**University of Technology Sydney**
**School of Mathematical and Physical Sciences**

Probability and Random Variables (37161) –
In-class Test
Tuesday 20 September 2016
Time allowed: 90 minutes
Test paper MUST NOT BE REMOVED from examination room

**Question 1.**

For each of the following variables, state whether it could reasonably be modelled by a Bernoulli variable, a Binomial variable or a Geometric variable. In each case, state the parameter(s) needed.

Where none of these distributions is appropriate, briefly explain why not.
**Hint:** Think of the range of each variable.

i) Out of a sample of 20 people selected uniformly at random from the Australian population, the number of females selected, assuming an equal proportion of both males and females in the population.

ii) The number of times a critically-ill patient dies in a given week, assuming that his/her illness is such that the patient has a 50% chance of surviving the whole week.

iii) Out of a sample of 20 members of the same family, the number who are colour blind, given that colour blindness is a genetically inherited condition which affects 7% of the population.

iv) The number of times a player simultaneously rolls two (regular fair six-sided) dice until the first time both land showing 5.

v) The number of times a player rolls one (regular fair six-sided) die until the second time it lands showing 5.
(e.g. if it landed 2,3,5,1,6,4,5, then the fact it took seven rolls until the second 5 would be recorded.)

**(5 Marks)**

**Question 2.**

Let $A$ and $B$ be events such that $P(A)=x$, $P(B)=y$ and $P(A\cup B)=z$.

a) Represent these events in a Venn diagram, clearly labelling the values of all four regions (That is, $P(A\cap B), P(A\cap B^c)$, $P(A^c\cap B)$ and $P(A^c\cap B^c)$)

**(4 Marks)**

b) Show that $P(A|B)=\dfrac{x+y-z}{y}$.

**(1 Mark)**

c) What condition (in terms of $x$, $y$ and $z$) must be satisfied if $A$ and $B$ are independent?

**(1 Mark)**

**Question 3.**

Tasks are assigned at a workplace to any one of five employees.

All tasks assigned to Employee 1 are completed on time.
Tasks assigned to Employee 2 are completed on time with probability 75%
Tasks assigned to Employee 3 are completed on time with probability 50%
Tasks assigned to Employee 4 are completed on time with probability 25%
No tasks assigned to Employee 5 are completed on time.
(Assume that the completion of each task is independent of the completion of all other tasks.)

One employee is selected at random with each of the five employees equally likely to be chosen. He/she is assigned two tasks to complete.

a) Calculate the probability that both tasks assigned to the selected employee are completed on time.

**(3 Marks)**

b) Given that both tasks are completed on time, show that the probability that the tasks were assigned to Employee 3 is $\frac{2}{15}$.

**(3 Marks)**

**Question 4.**

Let $X \sim Bin(10,0.7)$ and $Y \sim Geo(0.5)$ be independent random variables.

a) Calculate:

| | | | |
|---|---|---|---|
| i) | $P(X=2)$ | ii) | $P(Y=2)$ |
| iii) | $P(X=2 \mid Y=2)$ | iv) | $P(X^4=16)$ |
| v) | $E(X)$ | vi) | $E(Y)$ |
| vii) | $P(-Y<-4)$ | viii) | $P(XY=0)$ |

**(8 Marks)**

b) Show that $P\left(\frac{Y}{X}=0.2\right) \approx 0.0585$

**(2 Marks)**

**Question 5.**

A $Pareto(m,\alpha)$ variable has probability density function

$$f(x)=\begin{cases} \frac{\alpha m^{\alpha}}{x^{\alpha+1}} & x\in[m,\infty) \\ 0 & \text{otherwise} \end{cases}.$$

a) Show that the cumulative density function is given by

$$F(x)=P(X\le x)=\begin{cases} 0 & x<m \\ 1-\frac{m^{\alpha}}{x^{\alpha}} & x\in[m,\infty) \end{cases}.$$

**(3 Marks)**

**Note:** For the question below, you do not need to give your answers as decimals. If, for example, your answer is $\frac{e^{-2}2^5}{5!}$ you can leave it in this form.

**Question 6.**

Faults may be reported to a Maintenance Division either by phone or online. On average, 12 faults are reported by phone per hour and 15 faults are reported online per hour.

Each fault is classified according to how long it will take to resolve. 75% of faults are classified as quick to resolve. 25% of faults are classified as requiring ongoing maintenance.

Model the reporting of faults by phone or online with independent Poisson processes, and assume that the type of fault is independent of how it is reported.

a) Calculate the probabilities of the following events:

i) No faults are reported by phone between 1:00pm and 1:10pm.

ii) Exactly two quickly resolved faults are reported between 2:00pm and 3:00pm.

iii) Between 3:00pm and 3:05pm, exactly two faults are reported, one by phone and one online.

iv) The time between the first reported quickly resolved fault and the second reported quickly resolved fault is greater than 15 minutes.

v) The next two reported faults are, in order, a quickly resolved fault reported by phone then a fault requiring ongoing maintenance reported online.

vi) No faults are reported by phone between 2:00pm and 2:10pm, given that exactly two faults are reported between 1:00pm and 2:00pm.

viii) No faults are reported between 2:00pm and 3:00pm, given that exactly two faults are reported by phone between 2:00pm and 2:10pm.

**(7 Marks)**

b) Write down the distributions of the following:

i) The time between the tenth and eleventh faults reported in a day.

ii) Out of the first 50 faults reported, the number which are quickly resolved faults which are reported online.

**(2 Marks)**

**Question 7.**

A competition is played by a number of teams and each year exactly one of these teams is declared the premiership winner. It is assumed that each team has an equal chance of winning each year, independent of the outcomes of all other years.

That is, if $n_k$ teams compete in year $k$, then the number of premierships won by team $j$ $j \in \{1, 2, ..., n_k\}$ in that year is given by $C_j(k) \sim Bern\left(\frac{1}{n_k}\right)$.

i) Explain why, even though $C_j(k) \sim Bern\left(\frac{1}{n_k}\right)$,

$C_1(k) + C_2(k) + ... + C_{n_k}(k)$ is not binomially distributed.

In the 9 years between 2007 and 2015 inclusive, the same 16 teams have competed each year.

One team is selected uniformly at random and labelled as team 1.

ii) Write down the distribution of $C_1(2007) + C_1(2008) + ... + C_1(2015)$, the number of premierships won by team 1 over those 9 years.

A supporter follows a team (wearing black, white and blue) which has competed every year between 1967 and 2015 inclusive. The total number of teams competing during that period has varied through the years as shown in the table below.

| Number of Teams | Number of Years | Years |
|---|---|---|
| 10 | 1 | 1997 |
| 12 | 15 | 1967-1981 |
| 13 | 4 | 1984-1987 |
| 14 | 4 | 1982-1983 and 2000-2001 |
| 15 | 5 | 2002-2006 |
| 16 | 16 | 1988-1994 and 2007-2015 |
| 17 | 1 | 1999 |
| 20 | 3 | 1995-1996 and 1998 |

iii) Show that the expected number of premierships won by the supporter's team during its 49 years is approximately 3.49.

iv) Calculate the probability that the supporter's team has never won the premiership in any of its 49 years.

**(6 Marks)**