University of Technology Sydney
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Probability and Random Variables (37161) – Tutorial/Laboratory 4

1. Let $Y \sim Bin(10,0.3)$ and $Z \sim Geo(0.8)$ be independent random variables. Calculate:

i)	$P(Y=0)$	ii)	$P(Y>0)$	iii)	$P(Y>6)$
iv)	$P(Y>9)$	v)	$P(Y>9 \mid Y>8)$	vi)	$P(Y>9 \mid Z>8)$
vii)	$P(Z>8 \mid Z>9)$	viii)	$P(Y+Z=2)$	ix)	$E(Y+e^{\sqrt{\pi}}+Z)$.

2 Write down possible probability mass functions for discrete random variables *W*, *X* and *Y* such that:

i) $E(W)=-4,\ \mathrm{Var}(W)=0$;

ii) $E(X)=-4,\ \mathrm{Var}(X)>0$

iii) Y has range $\{0,1,-1,7\}$ and $E(Y)=1$

iv) Let *Z* be such that, for all values of *k*, $P(Z=k)=P(Z=-k)$.

Calculate $E(Z)$. Justify your answer.

3. Let *X* and *Y* be independent random variables with probability mass functions

$$P(X=k)=\begin{cases}0.25 & k=1\\ 0.5 & k=2\\ 0.25 & k=0\\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad P(Y=k)=\begin{cases}0.5 & k=0\\ 0.5 & k=0.5\\ 0 & \text{otherwise}\end{cases}.$$

i) Calculate $E(X)$

ii) Calculate $E(Y)$

iii) Calculate $E(7X+\pi Y+e^4)$. Leave your answer in exact form, not a decimal approximation.

iv) Find the probability mass function of XY.

iv) Calculate $E(XY)$

4. A player draws a card at random from a standard deck of 52, with each card equally likely to be chosen. After drawing a card and observing its value, it is returned to the deck and the deck is shuffled.

The player wins \$$X$ where X is the number of cards he/she draws until the first Diamond is drawn. That is, if the first card is a Diamond, he/she wins \$1. If the first card is not a Diamond, but the second is, he/ she wins \$2 etc.

i) What is the distribution of X?

ii) Write down $E(X)$.

The player is offered a financial option before the game. He/she can pay \$$Q$ in for the right (but not obligation) to swap his/her winnings for a fixed prize of \$10.
(The decision on whether or not to exercise this option is made at the end of the game.)

iii) Let the player's eventual prize money be \$$M$, assuming he/she exercises the option only if it increases his/her prize.
Calculate the probability mass function of M.

iv) Calculate the fair price of this option.
That is, find Q such that $E(X) = E(M) - Q$.

Hint: The partial sum of the geometric series
$A + Ar + Ar^2 + ... + Ar^k = A\left(\frac{1-r^{k+1}}{1-r}\right)$ (provided $r \neq 1$.)

5. A supermarket runs a promotion whereby each time a customer shops there, he/she is given a small toy. There are ten different toys to collect and each time the toy given is chosen at random with all ten possibilities equally likely.

A customer aims to collect all ten different toys. Let N_i be the number of additional shopping trips needed by a customer who already has $(i-1)$ different toys until he/she gains an i^{th} different toy.

i) Write down the probability mass of N_1, the number of shopping trips needed for a customer with no toys to gain his/her first toy.

ii) Calculate $E(N_1)$

iii) Write down the probability mass of N_2, the number of shopping trips needed for a customer with one toy to gain his/her second different toy.

iv) Calculate $E(N_2)$

v) Show that the expected number of trips until the customer has collected all the types of toy ≈ 30.

It is assumed that an equal proportion of a large population is born on each day of the year. Individuals are selected uniformly at random from the population and their birthdays noted.

vi) Show that you would expect to select approximately 2365 people before having picked at least one person from each of the 365 possible birthdays.

(Ignore 29 February. Those people don't deserve a birthday present anyway, as they make these combinatorics problems much more messy.)

Hint: For vi) you may need to use a spreadsheet to calculate the large sum required.