

University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 5 Preparation Work

1. Let X be a random variable with density function.

$$f(x) = \begin{cases} kx^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- i) Find the value of k and hence write down the cumulative distribution function of X .
- ii) Calculate the expected value of X .
- iii) Calculate $P(0.3 < X < 0.7)$.

2. Let Y be a random variable with density function

$$f(y) = \begin{cases} \frac{k}{y} & 1 < y < 10 \\ 0 & \text{otherwise} \end{cases}$$

where k is a real constant.

- a) Show that $k = \frac{1}{\ln(10)}$
- b) Find the following:
 - i) $P(Y > 2)$
 - ii) $E(Y)$
 - iii) $\text{Var}(Y)$.

3. Let Z be a uniform random variable $Z \sim U[1, M]$ with density function

$$f(z) = \begin{cases} 3 & 1 < z < M \\ 0 & \text{otherwise} \end{cases}$$

where M is a real constant.

- a) Find the value of M .
- b) Find the following:
 - i) $E(Z)$
 - ii) $\text{Var}(Z)$.

4. Let T be a random variable with density function.

$$f(t) = \begin{cases} k|t| & -1 < t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

where $|t|$ is the absolute value of t , i.e. $|t| = \max\{t, -t\}$ and k is a constant.

(Hint: It might help you to sketch the function $f(t)$)

- a) Show that $k = \frac{1}{5}$.
- b) Show that $P(T < 0) = 0.1$
- c) Calculate the following:
 - i) $E(T)$
 - ii) $P(T > 2 | T > 1)$
 - iii) $P(T > 3 | T > 2)$