University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) – Tutorial/Laboratory 1 SOLUTIONS

1.

a) Let Ω_i be the sample space for the outcome of *i* rolls of the die so that

$\Omega_1 = \{1,2,3,4,5,6\}$, $\Omega_2 = \{11,12,13,14,15,16,21,...,65,66\}$ etc.

i) $$P = \frac{|\{2,4,6\}|}{|\Omega_1|} = \frac{1}{2}$$

ii) $$P = \frac{|\{111,222,333,444,555,666\}|}{|\Omega_3|} = \frac{6}{6\times 6\times 6} = \frac{1}{36}.$$

iii) $$P = \frac{|\{11,12,13,14,15,16,22,24,26,33,36,44,55,66\}|}{|\Omega_2|} = \frac{7}{18}.$$

iv)
$$P = \frac{|\{112,123,134,145,156,213,224,235,246,314,325,336,415,426,516\}|}{|\Omega_3|}$$
$$= \frac{15}{216} = \frac{5}{72}$$

2.

a) $\Omega = \{FFF,FFM,FMF,FMM,MFF,MFM,MMF,MMM\}$.

i) $$P = \frac{|\{FFF,FFM,FMF,FMM\}|}{|\Omega|} = \frac{1}{2}$$

ii) $$P = \frac{|\{MMM\}|}{|\Omega|} = \frac{1}{8}$$

iii) $$P = \frac{|\{FFF,MMM\}|}{|\Omega|} = \frac{1}{4}$$

iv) $$P = \frac{|\{FFF,FFM,MFF,MMM,MMF,FMM\}|}{|\Omega|} = \frac{3}{4}$$

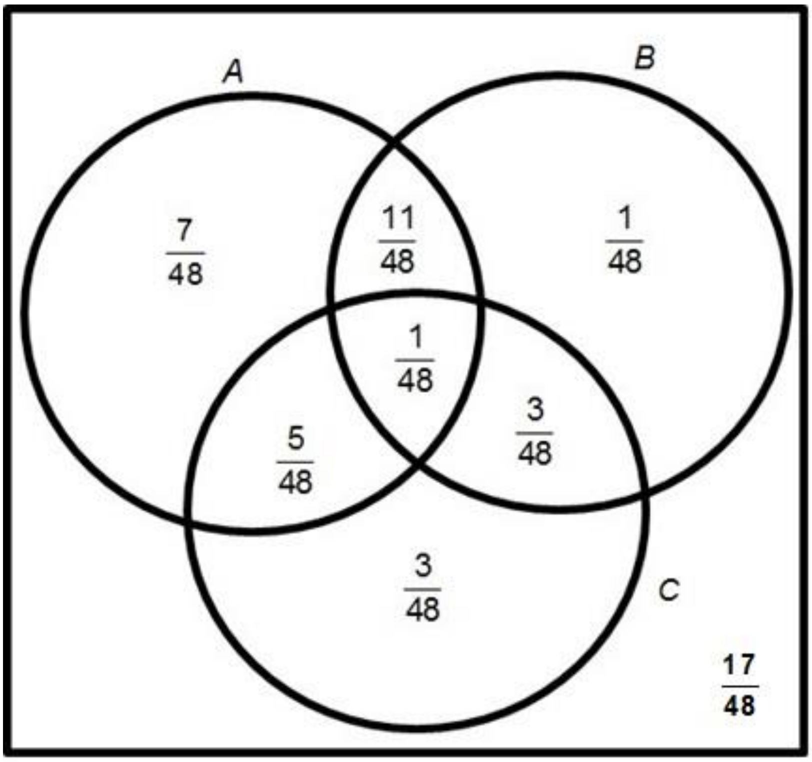
b) The conditional sample space, knowing there is at least one son is $\Omega_s = \{FFM, FMF, FMM, MFF, MFM, MMF, MMM\}$.

i) $P = \dfrac{|\{MMM\}|}{|\Omega_s|} = \dfrac{1}{7}$

ii) $P = \dfrac{|\{MMM, MMF, MFM, MFF\}|}{|\Omega_s|} = \dfrac{4}{7}$

iii) $P = \dfrac{|\{MFF, FMF, FFM\}|}{|\Omega_s|} = \dfrac{3}{7}$

3.



The probability that none of the events A, B or C occur is $\dfrac{17}{48}$.

4.

a)

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$ so $P(A \cap B) \leq P(A)$ and $P(A \cap B) \leq P(B)$.

These give $P(A \cap B) \leq \frac{9}{10}$ and $P(A \cap B) \leq \frac{1}{5}$ hence the more restrictive of these gives that $P(A \cap B) \leq \frac{1}{5}$.

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Here, this is $P(A \cup B) = \frac{9}{10} + \frac{1}{5} - P(A \cap B)$ however $P(A \cup B) \leq 1$ since it is a probability. This gives $P(A \cap B) \geq \frac{9}{10} + \frac{1}{5} - 1 = \frac{1}{10}$.

b)

$A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$ so $P(A) \leq P(A \cup B)$ and $P(B) \leq P(A \cup B)$. Here, this gives $\frac{9}{10} \leq P(A \cup B)$

Similarly, $P(A \cup B)$ is a probability so cannot exceed 1.