x^{-s}\cos(2\pi x)\,dx \to 0 \quad \text{as } N\to\infty \qquad \boxed{\Re(s)=1/2}$$

According what do we measure, the action and its speed will be determined. We think that the limit of the speed of light represents the limit of changing between the localized and non-localized form for each particle. in other words, If the process describes transferring energy from one place to another like annihilation and creation process, we think the maximum speed of this process will be the speed of light and all the associated properties for this process will also transfer by the same velocity and that achieves the locality principle in the Classical perception.
So this changing process obey the speed of light law, but if there is another type of process which depends on the strength of the connection not on the changing form of energy, in this case the speed of action will depend directly on the strength of the connection term, we think that this case is happing in the collapsing measurement process. This term in case of *Zeta-cosine* function has an infinite energy, therefore we can get an instantaneous action speed and that expresses the phenomenon
.

7.3.1. *Connection term for general function.*
We thing that this phenomenon is not happening in general, because the connection term maybe change according the function which conjugates with Zeta. In this case the connection term will be
(7.11)
$$\left(2\int_0^\infty x^{-s}\cos 2\pi N x\,dx\right)\left(\int_0^\infty u^{s-1}\sum_{n=1}^{\infty} f(nu)\,du\right) \quad = \int_0^\infty y^{-s}\,\mathcal{F}(Ny)\,dy$$

So, the strength of the connection will depend on the function $\mathcal{F}(Nx)$, as an instance, for the function $f(x)=e^{-\pi x^2}$, we have $\mathcal{F}(e^{-\pi u^2}) = f(x) = e^{-\pi x^2}$, therefore $\mathcal{F}(Nx) = e^{-\pi N^2 x^2} \to 0$ as $N\to\infty$, that means that system will not show any instantaneous actions. So we conclude that these *instantaneous* type of actions do not exist in general among system particles.

7.3.2. *The double slits experiment.*
In the model, the particle translates as a wave and the wave translates as a particle, this is the concept of motion. This property can explain how the gun of the particle which facing the double slits starts in a particle shape and ended on the screen with a particle shape as well.
The slits here are a kind of a device that is sensitive to the waves, so at these slits, the wave part will be affected by it causing changing in its path, they will be appeared as two sources for the wave, but in fact these two waves are not independent as shown in the behavior of regular waves, they still connected by their connection filed (by the connection term), so they will interfere with each other behind the

[15]We consider here the measurement process happens by solving the integrals of the equation. Solving the integrals are equivalent to disappear (x) variable (the space variable), so all the quantum phenomena will disappear and the function will only have the variable (n) and that gives a numerical result.

slits to construct the original wave again which will turn into a particle which finally falls on the screen. So, there are no destructive waves, but there are blank regions where the waves did not interfere each other. The evidence for that, every particle sent out is seen on the screen, so if there are destructive points at which the particle does not interfere, in this case, it may or may not appear in the screen and that was not observed in the experiment, therefore they are only constructive points. They take many many paths according to the incident angle when the wave meets the slits, these probabilities of paths perform the striped pattern.
When we detect or monitoring the particle this mostly causes the wave-wave interaction like the interaction of the self-operator with the particle wave form which causes the re-localization of the particle. Mathematically, the monitoring device will make a Fourier transform for the wave (or for the non-localized form) of the particle, which leads to the immediately disappearance of the particle wave and the return of the particle localized shape wherever the place of detection whether it was before or after the slits.

7.3.3. *The Stern-Gerlach experiment.*
In this part we will try to explain the Stern-Gerlach experiment by the new Model. This experiment demonstrated that the spatial orintation of angular momentum is quantized [16] In fact Zeta model has a structure that can exhibit both *Spin* and *two stats* ideas, that is the *Zeta-sine* form. This model has a condensing shape of energy where the very high frequencies ripples in *Zeta-cosine* (figure 5) condensing into two lines take the sawtooth shape (figure 6), and that gives more condensing particle properties.

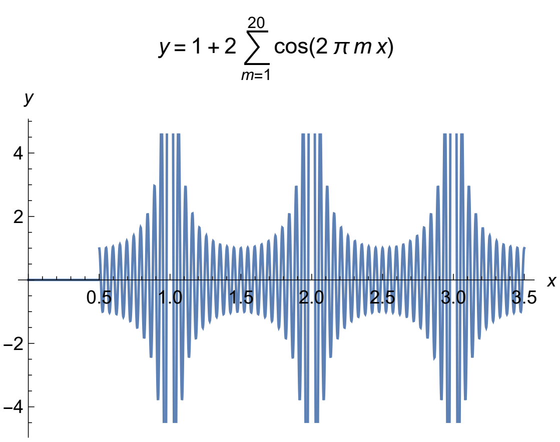

FIGURE 5. The high ripples in the *Zeta-cosine* form.

In fact, this shape can be used to explain the spin property of matter. If we look at the edges of the ramp function, it appears as very short waves in edges, if they are in a rotational motion, this motion will generate a *self-spin* in the direction of the ramp function. If the energy is moving in the positive x direction, the rotation in one end starts with a large amplitude and then decreases by moving forward, this generates a spin which is decreasing in the direction of the moving energy,

[16] https://en.wikipedia.org/wiki/Stern-Gerlach_experiment

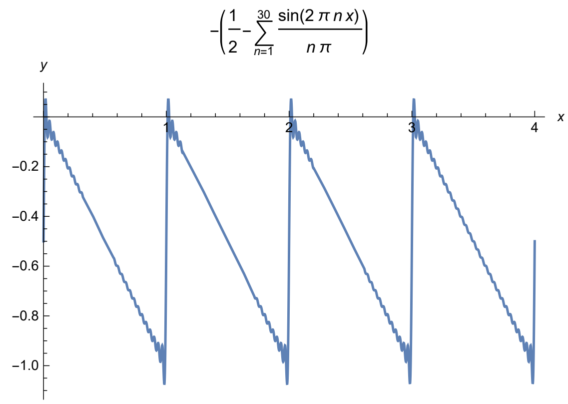

FIGURE 6. The correspond condensing in the *Zeta-sine* form

therefore the direction of the increasing spin will be in the opposite direction of the wave motion. As for the opposite side, the rotation starts with a small amplitude and increases with the same direction of the energy movement so the spin direction will be in the same direction of the energy movement. Thus, finally we have two spins opposite in direction to each other, (see the figure 7).

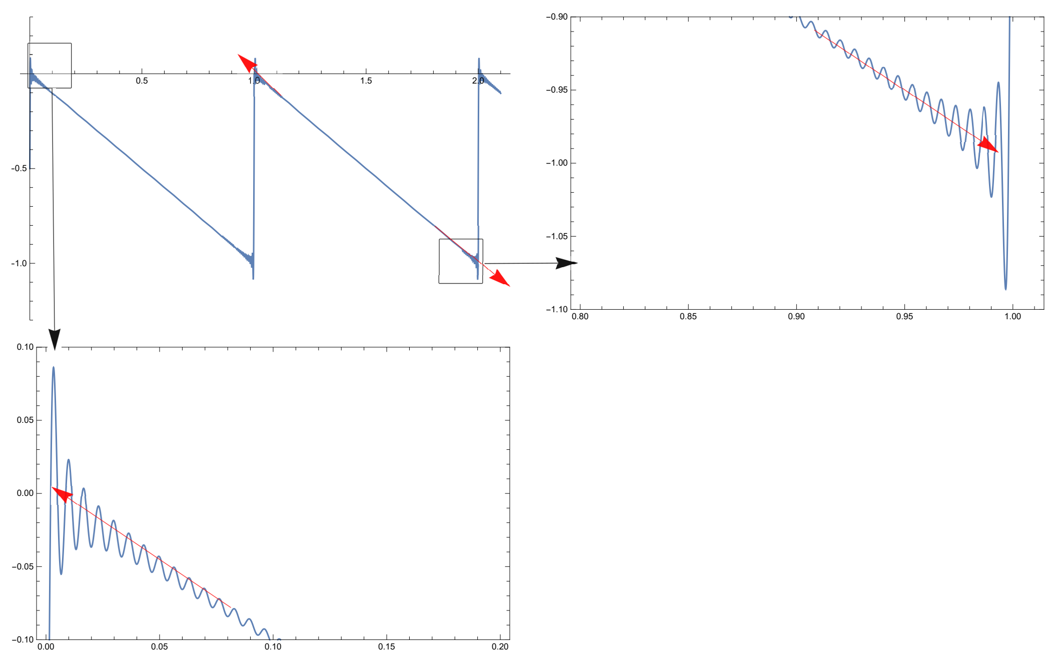

FIGURE 7. The generation of the spin in the matter

So, this model expresses the spin property, and that explains how particles can be affected by magnetic field as shown in the experiment, but this property alone cannot explain the spots up and down, this effect alone can cause spots take a

vertical line shape parallel to the magnetic field direction, not two spots up and down. The second part in the model can explain this quantization behavior, that is the jumping process which represents the motion on the vertical line function. It express two states for matter beside the spin. one state is up and the other is down with these two properties it can be explain the output of the The Stern-Gerlach experiment, one part of particle will be affected by the magnetic field taking the action to be aligned with its North and South direction, the second stage the energy suddenly changes its position from up to down or down to up by changing its orientation by assistance of the flipping property [section 4.3] . So the particle goes into two-only paths as two spots are separated from each other.
All the next aspects of the experiment can be explained by the same way. The particle has no memory and all its behavior is due to its self characteristics.

7.4. **The model for the Classical mechanics perspective.**
In this section we will again use our model to explain how this model can also express the properties of matter and energy as they are known in the Classical mechanics. We will also begin in the same way with a set of well-known principles that characterize Classical physics.

7.5. **The principle of the least action or stationary-action.**
If we start by the correspondence principle of Bohr which explain how Quantum systems reproduces Classical physics in the limit of large Quantum numbers [17] This idea has been demonstrated by the model in [7.2.3] by the equation 7.4 . So in the Classical mechanics; there is thinking that the smooth movement of energy is due to their huge numbers that are difficult to observe the quantum behavior in practice. This is one reason, but we think that the main reason for this smooth movement is achieving by what we say the *"principle of the least action or the stationary-action principle"* [18] We think that the Zeta model expresses also this property by its zeros. Our perspective here about the source of the the Quantum behavior of the energy is due to the unitary operator (self-operator) process, this process appears by the Zeta kernel periodicity [19] as a quantum energy transfer, this periodicity makes fluctuations in particle's motion, at Zeta zeros these quantum fluctuations will disappear and the movement will become smooth, putting the system in the state of *"The least action"*.

7.5.1. *The Lorentz invariant.*
We will assume here that the curvature/Graviton particle will be represented by the lightness function in the model which is the *Zeta-cosine* or *Zeta-sine* forms. In [7.1] we concluded that the direction of the time is in the vertical axis whereas the space takes the horizontal axis direction.
In addition, the connection of the two functions gives the meaning of the space-time (one word). The *Zeta-cosine* integral-pair that represent this idea has an

[17] Tipler, Paul: Liewellyn, Ralph (2008) Modern physics(5ed). W.H Freeman and company pp. 1600161

[18] https://en.wikipedia.org/wiki/Stationary-action_principle

[19] we mean here the periodicity of the whole summation function in Zeta kernel

isometric relationship at $\Re(s) = 1/2$, so what the energy travels through the horizontal axis x is the same as what it passes through vertical axis y which means (*space*) $x = t$ (*time*).
But, the model for the *Zeta-sine* form gives the shape of a right-angle triangle, in this shape the *space* axis is no longer perpendicular to the *time* axis. The energy moves through the ramp function x in the positive direction, then leaves the ramp function to moves up on the vertical axis through the time t axis, but it appears that the distance that the energy moves on the ramp function is not the same in the vertical direction, so ($x \neq t$), and the actual distance that the energy moves is equal to $\sqrt{x^2 - t^2}$ (figure 8). Thus, the function in this function shows the bend-

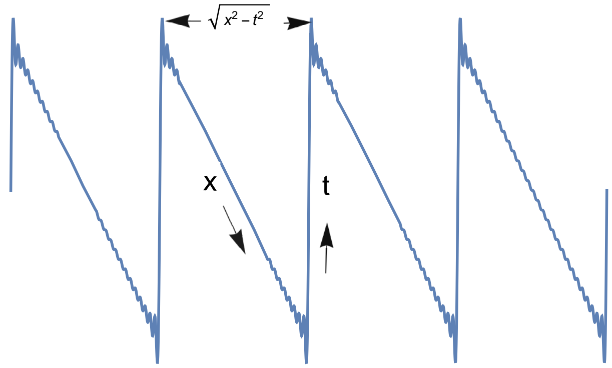

FIGURE 8. The actual distance for local action of moving objects in space-time

ing of the space on time. Now the self-operator expresses the annihilation-creation and that transfers energy in one period, this takes place continuously in a certain speed. Since this speed seems to be the fastest possible speed means the speed of light. If we now "imagine" that this object is moving at speed near to the annihilation-creation speed [20], in this case, the annihilation-creation will appear to be slower, thus the energy would take a longer distance and a longer time to be transmitted [21] (as in the figure) , and because the rate of the transmitted energy is constant [22] such that the horizontal movement distance does not change. Therefore $\sqrt{x^2 - t^2} = \sqrt{(x1)^2 - (t1)^2}$ or $x^2 - t^2 = (x1)^2 - (t1)^2$ (figure 9), this achieves the Lorentz invariant in spacetime. Then, the Lorentz invariant condition is achieved for the new model and for any function conjugate with it. Also, this proves to us that the relationship *Zeta-cosine* represents the state of the moving *(massless)* particle at the speed of light where $x = t$. And we also conclude that *Zeta-sine* function represents the state of matter *(masses)*. So, equation *Zeta-sine* answers the great mystery of the time sign in the relationship $x^2 - t^2 = x1^2 - t1^2$.

[20]Our perspective here, the initial frame is not stationary, but is moving by the maximum speed of the annihilation-creation process, which is the speed of light.

[21]Some physical phenomena began to change and by the language of Special Relativity the time began to slow down, and the transfer of energy takes longer, this is equivalent to increase the medium resistance or increasing the mass.

[22]This amount of energy is controlled by the identity operator action.

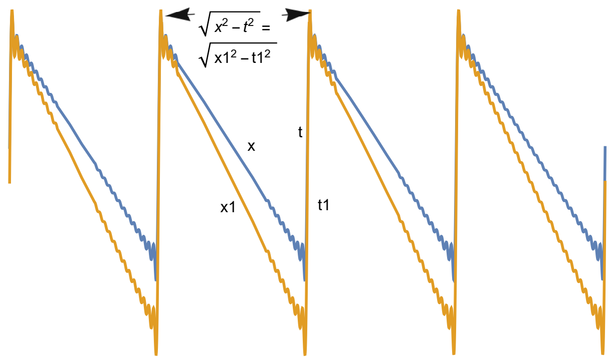

FIGURE 9. The affect in space-time of moving object by the speed compared to the speed of light

7.5.2. *The acceleration frame in the model and Einsteins general relativity.*
The identity operator (self-operator) in this model responsible for moving and transferring of the matter. Its kernel is the function $\cos(2\pi x)$, this function describes a uniform acceleration motion, which states that the normal motion of the matter across the universe takes place with a uniform acceleration. This can explain the reason for these accelerations of the bodies in the universe, and also may give an explanation to the universe expansion phenomenon [23].
Therefore, the model describes an acceleration frames and achieves the Lorentz invariant condition these are the main pillars on the Einstein's General Relativity theory.

7.5.3. *The force of Gravity.*
So we have mass that moves with an acceleration, which gives the concept of force. For the Gravity what's the source of this force? The gravity force can be understood as the attraction force between the object and its path, in the *Zeta-cosine* model, if delta function represents the mass object and the horizontal function "**1**" represents the path of the motion, then we get a gravity system, this object continuously moves on a continuous path, the source of gravity in this perspective is due to the internal connection between the function-pair of Zeta.
The Einstein's conception about the gravity force was **"b. Principle of Equivalence. Inertia and gravity are phenomena identical in nature".** [24] this definition of Einstein seems to be quite accurate, and here we will just add a more deep description for this inertia. Suppose that the particle is represented in the model by the masses particle (the kernel of *Zeta-sine* model), therefore, there is a translation of the ramp function to the function (**1**), the force of the Gravity will be understood as an attraction force between these functions. In the model, this translation is appeared by the identity operator process, which make a push to this triangle by continuous annihilation-creation processes, these processes do not happen instantaneously , but it takes time to accomplish, as well as, the creation

[23]One theory about the explanation of the universe expansion is the **the Dark matter** hypothesis. Zeta function model maybe give another answer to that question.

[24]Einstein's 1918 paper: On the Foundations of the General Theory of Relativity http://einsteinpapers.press.princeton.edu/vol7-trans/49

process takes the opposite direction appears as a reaction for the annihilation one, this achieves the concept of inertia. The fields here appear as potentials, these potentials express the Gravity force, so for the *Zeta-sine* model, we have

$$\begin{aligned}\zeta(s) &= \left[-s\int_0^\infty x^{-s-1}\ x\ dx + s\int_0^\infty y^{-s-1} \times \mathbf{1}\ dy\right] \\ &= s\left[\int_0^\infty x^{-1/2-it}\ (-1)\ dx + \int_0^\infty y^{-1/2-it}\ \left(\frac{1}{y}\right)\ dy\right]\end{aligned} \tag{7.12}$$

then the gravitational force can be determined by the potential derivative (the negative sign indicates that they are opposite in direction), so

$$\mathbf{F} = -\nabla U = -\nabla\left(\frac{1}{y} - 1\right) = -\frac{\partial}{\partial y}\frac{1}{y} + \frac{\partial}{\partial x}1 = \frac{1}{y^2} \tag{7.13}$$

The force here proportional to the distance in y-axis, but both functions are isometric, so it's also proportional to the distance in x-axis, or , in another way, by reciprocal the variables in the functions we get

$$\zeta(s) = s\left[\int_0^\infty x^{-1/2+it}\ \left(-\frac{1}{x}\right)\ dx + \int_0^\infty y^{-1/2+it}\ (1)\ dy\right] \tag{7.14}$$

Then

$$\mathbf{F} = -\nabla U = -\nabla\left(-\frac{1}{x} + 1\right) = -\frac{\partial}{\partial x}\frac{1}{x} + \frac{\partial}{\partial y}1 = \frac{1}{x^2} \tag{7.15}$$

Therefore $\left(\mathbf{F} \propto \frac{1}{x^2}\right)$ either. This result agrees with the Newton's Gravity law. This method maybe predict a type of gravity for the light or radiations, this can be derived by the same way from the *Zeta-cosine* model,

$$\zeta(s) = \left[\int_0^\infty x^{-s}\ \delta(x-n)\ dx - \int_0^\infty y^{-s} \times \mathbf{1}\ dy\right] \tag{7.16}$$

Therefore

$$\mathbf{F} = -\nabla U = -\nabla\left(\delta(x) - 1\right) = -\frac{\partial}{\partial x}\delta(x) = \frac{1}{x} \tag{7.17}$$

Here we considered the rule

$$\int_0^\infty f(x)\ \delta'(x-a)\ dx = -\int_0^\infty f'(x)\ \delta(x-a)\ dx \tag{7.18}$$

then

$$\int_0^\infty x^{-s}\ \delta'(x-a)\ dx = \int_0^\infty x^{-s}\left(s\ \frac{1}{x}\right)\ \delta(x-a)\ dx \tag{7.19}$$

The conclusion

Thus, the model has an ability to unify the theories of general and special relativity and quantum field theory in one framework, as well as, the possibility of generating models for particles, all of these nominate it to be a key for the unification forces or the *Grand Unified theory* which requires more effort and researches.

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