University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 10 Preparation Work

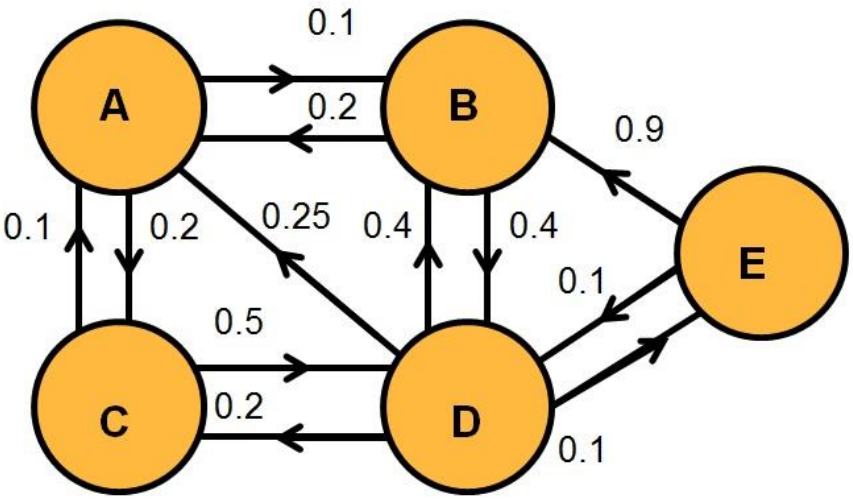
1.

a) Draw the state diagram corresponding to the transition matrix

$$P = \begin{pmatrix} 0.3 & 0.7 & 0 & 0 & 0 \\ 0.1 & 0.5 & 0.2 & 0.2 & 0 \\ 0 & 0.3 & 0.4 & 0.3 & 0 \\ 0 & 0.1 & 0.1 & 0.6 & 0.2 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

clearly labelling all possible transitions with directed arrows weighted with appropriate probabilities.

b) Write down the transition matrix corresponding to the state diagram given below:



2. A motorway is assessed each workday and classified as being in one of three states – clear, average or congested.

On 50% of workdays, the state of the road is the same as it was the previous day. On days when the road is clear, it is average the following workday with probability 50%. On days when the road is average, it is either clear or congested on the following workday each with probability 25%. On days when the road is congested, it is either clear or average on the following workday each with probability 25%.

a) Write down the transition matrix for this situation.

b) Calculate the 2-step transition matrix (i.e. that which contains the conditional probabilities for the states two days ahead given the present day's state.)

c) i) On a Monday, the road is classified as congested. What is the probability that the road is again congested on Wednesday of the same week.

ii) Assuming that you have no recent observations of the road's classification, what are the probabilities that the road is classified into each of the three states on a given day?