University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) –
Class 9 Preparation Work
SOLUTIONS

1.

i) Conditioning on the next move, we obtain that 80% of the time the fish is in the same location and 10% of the time it has moved either north one section or south one section. This gives $N_k = 0.1N_{k-1} + 0.1N_{k+1} + 0.8N_k$ hence $2N_k = N_{k-1} + N_{k+1}$.

ii) $N_1 = 0, N_{25} = 1$.

iii) We can see that the solution is of the form $N_k = A + Bk$ since if this is the case then $N_{k-1} = A + B(k-1)$ and $N_{k+1} = A + B(k+1)$. It is easy to verify that this satisfies the difference equation $2N_k = N_{k-1} + N_{k+1}$ since $2N_k = 2A + 2Bk$ and $N_{k-1} + N_{k+1} = A + B(k-1) + A + B(k+1) = 2A + 2Bk$ are clearly equal.

iv) Substituting in initial conditions gives that $N_k = \frac{k-1}{24}$.

Specifically, $N_{19} = \frac{19-1}{24} = 0.75$.

v) If the fish is at either extreme end, no further moves are required. $O_1 = 0, O_{25} = 0$.

vi) The maximum number of expected moves will be when the fish is in the middle of the habitat i.e. furthest from either stopping point. This gives a maximal value of O_k when $k = 13$.