University of Technology Sydney
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Probability and Random Variables (37161) – Tutorial/Laboratory 4
SOLUTIONS

1. i) $P(Y=0)=0.7^{10}$

ii) $P(Y>0)=1-P(Y=0)=1-0.7^{10}$

iii)
$$P(Y>6)=P(Y=7)+P(Y=8)+P(Y=9)+P(Y=10)$$
$$=\frac{10!}{7!3!}0.3^{7}0.7^{3}+\frac{10!}{8!2!}0.3^{8}0.7^{2}+\frac{10!}{9!1!}0.3^{9}0.7+0.3^{10}$$

iv) $P(Y>9)=P(Y=10)=0.3^{10}$

v) $$P(Y>9|Y>8)=\frac{P(Y>9\bigcap Y>8)}{P(Y>8)}=\frac{P(Y>9)}{P(Y>8)}=\frac{0.3^{10}}{10(0.3^{9})0.7+0.3^{10}}$$

vi) $P(Y>9|Z>8)=P(Y>9)=0.3^{10}$.

vii) $P(Z>8|Z>9)=1$.

viii)
$$P(Y+Z=2)=P(Y=0)P(Z=2)+P(Y=1)P(Z=1)$$
$$=(0.7)^{10}(0.2)(0.8)+10(0.7)^{9}(0.3)(0.8)$$

ix)
$$E(Y+e^{\sqrt{\pi}}+Z)=E(Y)+e^{\sqrt{\pi}}+E(Z)=(10)(0.3)+e^{\sqrt{\pi}}+\frac{1}{0.8}=4.25+e^{\sqrt{\pi}}.$$

2.

i) $\text{Var}(X)=0$ tell us that X is a constant, hence we have $P(X=k)=\begin{cases}1 & k=11\\ 0 & \text{otherwise}\end{cases}$. (Alternatively $X \sim Bin(11,1)$.)

ii) Many different answers.

We need to find possible $p_1, p_2, p_3 > 0$ such that $p_1+p_2+p_3=1$ and $[0]p_1+[4]p_2+[-3]p_3=0.5$.

One option, for example, is $p_1=0.7, p_2=0.2,, p_3=0.1$ hence

$$P(Y=k)=\begin{cases}0.7 & k=0\\ 0.2 & k=4\\ 0.1 & k=-3\\ 0 & \text{otherwise}\end{cases}.$$

iii) $P(Z=k)=P(Z=-k)$ for all possible k therefore

$$\begin{aligned}E(Z)&=\sum_k kP(Z=k)=\sum_{k>0}kP(Z=k)+\sum_{k<0}kP(Z=k)+0P(Z=0)\\ &=\sum_{k>0}kP(Z=k)+\sum_{k>0}-kP(Z=-k)+0\\ &=\sum_{k>0}kP(Z=k)+\sum_{k>0}-kP(Z=k)\end{aligned}$$

hence $E(Z)=0$.

3. i) $E(X)=1(0.25)+2(0.5)+0(0.25)=1.25$

ii) $E(Y)=0(0.5)+0.5(0.5)=0.25$

iii)

$$\begin{aligned}E(7X+\pi Y+e^4)&=7E(X)+\pi E(Y)+E(e^4)\\ &=7(1.25)+\pi(0.25)+e^4\\ &=8.75+0.25\pi+e^4\end{aligned}$$

iv)

$$P(XY=k)=\begin{cases}0.625 & k=0\\ 0.125 & k=0.5\\ 0.25 & k=1\\ 0 & \text{otherwise}\end{cases}$$

v) $E(XY)=0(0.625)+0.5(0.125)+1(0.25)=0.3125$.

Note, we could also calculate $E(XY)=E(X)E(Y)$, since X and Y are independent.

4.

i) $X \sim Geo(0.25)$

ii) $E(X) = 4$.

iii) If the first Diamond is drawn on the first, second, third,… ninth or tenth draw then the option is used and the player wins $10. This happens with probability

$$[0.25]+[0.75\times0.25]+\left[0.75^2\times0.25\right]+\ldots+\left[0.75^8\times0.25\right]+\left[0.75^9\times0.25\right].$$

This is the partial sum of a geometric series, with first term 0.25 and common ratio 0.75. This therefore sums to

$$[0.25]+[0.75\times0.25]+\left[0.75^2\times0.25\right]+\ldots+\left[0.75^8\times0.25\right]+\left[0.75^9\times0.25\right]$$

$$0.25\left[\frac{1-0.75^{10}}{1-0.75}\right]=1-0.75^{10}$$

$$P(M=k)=\begin{cases}1-0.75^{10} & k=10\\ 0.75^{k-1}0.25 & k\in\{11,12,13,\ldots\}\\ 0 & \text{otherwise}\end{cases}$$

iv) If no Diamond is drawn in the first 10 cards, then the option is valueless. If the first Diamond is drawn on the first card, the option is worth $9, if it is drawn on the second card, it is worth $8, if it is drawn on the third card, it is worth $7 etc. If drawn on the ninth card, the option is worth $1.

This gives the option an expected value of

$$\$9[0.25]+\$8[0.75\times0.25]+\ldots+\$2\left[0.75^7\times0.25\right]+\$1\left[0.75^8\times0.25\right]\approx\$6.23$$

(This also gives that the expected value gained if the option is bought for the fair price of around $6.23 is $E(M)\approx\$10.23$

E(M) = E(X) + Q = 6.23 + 4 = 10.23

5.

i) $$P(N_1 = k) = \begin{cases} 1 & k = 1 \\ 0 & \text{otherwise} \end{cases}$$

ii) $E(N_1) = 1$

iii) $$P(N_2 = k) = \begin{cases} 0.9(0.1)^{k-1} & k = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}.$$

iv) $N_2 \sim Geo(0.9)$ hence $E(N_2) = \dfrac{1}{0.9}$. = 1/ (1-P)

v) $N_1 \sim Geo(1),\ N_2 \sim Geo(0.9),\ N_3 \sim Geo(0.8), \ldots, N_{10} \sim Geo(0.1)$

N1 : get 10/10 toy choice, N2: get 9/10 toy choice, N3: get 8/10 toy choice …

Hence the expected number of trips until the customer has collected all the types of toy is

$$E(N_1) + E(N_2) + \ldots + E(N_{10}) = 1 + \frac{1}{0.9} + \frac{1}{0.8} + \ldots + \frac{1}{0.2} + \frac{1}{0.1} \approx 30.$$

vi) Similarly, the expected number of people selected until all birthdays have been observed is

$$\frac{365}{365} + \frac{365}{364} + \frac{365}{363} + \ldots + \frac{365}{2} + \frac{365}{1} \approx 2365.$$

Person1: 365/365 random pick 1 birthday in 365 day
Person2: 365/364 random 1 birthday in 364 other remaining day
Person3: 365/363 random 1 birthday in 363 other remaining day
Person4: 365/362 random 1 birthday in 362 other remaining day
…