University of Technology Sydney
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Probability and Random Variables (37161) – Tutorial/Laboratory 6

1. Let X be a random variable with density function

$$f(x) = \begin{cases} -\dfrac{x}{10} & x \in [-4,0] \\ \dfrac{x}{10} & x \in (0.2] \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $E(X) = -\dfrac{28}{15}$

b) Calculate:

i) $Var(X)$; ii) $P(X < 0 \mid X^2 > 4)$; iii) $P(X < -2 \mid X < 0)$.

2. Let Y be a continuous random variable with density function

$$g(y) = \begin{cases} \lambda e^{-\lambda y} & y \in [0,\infty) \\ 0 & \text{otherwise} \end{cases}$$

Consider the transformation $Z = \sqrt{Y}$

i) Write down the range of Z. Justify your answer.

ii) Showing all of your working, calculate $g(z)$, the probability density function of Z.

iii) Show that $\int_{-\infty}^{\infty} g(z)dz = 1$

3. One regular fair six-sided die is rolled repeatedly. Let T be the number of rolls until the first time the die shows a 2 and let S be the number of rolls until the first time the die shows a 6.

This means that $T \sim Geo\left(\frac{1}{6}\right)$ and $S \sim Geo\left(\frac{1}{6}\right)$ but the two variables are not independent.

i) Write down $E(T)$ and $E(S)$.

ii) Calculate $P(S = T)$. Justify your answer.

iii) In your own words, clearly explain why $E(T \mid S = 1) = 1 + E(T)$.

Note: You do not need to perform any calculation here.

4. Let W be an exponential random variable $W \sim \exp(\lambda)$.

That is, W has density function $f(w) = \begin{cases} \lambda e^{-\lambda w} & w \in [0,\infty) \\ 0 & \text{otherwise} \end{cases}$.

Consider the transformation $Z = W^{\frac{1}{\gamma}}$ where $\gamma > 0$.

i) Write down the range of Z. Justify your answer.

A random variable $T \sim Weibull(\beta,\eta)$ if it has probability density function

$$f(t) = \begin{cases} \dfrac{\beta}{\eta^{\beta}} t^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} & t \in [0,\infty) \\ 0 & \text{otherwise} \end{cases}$$

ii) Showing all of your working, calculate $g(z)$, the probability density function of Z.

iii) Hence show that Z is Weibull distributed, $Z \sim Weibull\left(\gamma, \left(\frac{1}{\lambda}\right)^{\frac{1}{\gamma}}\right)$.