University of Technology Sydney
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Probability and Random Variables (37161) – Tutorial/Laboratory 2

1. A bag contains two yellow balls and one green ball. During each draw, one ball is selected at random from the bag with all equally likely to be chosen. After being drawn, its colour is noted and it is placed back in the bag.

   a) Calculate the probabilities of the following events:

      i) The first ball drawn is green;

      ii) The first two balls drawn are both yellow;

      iii) The first four balls drawn are all the same colour

   b) Show that, given that the first three drawn are all the same colour, the probability that the first four balls drawn are all the same colour is $\frac{17}{27}$.

2. A box contains two coins. One of the coins is fair, the other has Heads on both sides. A player randomly selects one the coins (each with equal probability) and flips it three times.

   Without knowing which of the two coins the player has selected, calculate:

   i) the probability that the coin lands Heads all three times.

   ii) the probability that the player has selected the fair coin, given that it lands Heads all three times.

3. Consider two events A and B such that $P(A \cap B^c) = 0.1$ and $P(A^c \cap B) = 0.4$.

   Calculate $P(A \cap B)$:

   i) if $P(A \cup B) = 0.65$;

   ii) if A and B are mutually exclusive;

   iii) if A and B are independent.

4. Tasks are assigned at a workplace to any one of five employees.

All tasks assigned to Employee 1 are completed on time.
Tasks assigned to Employee 2 are completed on time with probability 75%
Tasks assigned to Employee 3 are completed on time with probability 50%
Tasks assigned to Employee 4 are completed on time with probability 25%
No tasks assigned to Employee 5 are completed on time.
(Assume that the completion of each task is independent of the completion of all other tasks.)

One employee is selected at random with each of the five employees equally likely to be chosen. He/she is assigned two tasks to complete.

i) Calculate the probability that both tasks assigned to the selected employee are completed on time.

ii) Given that both tasks are completed on time, show that the probability that the tasks were assigned to Employee 3 is $\frac{2}{15}$.

5.

a) Cards are selected at random from a standard deck of 52 with all cards equally likely to be chosen. After each card is drawn, its number is noted and it placed back into the deck and all cards shuffled.

i) What is the probability that the first card drawn is a 3 or a 7?

ii) What is the probability that the first two cards drawn are both picture cards (Jack, Queen or King)?

iii) Show that, given that the first card is a picture card, the probability that the first two cards drawn are both Kings is $\frac{1}{39}$.

b) Assume now that cards are still selected as before, except that they are **not** returned into the deck after being selected.

i) What is the probability that the first card drawn is a 3 or a 7?

ii) What is the probability that the first two cards drawn are both picture cards (Jack, Queen or King)?