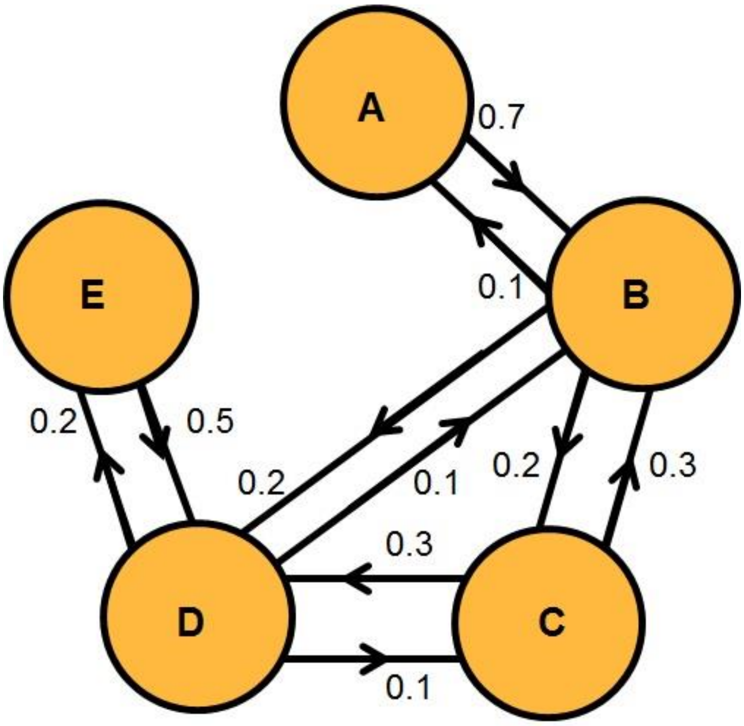
University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 10 Preparation Work
SOLUTIONS

1.

a)



b)

$$P = \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0.4 & 0 \\ 0.1 & 0 & 0.4 & 0.5 & 0 \\ 0.25 & 0.4 & 0.2 & 0.05 & 0.1 \\ 0 & 0.9 & 0 & 0.1 & 0 \end{pmatrix}$$

2.

a) $$P = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$$

b) $$P^2 = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}^2 = \begin{pmatrix} 0.375 & 0.5 & 0.125 \\ 0.3125 & 0.4375 & 0.25 \\ 0.3125 & 0.375 & 0.3125 \end{pmatrix}$$

c) i) On a Monday, the road is classified as congested. What is the probability that the road is again congested on Wednesday of the same week.

$$P(\text{Wednesday congested}|\text{Monday congested}) =$$

$$\begin{bmatrix} P(\text{Wednesday congested}|\text{Tuesday congested}) \\ \times P(\text{Tuesday congested}|\text{Monday congested}) \end{bmatrix}$$

$$+\begin{bmatrix} P(\text{Wednesday congested}|\text{Tuesday average}) \\ \times P(\text{Tuesday average}|\text{Monday congested}) \end{bmatrix}$$

$$+\begin{bmatrix} P(\text{Wednesday congested}|\text{Tuesday clear}) \\ \times P(\text{Tuesday clear}|\text{Monday congested}) \end{bmatrix}$$

$$= (0.5 \times 0.5) + (0.25 \times 0.25) + (0.25 \times 0) = 0.3125 .$$

(or could simply read off from the 2-step transition matrix in part b).)

ii) We need to find Π_{eq} such that $\Pi_{eq} P = \Pi_{eq}$.

Let $\Pi_{eq} = \begin{pmatrix} a & b & c \end{pmatrix}$ then we need $a+b+c=1$ such that

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix} = \begin{pmatrix} a & b & c \end{pmatrix}.$$

This gives

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix} = \begin{pmatrix} a & b & c \end{pmatrix}$$

$$\begin{aligned} 0.5a + 0.25b + 0.25c = a \quad &\Rightarrow \quad 2a = b + c \\ 0.5a + 0.5b + 0.25c = b \quad &\Rightarrow \quad 2a + c = 2b \\ 0.25b + 0.5c = c \quad &\Rightarrow \quad b = 2c \end{aligned}.$$

These are solved by $\Pi_{eq} = \begin{pmatrix} \frac{1}{3} & \frac{4}{9} & \frac{2}{9} \end{pmatrix}$.