University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 3 Preparation Work

1. Consider a random experiment which consists of flipping one (possibly biased) coin twice. Each time the coin is flipped is an independent trial such that the probability that it lands Heads is p and the probability that it lands Tails is $1-p$.

   Let N be the number of Heads obtained flipping the coin twice.

   i) Write down the probability mass function of N.

   ii) Show that $E(N) = 2p$.

   iii) Calculate $E(N^2)$ and hence show that $Var(N) = 2p(1-p)$

2. A game consists of three people (player A, player B and player C) rolling a regular fair six sided die, in the order A, B, C, A, B, C, A,… etc. The game ends when one player first rolls a five or a six.

   Let A be the event that Player A wins, B be the event that Player B wins and C be the event that Player C wins.

   i) Explain why $P(A)$ is given by the infinite series $P(A) = \frac{1}{3} + \frac{1}{3}\left(\frac{2}{3}\right)^3 + \frac{1}{3}\left(\frac{2}{3}\right)^6 + \frac{1}{3}\left(\frac{2}{3}\right)^9 + \frac{1}{3}\left(\frac{2}{3}\right)^{12} + ...$

   ii) Evaluate the series to find the value of $P(A)$.

   iii) Find similar series for $P(B)$ and $P(C)$.

   iv) Hence find the values of $P(B)$ and $P(C)$ and hence verify that $P(A) + P(B) + P(C) = 1$.

3. Consider a random experiment which consists of drawing a card at random from a standard deck of playing cards, where each card is equally likely to be selected.

Each time an even numbered card (2, 4, 6, 8 or 10) is chosen, the player scores a number of points equal to the number of the card chosen. Each time an odd numbered card (Ace, 3, 5, 7 or 9) is chosen he scores no points. Each time a picture card (Jack, Queen or King) is chosen, he scores -10 points.

The probability mass function for the number of points, X, he scores on one game is therefore

$$P(X=k)=\begin{cases} \dfrac{5}{13} & k=0 \\ \dfrac{1}{13} & k=2 \\ \dfrac{1}{13} & k=4 \\ \dfrac{1}{13} & k=6 \\ \dfrac{1}{13} & k=8 \\ \dfrac{1}{13} & k=10 \\ \dfrac{3}{13} & k=-10 \\ 0 & \text{otherwise} \end{cases}$$

i) Find the expected number of points scored in one game, $E(X)$.

ii) Write down the probability mass function of X^2.

iii) Calculate $E(X^2)$.

iv) Hence show that $Var(X)=40$.

4. Recall that two events A and B are independent if and only if $P(A \cap B) = P(A) \times P(B)$.

a) If $P(A) = 0$ or $P(A) = 1$, show that A is independent of all possible events B.

The above result should be reasonably intuitive, since if A either never happens ($P(A) = 0$) or always happens ($P(A) = 1$), then knowledge of whether or not it has occurred cannot give us any information about any other events.

b) If $P(A) > 0, P(B) > 0$ and $P(A) < P(A|B)$, show that $P(B) < P(B|A)$.

Again, the above result should be reasonably intuitive, since if $P(A) < P(A|B)$ then there is some positive correlation between A and B i.e. knowledge of one occurring makes the other more likely to occur.

For example, let our experiment be selecting one living person at random and let A be the event that we select an adult and B be the event that the person selected is over 1.50m tall.

Clearly, $P(A) < P(A|B)$ since we know that having selected a taller person makes it more likely that we have selected an adult. We also have that if we know that we have selected an adult means that we are more likely to have selected a taller person hence $P(B) < P(B|A)$.

c)

i) If the event A is independent of itself, what can we say about $P(A)$?

ii) In words, explain the result in part i)