

University of Technology Sydney
School of Mathematical and Physical Sciences

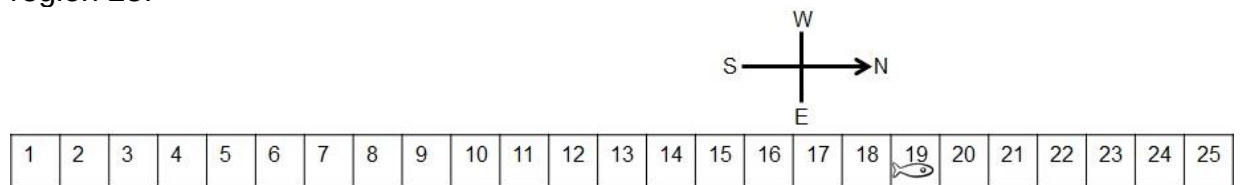
Probability and Random Variables (37161) –
 Class 9 Preparation Work

1.

The movement of a fish in a marine park is monitored over time. Its habitat is divided into 25 sections, each 1m long. These are arranged adjacently running north to south. Its position is measured every 10 seconds and its swimming speed is such that it cannot move more than one section in either direction between successive observations.

It is noted that, 80% of the time, it has not changed its section between successive observations. 10% of the time it has moved one section north and 10% of the time it has moved one section south.

The furthest section south is labelled region 1 and the furthest section north is region 25.



Let N_k be the probability that, starting from section k , the fish is first observed in the furthest north section (section 25) of the habitat before it is first observed in the furthest south (section 1)

- i) Show that N_k must satisfy $2N_k = N_{k-1} + N_{k+1}$. Clearly explain all steps in your calculation.
- ii) Write down the boundary conditions for this problem.
- iii) Verify that the difference equation in part i) can be solved by any function of the form $N_k = A + Bk$ where A and B are constants.
 Note: You do not need to derive this result, but can simply verify it by substituting in.
- iv) When the study begins, the fish is in section 19. Show that the probability that the fish is observed in section 25 before it is first observed in section 1 is exactly 75%.

Let O_k be the expected number of additional observations until the fish is first observed in either section 1 or section 25, given that it is in section k .

- v) Write down the boundary conditions for this problem.
- vi) O_k satisfies $2O_k = O_{k-1} + O_{k+1} + 10$.
 Which value for k maximises the expected time until the fish is observed at either end of the habitat? Justify your answer. (Note: you do not need to solve to find O_k explicitly)