University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 7 Preparation Work
SOLUTIONS

1.

a)

i) Number of online sales during a three minute period $\sim Poi\left((60+20)\times 0.8\times\frac{3}{60}\right)$ i.e. $\sim Poi(3.2)$. Therefore, the probability that there are no sales in this period is $e^{-3.2}$.

ii) Number of sports tickets sold in five minutes $\sim Poi\left(20\times\frac{5}{60}\right)$ i.e. $\sim Poi\left(\frac{5}{3}\right)$.

Number of concert tickets sold in five minutes $\sim Poi\left(60\times\frac{5}{60}\right)$ i.e. $\sim Poi(5)$.

Hence, by independence of the two processes, the probability that during a five minute window, two sports tickets are sold and one concert ticket is sold is $\frac{e^{-\frac{5}{3}}\left(\frac{5}{3}\right)^2}{2!}\frac{e^{-5}5^1}{1!}$.

iii) Each ticket (independently) is a sports ticket a sports ticket bought online with a debit card with probability $\left(\frac{20\times 0.8\times 0.3}{80}\right)$

Each ticket is a sports ticket bought via telephone with a credit card with probability $\left(\frac{20\times 0.2\times 0.7}{80}\right)$.

Hence the probability that the next two are, in order, a sports ticket a sports ticket bought online with a debit card then a sports ticket bought via telephone with a credit card is $\left(\frac{4.8}{80}\right)\left(\frac{2.8}{80}\right)$.

iv) During a one minute window, the only ticket sold is a concert ticket if the number of concert tickets sold is one and the number of non-concert (i.e. sports) tickets sold is zero.

Concert tickets in a minute $\sim Poi\left(60\times\frac{1}{60}\right)$ and sports tickets in a minute $\sim Poi\left(20\times\frac{1}{60}\right)$. The probability is therefore $\left(\frac{e^{-1}1^1}{1!}\right)\left(\frac{e^{-\frac{1}{3}}\left(\frac{1}{3}\right)^0}{0!}\right)=e^{-\frac{4}{3}}$.

v) The sales of the five previous tickets is irrelevant when considering future ticket sales. The probability that the next ticket sale is a debit card sale is simply 0.3.

b)

i) The time between successive online credit card sales $\sim \exp\left((60+20)\times 0.7\times 0.8\times \frac{1}{60}\right)$ i.e. $\sim \exp(0.747)$ (3 decimal places).

ii) The time between successive sports ticket sales, excluding those bought online with a credit card $\sim \exp\left(20\times(1-(0.7\times 0.8))\times \frac{1}{60}\right)$ i.e. $\sim \exp(0.147)$ (3 decimal places).

2.

a)

i) Number of drinks sold in a five minute window $\sim Poi\left(\frac{20+30}{12}\right)$ hence

P(no drinks are sold during the first five minutes of a day) $= e^{-\frac{25}{6}}$.

ii) The question is only asking about a five minute time window (i.e. between five and ten minutes) hence

P(one drink is sold during the first ten minutes of the day, given that none was sold during the first five minutes) $= \frac{e^{-\frac{25}{6}}\left(\frac{25}{6}\right)^1}{1!} = \frac{25e^{-\frac{25}{6}}}{6}$.

iii) Again, the question is only asking about a five minute time window (i.e. between ten and fifteen minutes) and is only asking for the chance of one more drink sale. Hence

P(eleven drinks are sold during the first fifteen minutes of a day, given that ten were sold during the first ten minutes of the day) $= \frac{e^{-\frac{25}{6}}\left(\frac{25}{6}\right)^1}{1!} = \frac{25e^{-\frac{25}{6}}}{6}$.

iv) Number of cold drinks sold to men in an hour $\sim Poi\left(20\times 0.4\right)$ hence P(exactly four cold drinks are sold to men during the third hour of the day) $= \frac{e^{-8}8^4}{4!}$.

v) the next four sales are, in order, a cold drink sold to a man, a hot drink sold to a man, a hot drink sold to a man, then a hot drink sold to either a man or a woman.

$$= \left(\frac{20\times 0.4}{50}\right)\left(\frac{30\times 0.4}{50}\right)\left(\frac{30\times 0.4}{50}\right)\left(\frac{30}{50}\right) = \left(\frac{432}{78125}\right)$$

b)

i) The number of drinks sold to women in a week $\sim Poi\left(0.6\times(20+30\times 7\times 24\right)$ i.e. $\sim Poi\left(5040\right)$.

ii) The number of cold drinks sold to women per hour $\sim Poi\left(0.6\times 20\right)$ hence the time between successive sales of cold drinks to women (in minutes) $\sim \exp\left(\frac{0.6\times 20}{60}\right)$ i.e. the number $\sim \exp(0.2)$ ..

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