University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) – Tutorial/Laboratory 1

1. Consider rolling a regular (fair, six-sided) die three times. Assume that the outcome of each roll is independent of that of all others.

   What is the probability of:

   i) obtaining an even number on your first roll?

   ii) obtaining the same outcome all three times?

   iii) obtaining a number on your second roll which is divisible by the number obtained on your first?

   iv) obtaining a number on your third roll which is equal to the sum of the values obtained on the first two rolls?

2. Assume that each child is born male with probability 50% and born female with probability 50% and that that sex of each child is independent of that of all others.

a) What is the probability that a given family with three children:

   i) has an eldest child who is female?

   ii) has no daughters, but three sons?

   iii) has only one sex of child?

   iv) has two consecutively born children of the same sex?

b) If you know that the family has three children, and at least one child is male, what is the probability that:

   i) all three of the children are male?

   ii) the eldest child is male?

   iii) the family has two daughters?

3. Let *A, B* and *C* be events such that:

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3} \text{ and } P(C) = \frac{1}{4}$$

$$P(A \cap B) = \frac{1}{4},\ P(A \cap C) = \frac{1}{8} \text{ and } P(B \cap C) = \frac{1}{12}$$

$$P(A \cap B \cap C) = \frac{1}{48}.$$

Calculate the probability that none of the events *A, B* and *C* occur.

Hint: It may help to draw a Venn diagram.

4. Let *A* and *B* be events such that $P(A) = \frac{9}{10}$ and $P(B) = \frac{1}{5}$.

a) Show that $\frac{1}{10} \leq P(A \cap B) \leq \frac{1}{5}$.

Explain your reasoning clearly.

b) Find a similar inequality for the maximum and minimum value that $P(A \cup B)$ can take.

Explain your reasoning clearly.