University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161)
Class Test Group 3 (5:00pm – 6:30pm Tuesday)
Please submit solutions via Canvas within five minutes of the completion of the tutorial time. Work arriving late may not be marked. It is your responsibility to ensure that all files you submit are readable. This is an individual assessment item and your work must be your own, however you may confer or consult with other students throughout the tutorial time.

Question 1.

a) For each of the following variables, find the range of the random variable:

i) The number of cards drawn from a standard deck of 52 playing cards until the first Queen is selected, assuming cards are not returned to the deck after each draw.

ii) The number of times a regular fair six-sided die is rolled until the first time it lands on a number greater than 4.

iii) The number of Heads shown on a single flip of a coin which lands Tails with probability 70%.

iv) From a household of 5 people, the number of people whose surname begins with a vowel, assuming 15% of the population has a surname beginning with a vowel.

v) Out of 10 trials of flipping two fair coins simultaneously, the number of times that both coins land Tails.

vi) When flipping a fair coin repeatedly, the number of additional flips between the seventh time the coin lands Heads and the seventeenth time the coin lands Heads.

(6 Marks)

b) For each of these, state whether it could reasonably be modelled by a Bernoulli variable, a Binomial variable, a Geometric variable, or none of the above. In each case, state the parameter(s) needed. Where none of these distributions is appropriate, briefly explain why not.

(6 Marks)

Question 2.

Let X, Y and Z be events in a sample space Ω such that

$$P(X) = 0.4,\ P(Y) = 0.5 \text{ and } P(X \cup Y) = 0.75.$$

a) Calculate the following:

i) $P(X \cap Y)$; ii) $P(Y^c)$; iii) $P(X|Y)$;

iv) $P(Y|X)$; v) $P(X^c \cap Y^c)$; vi) $P(Y|X^c \cap Y^c)$.

Hint: It may help to draw a Venn diagram.

(6 Marks)

Let Z be such that $P(X \cap Y \cap Z) = 0.15$, $Z \subseteq X$ and $Z \subseteq Y$.

b) Calculate the following:

i) $P(Z \mid X)$; ii) $P(X \mid Z)$; iii) $P(Z^c)$.

(5 Marks)

Question 3.

Let X be a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{\sin(x)+4}{8\pi} & -\pi < x < \pi \\ 0 & \text{otherwise} \end{cases}.$$

Explaining your reasoning, find:

i) $E(X)$; ii) $P\left(x > \frac{\pi}{2} \middle| x > 0\right)$.

Hint: You may not need to perform any calculations for these, but can answer via written explanation if you prefer.

(4 Marks)

Question 4.

Consider a random experiment which consists of rolling a regular fair six-sided die once and observing the number shown. This has sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$.

Write down three possible events A, B and C such that A and B are independent and A and C are mutually exclusive.

Your answer may be in the form of sets (e.g. $X = \{6\}$) or in words (e.g. "X is the event that the die shows a 6.")

(3 Marks)

Question 5.

Let Y be a random variable with probability mass function

$$P(Y=k)=\begin{cases} 0.1 & k=0 \\ \dfrac{1}{k} & k\in\{2,4,8\} \\ 0.025 & k=-4 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that:

i) $E(Y)=2.9$; ii) $E(Y^2)=14.4$.

(2 Marks)

b) Showing all your working, calculate:

i) $Var(Y)$; ii) $P(Y>0)$; iii) $P(Y^2>0)$;

iv) $P(Y=4\,|\,Y^2=16)$; v) $P(Y=6\,|\,Y^2=36)$.

(6 Marks)

Question 6.

A player rolls a regular fair six-sided die repeatedly and notes the number shown each time. He/she also draws one of three cards with each card equally likely to be selected.

If the player selects Card A, he/she scores a point every time the die shows an even number and zero points otherwise.

If the player selects Card B, he/she scores zero points every time the die shows a number divisible by 3 and one point otherwise.

If the player selects Card C, he/she scores zero points every time the die shows a number divisible by 6 and one point otherwise.

a) The player selects a card once, then rolls the die three times.

i) Show that the probability that the player scores zero points off his/her three rolls is $\frac{1}{18}$.

ii) Given that the player rolls the die three times and scores zero points, calculate the probability that the card chosen was Card B.

(4 Marks)

b) The player selects a card once, then rolls the die three times and is told the number of points he/she has scored.

There are two possible sequences of three outcomes for which the conditional probability of having selected each card given the observed score is the same as the unconditional probability without knowing the score? Write down either of these two sequences and justify your answer.

(3 Marks)