University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 2 Preparation Work
SOLUTIONS

1. i) $P(\text{first three red} \mid \text{first one red}) = \dfrac{\frac{1}{2}\times\frac{25}{51}\times\frac{24}{50}}{\frac{1}{2}} = \dfrac{12}{51}$

ii) $P(\text{first three red} \mid \text{first one diamond}) = \dfrac{\frac{1}{4}\times\frac{25}{51}\times\frac{24}{50}}{\frac{1}{4}} = \dfrac{12}{51}$

iii) $P(\text{fifth and six are aces} \mid \text{one ace in first four}) = \dfrac{3}{48}\times\dfrac{2}{47} = \dfrac{1}{376}$

iv) Of the 12 picture cards which could be drawn first, only four are jacks (the conditional information is irrelevant here.)
$P(\text{first picture card jack} \mid \text{first four 2, 3 or 4}) = \dfrac{1}{3}$.

2.

i) $P(F|A) = 0.1, P(F|B) = 0.15$ and $P(F|C) = 0.25$.

ii)
$$P(C|F^c) = \frac{P(C\cap F^c)}{P(F^c)} = \frac{P(C\cap F^c)}{P(A\cap F^c)+P(B\cap F^c)+P(C\cap F^c)}$$
$$\frac{0.25\times 0.75}{(0.5\times 0.9)+(0.25\times 0.85)+(0.25\times 0.75)}$$
$$\approx 0.22$$

3.

i) Let the proportion of *t* alleles ("non-taster" alleles) in the population be *p*. A person is a "non-taster" if and only if he/she has two *t* alleles. If a proportion *p* of alleles are "non-taster" alleles, then someone is "non-taster" is both his/her maternal allele and paternal allele are both *t*. This occurs with probability $p \times p = p^2$. Since 30% of the population are "non-tasters", $p^2 = 0.3$ hence $p \approx 0.548$.

ii) If both parents "non-tasters" (*tt*) it is certain (i.e. probability of 1) that all children they have will be "non-taster" (*tt.*)

iii) If the mother is *tt*, all of her children will have at least one *t*. If the father has alleles *Tt,* 50% of his children will inherit a *T* and the other 50% will inherit a *t*. Thus, each child (independently) has a 50% chance of being a "non-taster". The probability that all three children will be "non-tasters" is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

4. (See http://en.wikipedia.org/wiki/Simpson%27s_paradox)

a)

i) For Derek Jeter,

$$P(H) = P(H|Y = 1995)P(Y = 1995) + P(H|Y = 1996)P(Y = 1996) + P(H|Y = 1997)P(Y = 1997)$$

$$P(H) = 0.250\left(\frac{48}{1284}\right) + 0.314\left(\frac{582}{1284}\right) + 0.291\left(\frac{654}{1284}\right) \approx 0.300\,.$$

ii) For David Justice,

$$P(H) = P(H|Y = 1995)P(Y = 1995) + P(H|Y = 1996)P(Y = 1996) + P(H|Y = 1997)P(Y = 1997)$$

$$P(H) = 0.253\left(\frac{411}{1046}\right) + 0.321\left(\frac{140}{1046}\right) + 0.329\left(\frac{495}{1046}\right) \approx 0.298\,.$$

b)

i) Each year, David Justice had the better batting average.

ii) Averaged over all three years, however, Derek Jeter had the better batting average.

c) Although Justice had the better year on year average, he did take over 40% of his bats in 1995 when he only batted at 0.253. Compare this with Jeter, whose worst year (1995) consisted of only around 4% of his total times at bat. In other words, Jeter had more bats in his better years, whereas Justice had a much greater proportion in his weakest year.