University of Technology Sydney School of Mathematical and Physical Sciences

Probability and Random Variables (37161) – Class 7 Preparation Work

1. A ticket agency sells tickets to music concerts and sporting events. Bookings may be made either online or via telephone and payment can be made with a credit card or a debit card.

The time between successive concert ticket sales (in hours) ~ $\exp(60)$ and the time between successive sports ticket sales (in hours) ~ $\exp(20)$. 80% of all sales (independently of the type of ticket or method of payment) are made online, the rest are made via telephone. 70% of sales (independently of type of ticket or method of purchase) are made by credit card and the rest are made by debit card.

- a) Calculate the probabilities of the following events:
 - i) During a three minute period, no sales are made online.
 - ii) During a five minute window, two sports tickets are sold and one concert ticket is sold.
 - iii) The next two ticket sales are, in order, a sports ticket bought online with a debit card then a sports ticket bought via telephone with a credit card.
 - iv) During a one minute window, the only ticket sold is a concert ticket.
 - v) Given that the last five tickets sold were all debit card sales, the next ticket sale is also a debit card sale.
- b) What are the distributions of the following in minutes?
 - i) The time between successive online credit card sales.
 - ii) The time between successive sports ticket sales, excluding those bought online with a credit card.

2. A shop sells both hot and cold drinks.

Hot drink sales occur at the instants of a Poisson process with expectation 30 drinks per hour.

Cold drink sales occur at the instants of a Poisson process with expectation 20 drinks per hour.

60% of customers purchasing a drink are female, 40% of customers purchasing a drink are male.

(Assume the shop is open 24 hours per day and that both men and women have the same probabilities of selecting a hot drink over a cold drink).

- a) Calculate the probability that:
 - i) no drinks are sold during the first five minutes of a day.
 - ii) one drink is sold during the first ten minutes of the day, given that none was sold during the first five minutes.
 - iii) eleven drinks are sold during the first fifteen minutes of a day, given that ten were sold during the first ten minutes of the day.
 - iv) exactly four cold drinks are sold to men during the third hour of the day.
 - v) the next four sales are, in order, a cold drink sold to a man, a hot drink sold to a man, a hot drink sold to a man, then a hot drink sold to either a man or a woman.
- b) Find the distribution of:
 - i) the number of drinks sold to women in a week.
 - ii) the time in minutes between successive sales of cold drinks to women.