## University of Technology Sydney School of Mathematical and Physical Sciences

Probability and Random Variables (37161) – Class 1 Preparation Work

- 1. Two fair regular six-sided dice are rolled. The sample space,  $\Omega$ , therefore contain s 36 equally likely possible outcomes  $\Omega = \{11, 12, 13, 14, 15, 16, 21, 22, ..., 64, 65, 66\}$
- i) The probability that the second die shows a larger number than the first is  $\frac{\left|\{12,13,14,15,16,23,24,25,26,34,35,36,45,46,56\}\right|}{|\Omega|} = \frac{15}{36} = \frac{5}{12}.$

 $|\Omega| = \frac{1}{36} = \frac{1}{12}.$ 

- ii) The probability that the sum of the values is 8 is  $\frac{|\{26,35,44,53,62\}|}{|\Omega|} = \frac{5}{36}.$
- iii) The probability that the product of the two values is a multiple of 10 is  $\frac{|\{25, 45, 65, 52, 54, 56\}|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}.$
- If 80% have passed A, including 35% who have passed A and B, then 45% must have passed only A.
  100% of students have passed A or B. If 80% have passed A, then the remaining 20% must have passed B but not A.
  Together, these give a total of 55% who have passed B.



3. A random experiment consists of selecting one person at random. The sample space for this experiment is the set of all living people.

Consider the following events:

- $X_1$ : The set of all people living in Australia
- $X_2$ : The set of all people enrolled at UTS
- $X_3$ : The set of all people enrolled in a university course
- $X_4$ : The set of all adults
- $X_{s}$ : The set of all people living in Australia or New Zealand
- $X_{6}$ : The set of all children living in New Zealand
- $X_7$ : The set of all adults living in Australia
- $X_{s}$ : The set of all children living in Australia
- i) The following pairs are mutually exclusive:
- $X_1 \& X_6$  (a child living in New Zealand cannot also be living in Australia)
- $X_4 \& X_6$  (an adult cannot also be a child living in New Zealand)
- $X_4 \& X_8$  (an adult cannot also be a child living in Australia)
- $X_6 \& X_7$  (an child living in New Zealand cannot also be an adult living in Australia)
- $X_{_{6}} \& X_{_{8}}$  (an child living in New Zealand cannot also be a child living in Australia)

(Arguably also others, depending on if you believe all people enrolled at university are adults and/or if you believe all UTS students are living in Australia.)

 $\begin{array}{ll} \mathsf{ii} \mathsf{)} & X_7 \subseteq X_1 & X_8 \subseteq X_1 & X_1 \subseteq X_5 \\ & X_6 \subseteq X_5 & X_7 \subseteq X_5 & X_8 \subseteq X_5 \\ & X_2 \subseteq X_3. \end{array}$ 

(Arguably also others, depending on if you believe all people enrolled at university are adults and/or if you believe all UTS students are living in Australia.)

- iii)  $X_1 = X_7 \cup X_8$
- iv)  $X_7 = X_1 \cap X_4$
- $\mathbf{V}) \qquad \mathbf{X}_{6} = \mathbf{X}_{1}^{c} \cap \mathbf{X}_{4}^{c} \cap \mathbf{X}_{5}.$