University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
In-class Test
SOLUTIONS

Question 1.

i) Out of a sample of 20 people selected uniformly at random from the Australian population, the number of females selected, assuming an equal proportion of both males and females in the population $\sim Bin(20, 0.5)$ (assuming everyone is exactly one of male or female.)

ii) The number of times a critically-ill patient dies in a given week, assuming that his/her illness is such that the patient has a 50% chance of surviving the whole week $\sim Bern(0.5)$.

iii) Out of a sample of 20 members of the same family, the number who are colour blind, given that colour blindness is a genetically inherited condition which affects 7% of the population.
None of the three distributions is appropriate as each person's status is dependent on others as the condition is genetically inherited.

iv) The number of times a player simultaneously rolls two (regular fair six-sided) dice until the first time both land showing 5 $\sim Geo\left(\frac{1}{36}\right)$

v) The number of times a player rolls one (regular fair six-sided) die until the second time it lands showing 5.
(e.g. if it landed 2,3,5,1,6,4,5, then the fact it took seven rolls until the second 5 would be recorded.)
None of the three distributions is appropriate (The range of the variable is $\{2, 3, 4, ...\}$

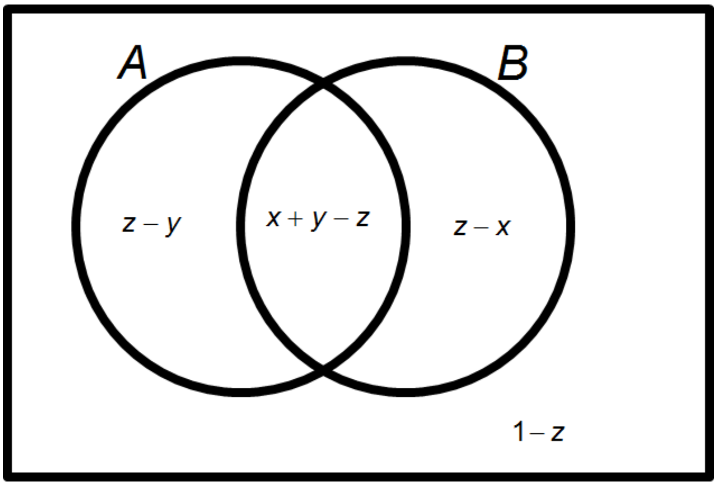
(5 Marks)

Question 2.

a)

$$P(A^c \cap B^c) = P(A \cup B)^c = 1 - z$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= x + y - z$$

$$P(A \cap B^c) = P(A) - P(A \cup \cap B) = x - (x + y - z) = z - y$$ and $$P(B \cap A^c) = P(B) - P(A \cup \cap B) = y - (x + y - z) = z - x$$



(4 Marks)

b) $P(A \cap B) = x + y - z$ hence $P(A|B) = \dfrac{x + y - z}{y}$.

(1 Mark)

c) If A and B are independent then $P(A)P(B) = P(A \cap B)$ so $xy = x + y - z$.

(1 Mark)

Question 3.

Let T be the event that the task is completed on time and let $E_1,...,E_5$ be the events that the jobs were assigned to Employee 1,…, Employee 5 respectively.

a) $$P(T)=P(T|E_1)P(E_1)+...+P(T|E_5)P(E_5)$$
$$=\frac{1}{5}\left(1^2+0.75^2+0.5^2+0.25^2+0^2\right)=\frac{3}{8}.$$

(3 Marks)

b) $$P(E_3|T)=\frac{P(E_3\cap T)}{P(T)}=\frac{\frac{1}{5}\left(0.5^2\right)}{\frac{3}{8}}=\frac{2}{15}.$$

(3 Marks)

Question 4.

a) i) $P(X=2)=\binom{10}{2}(0.7)^2(0.3)^8\approx 0.00145$

ii) $P(Y=2)=(0.5)^2=0.25$

iii) $P(X=2|Y=2)=P(X=2)=\binom{10}{2}(0.7)^2(0.3)^8\approx 0.00145$

iv) $P(X^4=16)=P(X=2)=\binom{10}{2}(0.7)^2(0.3)^8\approx 0.00145$

v) $E(X)=10\times 0.7=7$

vi) $E(Y)=\dfrac{1}{0.5}=2$

vii) $P(-Y<-4)=P(Y>4)=(0.5)^4=0.0625$

viii) $P(XY=0)=P(X=0)$ (since Y cannot be zero.)

$$P(X=0)=(0.3)^{10}$$

(8 Marks)

b) $$P\left(\frac{Y}{X}=0.2\right)=P(Y=1)P(X=5)+P(Y=2)P(X=10)\approx 0.0585$$
$$=(0.5)\binom{10}{5}(0.3)^5(0.7)^5+(0.5)^2(0.7)^{10}\approx 0.0515+0.007\approx 0.0585$$

(2 Marks)

Question 5.

a)

$$F(x)=P(X\le x)=\int_{-\infty}^{x}f(t)dt=\begin{cases}\int_m^x \frac{\alpha m^\alpha}{t^{\alpha+1}}dt & x\in[m,\infty)\\ 0 & x<m\end{cases}.$$

$$=\begin{cases}\alpha m^\alpha\left[\frac{-1}{\alpha t^\alpha}\right]_m^x & x\in[m,\infty)\\ 0 & x<m\end{cases}=\begin{cases}1-\frac{m^\alpha}{x^\alpha} & x\in[m,\infty)\\ 0 & x<m\end{cases}$$

(3 Marks)

Question 6.

a)

i) Number of faults reported by phone during a ten minute window $\sim Poi\left(\frac{12}{6}\right)$ hence the probability of no faults are reported by phone between 1:00pm and 1:10pm is $\dfrac{e^{-\frac{12}{6}}\left(\frac{12}{6}\right)^0}{0!} = e^{-2}$.

ii) Number of quickly resolved faults during an hour window $\sim Poi\left(0.75 \times (12+15)\right)$ hence the probability of getting exactly two is $\dfrac{e^{-\frac{81}{4}}\left(\frac{81}{4}\right)^2}{2!}$.

iii) Number of faults reported by phone during 5 minute window $\sim Poi\left(\frac{12}{12}\right)$ and number of faults reported online during 5 minute window $\sim Poi\left(\frac{15}{12}\right)$.

The probability both these variables equal one is $\left(\dfrac{e^{-1}1^1}{1!}\right)\left(\dfrac{e^{-\frac{5}{4}}\left(\frac{5}{4}\right)^1}{1!}\right) = \dfrac{5e^{-\frac{9}{4}}}{4}$.

iv) The probability that the time between the first reported quickly resolved fault and the second reported quickly resolved fault is greater than 15 minutes is equal to the probability that there are no quickly resolved faults during a 15 minute period. Number of quickly resolved faults during 15 minutes $\sim Poi\left(\dfrac{0.75 \times (12+15)}{4}\right)$ hence this probability is $\dfrac{e^{-\frac{81}{16}}\left(\frac{81}{16}\right)^0}{0!} = e^{-\frac{81}{16}}$.

v) The probability that next two reported faults are, in order, a quickly resolved fault reported by phone then a fault requiring ongoing maintenance reported online is $\left(\dfrac{0.75 \times 12}{27}\right)\left(\dfrac{0.25 \times 15}{27}\right)$

vi) The number of faults are reported by phone between 2:00pm and 2:10pm is independent of the fact that that exactly two faults are reported between 1:00pm and 2:00pm. The number of faults by phone in a ten minute period is (as in part i)) e^{-2}.

viii) We already know that exactly two faults are reported by phone between 2:00pm and 2:10pm, so the probability that no faults are reported between 2:00pm and 3:00pm is 0.

(7 Marks)

b)

i) The time between the tenth and eleventh faults reported in a day (in hours) $\sim Exp(27)$

ii) Each call (independently) is a quickly resolved fault which is reported online with probability $\frac{0.75\times 15}{27}$ hence the number out of the first 50 calls which meet this criterion $\sim Bin\left(50, \frac{5}{12}\right)$.

(2 Marks)

Question 7.

i) Even though $C_j(k) \sim Bern\left(\frac{1}{n_k}\right)$,

$C_1(k)+C_2(k)+...+C_{n_k}(k)$ is not binomially distributed because the Bernoulli variables are dependent, as they must certainly sum to 1 (knowing that, for example, team 1 has won the premiership, we know for certain that the number of premierships won that year by the other $n_k - 1$ teams is zero.)

ii) $C_1(2007)+C_1(2008)+...+C_1(2015) \sim Bin\left(9, \frac{1}{16}\right)$

iii) Let the team in question be labelled team *j*.

$E(C_j(1967)+C_j(1968)+...C_j(2015))$

$$=\left(\frac{1}{10}\right)+\left(\frac{15}{12}\right)+\left(\frac{4}{13}\right)+\left(\frac{4}{14}\right)+\left(\frac{5}{15}\right)+\left(\frac{16}{16}\right)+\left(\frac{1}{17}\right)+\left(\frac{3}{20}\right)\approx 3.49$$

iv) Each year when n_k compete, each team fails to win the premiership with probability $\frac{n_k - 1}{n_k}$.

$P(C_j(1967)+C_j(1968)+...C_j(2015)) = 0$

$$=\left(\frac{9}{10}\right)\left(\frac{11}{12}\right)^{15}\left(\frac{12}{13}\right)^{4}\left(\frac{13}{14}\right)^{4}\left(\frac{14}{15}\right)^{5}\left(\frac{15}{16}\right)^{16}\left(\frac{16}{17}\right)\left(\frac{19}{20}\right)^{3}\approx 0.0268$$

(6 Marks)