University of Technology Sydney
School of Mathematical and Physical Sciences

Probability and Random Variables (37161) – Tutorial/Laboratory 2
SOLUTIONS

1.

a)

i) The first ball drawn is green with probability $\frac{1}{3}$.

ii) The first two balls drawn are both yellow with probability $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9}$.

iii) The first four balls drawn are all the same colour with probability $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)=\frac{17}{81}$.
(Note the first term of this sum is the chance all four are yellow and the second term is the chance all four are green.)

b) The probability that he first four balls drawn are all the same colour, given that the first three are all the same colour is

$$\frac{\left(\frac{17}{81}\right)}{\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}=\frac{17}{27}.$$

2.

Let *HHH* be the event that three heads are obtained and *F* be the event that the fair coin is selected.

i) $P(HHH)=P(HHH|F)P(F)+P(HHH|F^c)P(F^c)=\frac{1}{8}\times\frac{1}{2}+1\times\frac{1}{2}=\frac{9}{16}$.

ii) $P(F|HHH)=\frac{P(F\cap HHH)}{P(HHH)}=\frac{\frac{1}{2}\times\frac{1}{8}}{\frac{1}{8}\times\frac{1}{2}+1\times\frac{1}{2}}=\frac{1}{9}$.

3.

i) $P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$ hence if $P(A \cup B) = 0.65$ then $P(A \cap B) = 0.65 - 0.1 - 0.4 = 0.15$.

ii) If A and B are mutually exclusive then $P(A \cap B) = 0$.

iii) Let $P(A \cap B) = x$.

This then gives $P(A) = [0.1 + x]$ and $P(B) = [0.4 + x]$.

For independence, we need $P(A \cap B) = P(A) \times P(B)$ i.e. we solve to find x such that $x = [0.1 + x] \times [0.4 + x]$.
This gives $x^2 - 0.5x + 0.04 = 0$, hence $x = P(A \cap B) = 0.1$ or 0.4.

4.

Let T be the event that the task is completed on time and let $E_1, ..., E_5$ be the events that the jobs were assigned to Employee 1,…, Employee 5 respectively.

i) $P(T) = P(T | E_1)P(E_1) + ... + P(T | E_5)P(E_5)$
$= \frac{1}{5}\left(1^2 + 0.75^2 + 0.5^2 + 0.25^2 + 0^2\right) = \frac{3}{8}$.

ii) $P(E_3 | T) = \frac{P(E_3 \cap T)}{P(T)} = \frac{\frac{1}{5}\left(0.5^2\right)}{\frac{3}{8}} = \frac{2}{15}$.

5.

a)

i) The probability that the first card drawn is a 3 or a 7: $\frac{2}{13}$

ii) The probability that the first two cards drawn are both picture cards (jack, queen or king): $\frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

iii) The first two cards drawn are both kings, given that the first card is a picture card :

$$\frac{P(\text{first two Kings} \cap \text{first card picture})}{P(\text{first card picture})} = \frac{P(\text{first two Kings})}{P(\text{first card picture})} = \frac{\frac{1}{13} \times \frac{1}{13}}{\frac{3}{13}} = \frac{1}{39}.$$

b)

i) The probability that the first card drawn is a 3 or a 7: $\frac{2}{13}$

ii) The probability that the first two cards drawn are both picture cards (jack, queen or king): $\frac{3}{13} \times \frac{11}{51} = \frac{11}{221}$